STRESS WAVE PROPAGATION IN ELASTIC-ELASTIC AND ELASTIC-VISCOELASTIC BILAMINATES

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ABSTRACT

In the present study normal plate-impact experiments are conducted to investigate the propagation of acceleration waves in 2-D layered material systems. The objective of these experiments is to understand the role of material architecture and material inelasticity in controlling precursor decay and late-time dispersion of acceleration waves in elastic-elastic and elastic-viscoelastic laminates. The experiments are conducted using the 82.5 mm single-stage gas-gun facility at CWRU. The history of the free-surface particle velocity at the rear surface of the target plate is measured by employing a VALYN VISAR. In order to understand the effects of layer architecture, experiments are conducted on elastic-elastic bilaminates fabricated with different layer thicknesses and impedance mismatch. Moreover, in order to understand the effects of material inelasticity, experiments on elastic-viscoelastic bilaminates are utilized. The results of the study indicate that the structure of acceleration waves is strongly influenced by impedance mismatch of the layers constituting the bilaminates, density of interfaces, distance of wave propagation, and the material inelasticity.

INTRODUCTION

The understanding of dynamic behavior of materials is vital to many areas of both civil and military applications. Better understanding of dynamic response has important practical implications connected with impact and blast mitigation, design of lightweight armor, and optimal design of engineered structures with potential danger of shock loading. A number of material systems ranging from metal, ceramics, and polymers in both monolithic and composite forms are currently being used to achieve a combination of characteristics to meet the desired goals. Some of the recent examples highlighting the success of these systems include, layered materials, woven composites and functionally graded materials. These material systems promise lightweight armors which are structurally robust, and are being contemplated for use in future combat vehicles and other defense applications.

A large body of knowledge currently exists in the literature on the propagation of acceleration waves and finite amplitude shock waves in heterogeneous materials. For such systems, scattering, dispersion and attenuation play a critical role in determining the thermo-mechanical response of the media. This phenomenon can be attributed to a number of non-linearities arising from loading conditions, material heterogeneity at various length scales, and material inelasticity and failure. However, the phenomenon of wave scattering and dispersion during propagation of shock waves in heterogeneous material systems continues to be poorly understood.

In most homogeneous materials shock waves in the absence of phase transitions are understood to have a one-wave structure. However, upon dynamic loading of bilaminates, a two-wave structure is obtained— a leading shock front followed by a complex pattern that varies with time. This complex pattern is generated by a continuous interaction of compression and rarefaction waves due to the presence of inter-laminar interfaces. To date, only a limited number of experiments have been conducted that concern the propagation of finite amplitude shock waves in heterogeneous materials. Barker et al. [1] conducted experiments on periodic laminates, and found that below certain critical input amplitude, the stress waves decayed exponentially with distance. However, above the critical amplitude a structured shock-wave was obtained. Lundegar and Drumhellar [2] and Oved et al. [3] conducted shock-wave experiments on layered stacks. These experiments showed that the oscillatory nature of the stress waves due to layering. Nesterenko et al. [4-6] observed an anomaly in the precursor decay for the case of propagation of strong shock waves in periodic bilaminates with a relatively small cell size. They noted that for
bilaminates with a relatively small layer thickness the jump in particle velocity at the wave front is essentially higher than the jump obtained with the larger thickness layer at the same distance of propagation. Comparison of experimental results and computer simulations indicated that this effect is primarily due to interactions of secondary compression waves with the leading shock front. More recently, Zhuang [7] has conducted a combined experimental and computational investigation to study the effects of interface scattering on shock wave propagation in heterogeneous material systems. In the study they investigated high velocity plate-impact experiments on bilaminates to investigate the effect of impedance mismatch and the density of interfaces on the structure of strong shock waves.

The focus of the present research is to better understand wave scattering and dispersion at material interfaces and the role of material inelasticity in determining the structure of acceleration waves in 2-D layered material systems. The 2-D layered systems offer a unique opportunity for designing interpretable experiments as-well-as for providing insights into wave propagation in much more complicated microstructures, e.g. fiber reinforced, particulate and woven composites. In the present study wave propagation in both elastic-elastic and elastic-viscoelastic bilaminates is analyzed. The analysis makes use of the Laplace transform and Floquet theory for ordinary differential equations with periodic coefficients. In addition, normal plate impact experiments are conducted using the 82.5 mm single-stage gas-gun facility at CWRU. In these experiments the particle velocity profile at the free surface of the target plate is measured by using a multi-beam VALYN VISAR and compared with the predictions of the analytical solutions. These comparisons are used to understand the effects of layer thickness, impedance mismatch, and material inelasticity on precursor decay and late-time dispersion.

