**Automatic Analysis of Photomechanics Interferogram Using Wavelet Transform**

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**ABSTRACT**

An advanced continuous wavelet transform technique has been developed to automatically analyze photomechanics interferograms. This technique utilizes all possible wavelets to match the local interferogram fringes, and it can automatically detect local fringe frequencies and orientations with high accuracies. The advanced wavelet transform technique has the capability to directly determine gradients of fringe orders (e.g., strains) from a raw interferogram pattern, and it is also very suitable for interferogram filtering. To reduce the processing time, the wavelet transform can be implemented by employing a fast Fourier transform scheme. The technique has been successfully applied to the analyses of various photomechanics interferograms.

**INTRODUCTION**

Since the 1980s, numerous computer-aided digital interferogram processing algorithms have been developed to complement optical interferometric measurement techniques. Among the algorithms capable of extracting whole-field information of phase distributions (or fractional fringe orders) from one or more interferograms, the most predominantly used ones are those that can automatically analyze interferograms. Typical automatic phase analysis methods include phase shifting method, Fourier transform method and wavelet transform method. Compared with phase shifting method where a series of accurately or randomly phase-shifted interferograms are required [1], Fourier transform and wavelet transform methods have the capability to extract phase information from one single interferogram.

Unlike Fourier transform which yields the spectra of the entire interferogram and the spectra do not accurately reflect the local fringe information, wavelet transform can detect the characteristics of local signals. For this reason, wavelet transform is particular helpful for the analysis and processing of interferograms where local fringe information is important. Research of wavelet transform on interferogram processing has been active for a decade and different schemes have been proposed. Early work [3]-[8] usually utilizes 1-D wavelet transform for interferogram processing; the accuracies are limited due to the simplification of 2-D interferograms to 1-D transformations. To analyze moiré interferograms in a 2-D way, Kadooka et al. [9] proposed an algorithm using 2-D continuous wavelet transform to improve the processing accuracies. However, their algorithm is complicated and is only suitable for interferograms with relatively uniform fringes. It should be noted that all the existing methods require very long computation time, and none of the methods have demonstrated examples with analysis results that are comparable with the ones obtained using other widely-used processing techniques such as phase shifting method.

This paper will demonstrate a novel advanced 2-D continuous wavelet transform technique to cope with the limitations in existing methods. The technique chooses an appropriate wavelet function and a proper normalization parameter based on the characteristics of typical photomechanics interferograms. The technique is simple and fast; it is very effective for general interferogram processing like interferogram filtering, phase extractions and strain determinations.

**ADVANCED WAVELET TRANSFORM TECHNIQUE**

The intensity of a real interferogram can be expressed as
\[ I(\vec{x}) = I_0(\vec{x}) + I_m(\vec{x}) \cos[\phi(\vec{x})] \]  

(1)

where \( \vec{x} = (x, y) \) is the 2-D coordinates of the individual pixel in the interferogram, \( I_0(\vec{x}) \) is the background or mean intensity, \( I_m(\vec{x}) \) is the modulation amplitude, and \( \phi(\vec{x}) \) is the angular phase information.

The wavelet transform of an interferogram \( I(\vec{x}) \) is defined as

\[ W(\vec{b}, a, \theta) = a^{-n} \int_{-\infty}^{\infty} I(\vec{x}) \psi^* \left[ a^{-1} r_\theta (\vec{x} - \vec{b}) \right] d^2 \vec{x} \]  

(2)

where \( a > 0 \) is a scale dilation parameter corresponding to the width of the wavelet, \( \vec{b} \) is a 2-D translation parameter corresponding to the position of the wavelet, \( \theta \) is a rotation parameter, \( r_\theta \) is the conventional 2 by 2 rotation operator matrix, \