WAVE PROPAGATION IN ELASTIC-VISCOELASTIC BILAMINATES

Consider bilaminates consisting of alternating elastic and viscoelastic layers of uniform thickness and infinite lateral extent. The elastic layers occupy odd numbered layers, i.e. $n = 1, 3, 5...$, and the viscoelastic layers occupy even numbered layers, i.e. $n = 2, 4, 6...$. Consider the individual layers to be homogeneous and isotropic, and the layer thickness of the both constituents to be the same, $l_1 = l_2 = 0.5d$, where $l_1$ and $l_2$ are the thickness of the elastic and viscoelastic layers, respectively, and $d$ is the thickness of a typical bilaminate.

For infinitesimal deformation, longitudinal waves propagating in the $x$-direction are governed by the balance of linear momentum and continuity for elastic and viscoelastic laminae

$$
\rho_1 \frac{\partial u_1}{\partial t}(x,t) - \frac{\partial \sigma_1}{\partial x}(x,t) = 0 \quad \text{and} \quad \frac{\partial \varepsilon_1}{\partial t}(x,t) = \frac{\partial u_1}{\partial x}(x,t)
$$

(1)

$$
\rho_2 \frac{\partial u_2}{\partial t}(x,t) - \frac{\partial \sigma_2}{\partial x}(x,t) = 0 \quad \text{and} \quad \frac{\partial \varepsilon_2}{\partial t}(x,t) = \frac{\partial u_2}{\partial x}(x,t)
$$

(2)

and, the constitutive equations for elastic and viscoelastic laminae

$$
\sigma_1(x,t) = E \varepsilon_1(x,t), \quad \text{and} \quad \sigma_2(x,t) = \int_{-\infty}^{t} G(t - \tau) d\varepsilon_2.
$$

(3)

In Eqs. (1) to (3), $\sigma_1$ and $\sigma_2$ are the longitudinal components of the stress tensor in the elastic and viscoelastic laminae, $u_1$ and $u_2$ are the longitudinal component of the particle velocity in the elastic and viscoelastic laminae, $\varepsilon_1$ and $\varepsilon_2$ are the longitudinal component of the strain tensor in the elastic and viscoelastic laminae, $E$ and $G(t)$ represent the elastic and the viscoelastic modulus, respectively, and $\rho_1$ and $\rho_2$ are the mass density of the elastic and the viscoelastic laminae. The relaxation function for the viscoelastic material behavior is assumed to be described by an exponential function of the type

$$
G(t) = [G(0) - G(\infty)] e^{-t/\tau} + G(\infty),
$$

(4)

where, $G(0)$ denotes the “glassy” modulus at $t = 0$, $G(\infty)$ denotes the “rubbery” modulus at $t = \infty$, and $\tau$ denotes the relaxation time.
We seek solution to Eqs. (1) to (3) which satisfy zero stress and zero particle-velocity initial conditions, and boundary conditions given by $\sigma_1(0, t) = -\sigma_0 H(t)$. Application of the Laplace transform yields a system of four algebraic equations in the transformed plane. These equations contain four complex constants associated with the solution for the longitudinal component of stress. Two conditions on the four complex constants are obtained by requiring that the particle velocity and stress be continuous across the interface between two adjacent layers comprising the bilaminates. The remaining conditions are obtained by the application of Floquet theory for periodic structures.

**SOLUTION AT WAVEFRONT: ELASTIC PRECURSOR DECAY**

Let the longitudinal wave-fronts propagate with speeds $c_1$ and $c_2$ in the elastic and the viscoelastic laminae, respectively. An average wave speed for the longitudinal wave-fronts can be defined as

$$c_{\text{wave}} = \frac{d}{l_1/c_1 + l_2/c_2}.$$  \hspace{1cm} (5)

At the arrival of the longitudinal wave at $x = x_n$, where $x_n = (n/2)d$ is the distance from $x = 0$ to the interface between the $n^{th}$ and the $(n+1)^{th}$ layers, the stress is given by

$$\sigma(x_n, x_n^+ / c_{\text{wave}}) = -\sigma_0 \left\{ \exp \left[ \frac{l_2 G'(0)}{2c_2 G(0)} \right] \right\}^{n/2} \left[ \frac{1}{2} + \frac{1}{4} \left( \frac{\rho_2 c_2}{\rho_1 c_1} + \frac{\rho_1 c_1}{\rho_2 c_2} \right) \right]^{-n/2}.$$  \hspace{1cm} (6)

For the case of elastic-elastic bilaminates the argument of the exponential function is zero and the last term on the RHS of Eq. (6) can be interpreted as an average transmission coefficient for propagation of an elastic wave through a cell of length $d$. Then, the attenuation of the amplitude at the wave-front is the decay primarily due to successive elastic wave reflections. For the elastic-viscoelastic bilaminates the argument of the exponential function gives rise to additional attenuation due to material inelasticity. The rate of decrease in stress is often so rapid that the stress at the wavefront can become negligibly small at remote positions. Figure 1 shows the magnitude of the elastic precursor as a function of number of layers and the impedance mismatch. A strong decay in the elastic precursor is observed with an increase in the number of layers and/or the increase in impedance mismatch. For the case of elastic-viscoelastic bilaminates the argument of the exponential function gives rise to additional attenuation due to material inelasticity. The rate of decrease in stress is often so rapid that the stress at the wavefront can become negligibly small at remote positions. Figure 1 shows the effect of material mismatch and the number of layers on the elastic precursor decay. Figure 2 shows the effect of viscoelasticity on the rate of decay of the elastic precursor after wave propagation through 10 layers. The x-axis represents the ratio of the time taken for the longitudinal wave to travel the thickness of a viscoelastic layer to the relaxation constant for the material. It is to be noted that when the relaxation time is large and/or the viscoelastic layer thickness is small, the effect of material inelasticity on elastic precursor decay is small. Also, when the ratio between the instantaneous modulus and the rubbery modulus, i.e. $\gamma^2 = G'(0) / G(\infty)$, is close to one, the effect of material inelasticity on the elastic precursor decay is negligible.

![Figure 1: Effect of material mismatch and the number of layers on the elastic precursor decay.](image1)

![Figure 2: Effect of material inelasticity on the elastic precursor decay.](image2)
LATE TIME ASYMPTOTIC SOLUTION

At sufficiently late times after the arrival of the wavefront, the stress at a remote position is expected to reach a level $\sigma_o$, which corresponds to the applied stress boundary condition at $x = 0$. The transition from the low-amplitude stress at the wavefront to this equilibrium state at late-times can be characterized by obtaining the late-time asymptotic evaluation of the integral

$$\sigma(x_n, t) = \frac{1}{2\pi i} \int_{\gamma + i\infty}^{\gamma - i\infty} \tilde{\sigma}(x_n, s) e^{st} ds. \quad (7)$$

The integral in Eq (7) can be evaluated asymptotically for large $t$ by using the method of steepest descent. To this end, it is convenient to introduce $\delta = t - x_n / c_{Lave}$, in which $c_{Lave}$ denotes the average wave-speed at which the main parts of the longitudinal disturbance propagates at late time and is given by

$$c_{Lave} = \frac{d}{\left( \frac{l_1}{c_1} \right)^2 + \left( \frac{l_2}{c_2} \right)^2 + \left( \frac{\rho_2 c_2}{\rho_1 c_1} \right) \left( \frac{l_1}{c_1} \right) \left( \frac{l_2}{c_2} \right)^2 \gamma \gamma} \left( \frac{\rho_1 c_1}{\rho_2 c_2} \right) \left( \frac{l_1}{c_1} \right) \left( \frac{l_2}{c_2} \right)^2 \gamma \gamma}. \quad (8)$$

It is interesting to note that $c_{Lave}$ depends on the impedance mismatch between the layers. For laminate architectures in which the impedance mismatch is close to one, the late time dispersion wave and the elastic precursor arrive at a particular location at the same time. However, for laminates in which the impedance mismatch is large, $c_{Lave}$ is considerably less than $c_{Wave}$.

After certain algebraic manipulations it can be shown that the solution to the integral in Eq. (7) is

$$\sigma(x_n, t) = \sigma_o e^A \left[ \frac{1}{3} + \sum_{n=0}^{\infty} \frac{\Gamma \left( \frac{m+1}{3} \right)}{(m+1)!} B^{m+1} \sin \left( \frac{\pi}{3} (m+1) \right) \right]. \quad (9)$$

where,

$$A = -\left( t - x_n / c_{Lave} \right) h^{(0)} + \frac{1}{3} \left[ h^{(0)} \right]^3 \left( x_n / c_{Lave} \right), \quad (10)$$

and

$$B = \left( t - x_n / c_{Lave} \right) - \frac{1}{2} \left[ h^{(0)} \right]^3 \left( x_n / c_{Lave} \right) \left( \frac{6}{h^{(0)} \left( x_n / c_{Lave} \right)} \right)^{1/3}. \quad (11)$$

In Eqs. (10) and (11),

$$h^{(0)} = \frac{c_{Lave}^2}{d^2} \left[ \left( \frac{l_2}{c_2} \right)^2 (y^2 - 1) - \frac{l_1 l_2}{c_1 c_2} \left[ \tau (y^2 - 1) \left( \frac{\rho_2 c_2}{\rho_1 c_1} \right) \right] \right]. \quad (12)$$

and
For infinitesimal deformation, solutions for elastic precursor decay and late-time dispersion which satisfy zero stress and particle velocity initial conditions, and boundary conditions given by a step loading function in time, are summarized in Figure 1. Upon impact of the laminate, a two wave structure is obtained. The leading elastic precursor propagates at a speed dictated by the average wave speed in the two constituents given by \( c_{wave} \), while the late-time dispersed front arrives at a speed \( c_{Lave} \). The late time stress wave oscillates about a mean level dictated by the amplitude of the input stress pulse.

For the case of Mo-Ti bilaminates a larger precursor decay as well as a higher frequency of the late time dispersive waves is observed. Also consistent with Eqns. (5) and (8), the time difference between the arrival of the leading wave front and the late time dispersive wave is much longer in the case of Mo-Ti laminates when compared with the Fe-Ti laminates. It is interesting to note that the late-time dispersive waves show steady wave profiles with increasing distance of propagation into the bilaminates.
Figure 2. Effect of distance of propagation on the elastic precursor and late-time dispersion for Fe-Ti laminates.

Figure 3. Effect of distance of propagation on the elastic precursor and late-time dispersion for Mo-Ti laminates.

Figure 4 shows the effect of the layer thickness on the elastic precursor decay and late time dispersion during stress-wave propagation in Ti-Fe bilaminates. Results for three different layer thicknesses are presented: 0.75 mm, 1.5 mm and 2.25 mm. As expected, the arrival of the elastic precursor at $x_n = 9$ mm occurs at the same time for the three different laminate architectures. However, the laminates with largest layer thickness, i.e. 2.25 mm, shows the smallest elastic precursor decay, while the laminate with the smallest layer thickness, i.e. 0.75 mm, shows the highest precursor decay. The late-time dispersive wave for the smallest layer thickness laminates contain the highest frequency oscillations, while the largest layer thickness laminates contain the lowest frequency oscillations. Also, the rise-time associated with the late-time dispersive wave is the shortest for laminates with the smallest layer thickness and decreases with an increase in the density of interfaces.

Figure 5 compares the late time dispersion for several select material pairs with various impedance bilaminates. Because of the dependence of $c_{\text{mean}}$ with impedance mismatch, the dispersion profiles have been shifted in time so as to start at the same point in time. The late time dispersion profiles can be characterized by the rise time and the frequency of the oscillations contained in the wave profiles. It is observed that the rise times of the dispersion waves increases with an increase in impedance mismatch, while the frequency of the oscillations is observed to decrease with an increase in the impedance mismatch. Also, the late-time dispersive wave oscillates about the mean level corresponding to the input stress. The maximum amplitude of the dispersion wave is $\sim 1.3 \sigma_0$, and is independent of the impedance mismatch of the bilaminates.

Figure 4: Effect of layer thickness on the elastic precursor and the late-time dispersion for Fe-Ti laminates.

Figure 5: Characteristics of late-time dispersion for select material pairs with different impedance mismatch.
EXPERIMENTAL METHODS

The experiments described in this paper are designed to illustrate the effect of impedance mismatch, layer thickness and material inelasticity on elastic precursor decay and late-time dispersion of weak shock waves. In view of this, plate impact experiments are conducted on bilamimates comprising alternating Ti-Fe (elastic-elastic) and the Al-PC (elastic-viscoelastic) layers. The experiments are conducted using the 82.5mm single-stage gas-gun at Case Western Reserve University. The experiment involves the impact of an elastic flyer plate with the target assembly at normal incidence. The target assembly is a sandwich in which the laminate under investigation is confined between two metal plates that remain elastic under impact. Impact takes place on the front target plate; the waves transmitted through the layered specimen are monitored on the free surface of the rear target plate by means of laser interferometry. The measured motion at the free surface of the rear target plate and the known elastic properties of the front and the rear target plates are used to obtain the wave characteristics as the shock wave propagates through the layered specimen.

The schematic of the plate-impact experimental configuration is shown in Figure 6. A fiberglass projectile carrying the flyer plate is accelerated down the gun barrel by means of compressed nitrogen gas. The rear end of the projectile has sealing O-ring and a plastic (Teflon) key that slides in a key-way inside the gun barrel to prevent any rotation of the projectile. In order to reduce the possibility of an air cushion between the flyer and target plates, impact takes place in a target chamber that has been evacuated to 50 µm of Hg prior to impact. A laser based optical system utilizing a UNIPHASE Helium-Neon 5mW laser (Model 1125p) and a high frequency photo-diode is used to measure the velocity of the projectile. To ensure the generation of plane waves with wave-front sufficiently parallel to the impact face, the flyer and the target plates are carefully aligned to be parallel to within $2 \times 10^{-5}$ radians by using an optical alignment scheme. The actual tilt between the two plates is measured by recording the times at which four, isolated, voltage-biased pins, that are flush with the surface of the target plate, are shorted to ground. The acceptance level of the experiments is of the order of 0.5 mrad. A VALYN VISAR laser interferometer is used to measure the history of the normal particle velocity at the rear surface of the target plate. A COHERENT VERDI 5 Watt solid-state diode-pumped frequency doubled Nd:YVO$_4$ CW laser with wavelength of 532 nm is used to provide a coherent monochromatic light source.

EXPERIMENTAL RESULTS AND DISCUSSION

In the present study four series of experiments were designed. The first series of experiments involved the investigation of the effect of distance of propagation on precursor decay and late-time dispersion. The experiments were conducted on Fe-Ti laminates. The thickness of each Fe and Ti layer in the laminate was 0.75 mm. A schematic of the experimental configurations for this series of experiments is shown in Figure 7. In the first experiment (Shot LT06) 12 alternating layers of Fe and Ti with a total laminate thickness of 9 mm were utilized. In the second experiment (Shot LT07) 4 alternating layers of Fe and Ti with a total laminate thickness of 3 mm were utilized. Figure 8 shows experimental results on the Ti-Fe laminates.

Figure 6: Schematic of the plate impact configuration employed in the present investigation. The layered specimen is sandwiched between two hard plates of the target assembly.
The impact velocities for Shots LT06 and LT07 were 74 m/s and 79 m/s, respectively. In the figure the abscissa represents the time after impact while the ordinate represents the normalized particle velocity measured at the free surface of the target plate. For both experiments the impact velocity is used as the normalization factor. Also, for both experiments the flyer plate was made from hardened CH tool-steel, and was approximately 16 mm in thickness. The thicknesses of the front and rear CH tool-steel plates that sandwich the laminate were 8.65 mm and 5.14 mm, respectively. The relatively large thickness of the flyer plate precludes any unloading waves from the back surface of the flyer plate to reach the flyer/target interface during the experiment. The total window time is restricted by one reverberation of the longitudinal wave within the sandwiched laminate. The elastic precursor arrives at the free surface of the target plate at approximately 4.2 µs for Shot LT06 and at 5.65 µs for Shot LT07. The later arrival of the elastic precursor in the case of Shot LT07 is consistent with the larger thickness of the laminate employed in Shot LT07. It is interesting to note that the general characteristics of the wave structure are strikingly similar to the analytical predictions shown in Figure 2. The precursor decay is much higher for Shot LT07 indicating that the decay increases with the distance of distance of propagation. The late-time dispersion arrives after the arrival of the elastic precursor. The highest particle-velocity attained observed in the measured wave profiles is approximately 1.2 times the impact velocity and the late-time dispersive wave is observed to oscillate about a mean level 1.0. As expected, the frequency of the oscillations is not affected by the distance the wave propagates into the laminates and is essentially the same for the two experiments. Also, the amplitude of these oscillations is observed to decay with time.

Figure 7: Schematic of the laminate architecture employed to conduct the first series of plate impact experiments.

Figure 8: Results of plate impact experiments on Fe-Ti laminates showing the effect of distance of propagation on precursor decay and late time dispersion.

The second series of experiments was conducted to understand the effects of density of interfaces on the elastic precursor decay and late time dispersion. Again, in this series of experiments Fe-Ti laminates were utilized. A schematic of the layer architectures employed in this series of experiments is shown in Figure 9. For the two experiments the overall thickness of the laminate was 9 mm. In the first experiment (LT07) 12 alternating layers of Fe and Ti with an individual layer thickness of 0.75 mm were utilized. In the second experiment 6 alternating layers of Fe and Ti with an individual layer thickness of 1.5 mm were utilized. Figure 10 shows the results of the two experiments conducted in this series on Ti-Fe laminates. The impact velocities for Shot LT07 and LT09 were 79 m/s and 71 m/s, respectively. It can be seen that the precursor decay for Shot LT07 is much greater than that obtained for Shot LT09. Moreover, the rise-time associated with the late-time dispersive wave is shorter for Shot LT07 when compared with Shot LT09. The frequency of oscillations in the late-time dispersive wave is higher for the Shot LT07 when compared with Shot LT09. These results are all consistent with the analytical results shown in Figures 3 and 4 for Fe-Ti laminates.
Figure 9: Schematic of the laminate architecture employed to conduct the second series of plate-impact experiments.

Figure 10: Results from plate impact experiments on Ti-Fe laminates showing the effect of layer thickness on precursor decay and late-time dispersion.

The objective of the third series of experiments was to understand the effects of impedance mismatch, density of interfaces, and the material inelasticity on precursor decay and late-time dispersion. These experiments were conducted by employing Al-PC laminates. A schematic of the laminate architectures used in the series of experiments is shown in Figure 11. For shot LT23 the thickness of the Al and PC layers were 0.75 mm and 0.8 mm, respectively. The total number of layers was 4, with an overall thickness of 3.1 mm. The laminate was sandwiched between impedance matched 7075-T6 Al front and rear target plates. Moreover, the layers in the laminate were bonded to each other by using a low viscosity epoxy. For shot LT24 the thickness of the Al and PC layers was 0.25 mm. A total of 12 Al and PC layers were used to make the laminate, with a total laminate thickness of 3.0 mm. Again, for this experiment the laminate was sandwiched between impedance matched 7075-T6 Al front and rear target plates. Also, the layers in the laminate were bonded to each other with a low viscosity epoxy.

Figure 11: Schematic of the laminate architecture employed to conduct the third series of plate-impact experiments.

Figure 12 shows the oscillographs of the recorded VISAR signal for Shot LT24. The resonant oscillations induced by the multiple reflections of the shock wave at the material interfaces are clearly seen in the record, which is made possible by the high velocity resolution (low velocity fringe constant) of the VALYN VISAR system. Part of the data reduction procedure involves plotting the amplitudes of the two fringe records against each other at each data reading time, thereby forming a
Lissajous plot of the data. This plot for Shot LT24 is shown in Figure 13. The Lissajous plot allows the user to judge whether certain corrections to the input data are warranted. The judgement is based in part on how nearly the Lissajous plot resembles a perfect circle, which would be the result if the fringe amplitudes of the two data sets were equal, and if the phase difference were 90 degrees. Judging from Figure 13, the Lissajous of this experiment is a near perfect circle, which confirms the high quality of the interference signal.

![Figure 12: Oscillographs of VISAR signals for Shot LT24.](image)

![Figure 13: Lissajouss showing the amplitude of the VISAR fringe records plotted against each other at each time data.](image)

Figures 14 and 15 show the experimental results obtained from the two experiments on the Al-PC laminates. The impact velocities for Shots LT23 and LT24 were 84 m/s and 82 m/s respectively. For both experiments the flyer plate and the front and rear target plates are made from 7075-T6 Al alloy. The relatively large thickness of the flyer plate precludes any unloading waves from the back surface of the flyer plate to reach the flyer-target interface during the window time of the experiment. In both plots the abscissa represents the time after impact while the two ordinates represent the normalized particle velocity measured at the free surface of the target plate and the normalized stress obtained by using Eq. (9), respectively. It is interesting to note that the velocity versus time profiles obtained in the experiments are much smoother when compared with the particle velocity profiles obtained for the elastic-elastic (Fe-Ti) bilaminates. In the case of the Fe-Ti bilaminates a high frequency oscillatory signal was observed to be riding on top of the particle-velocity profiles. The presence of air gaps of several microns between the Fe-Ti layers are understood to be the origin of the high frequency oscillations observed in the signal. The reason for the absence of these oscillations in the Al-PC laminates is understood to be due to the use of epoxy in preparing the Al-PC laminates. The use of epoxy to bond the individual layers precludes any air gaps in between the layers. The parameters of the visco-elastic material model for PC that are used in the simulations correspond to nearly elastic behavior, i.e. $\gamma - 1$. It is interesting to note that the predicted and the experimentally observed arrival times of the late-time dispersive waves are very close. Also, the frequency of the oscillations contained in the late-time dispersive wave corresponds closely to the analytical predictions with $\gamma - 1$ for PC.

It is also interesting to compare the experimental results for Shot LT06 and Shot LT23. For Shot LT06 experiments were conducted using Fe-Ti bilaminates with layer thickness of 0.75 mm. The total laminate thickness was 3 mm. For Shot LT23 the laminate comprises Al-PC layers with layer thickness of 0.75 mm and an overall laminate thickness 3 mm. The main difference between the two architectures is the impedance mismatch at the layer interface. For the case of Al-PC laminates the impedance mismatch is 8.3, while for the case of the Fe-Ti laminates the impedance mismatch is 1.75. It is interesting to note the relatively long rise-time associated with the late-time dispersive waves for the case of the higher impedance mismatch Al-PC laminates. Also, as predicted by the analytical results, the frequency of the late-time dispersive waves is much smaller for the case of the higher impedance Al-PC laminates as compared with the Fe-Ti laminates.
The fourth series of experiments was conducted to understand wave propagation in elastic-viscoelastic bilaminates confined by high impedance elastic plates. In this series of experiments Al-PC bilaminates were utilized. The thickness of each Al and PC layer is 0.125 mm. In the first experiment 4 alternating layers of Al and PC with a total laminate thickness of 0.5 mm were utilized. For the second experiment 8 alternating layers of Al and PC with a total laminate thickness of 1.0 mm were utilized. For the third experiment 12 alternating layers of Al and PC with a total laminate thickness of 1.5 mm were utilized. A schematic of the experimental configurations for this series of experiments is shown in Figure 15. Figure 16 shows experimental results on the Al-PC laminates. The impact velocities for the three experiments, LT19, LT18, and LT20 were 81 m/s, 74 m/s and 75 m/s, respectively. The abscissa shows the time after impact while the ordinate shows the normalized particle velocity. It is seen that in each case, the normalized particle velocity (or the stress) builds up to the impact velocity in a series of steps. Each step represents one reverberation of the stress wave between the CH steel plates. As expected the time duration of each step is longest for the laminate with the greatest thickness, i.e., Shot LT18 with 12 alternating layers of PC and Al. Also, it is interesting to note the oscillatory nature of the particle velocity profile at each step.

Figure 14: Results from plate impact experiments on Al-PC laminates showing the late time dispersion wave.

Figure 15: Results from plate impact experiments on Al-PC laminates showing the late-time dispersion wave.

Figure 16: Results from plate impact experiments on thin Al-PC laminates sandwiched between two hard CH tool steel plates.

Figure 15: Schematic of the laminate architecture employed to conduct the fourth series of plate-impact experiments.
CONCLUSIONS

In the present study experiments on elastic-elastic and elastic-viscoelastic bilaminates are conducted to understand the role of material architecture and material inelasticity in governing elastic precursor decay and late-time wave dispersion. The measured particle velocity at the free-surface of the target plate is compared with analytical predictions stress wave propagation in bilaminates obtained by using asymptotic methods and Floquet theory for periodic structures. It was observed that

(a) Speed of the elastic precursor is independent of the impedance mismatch of the individual laminae constituting the bilaminates and is equal to the average wave speed within the bilaminates.
(b) Speed of late-time dispersion wave decreases with an increase in impedance mismatch; however, it is found to be independent of the density of interfaces.
(c) The elastic precursor decay increases with an increase in impedance mismatch, the density of interfaces, and the distance of wave propagation.
(d) Rise-time of the late-time dispersion wave increases with an increase in impedance mismatch; however, it is observed to decrease with an increase in the density of interfaces.
(e) The frequency of oscillations of the late-time dispersive wave is observed to decrease with an increase in impedance mismatch; however, it is observed to increase with an increase in the density of interfaces.

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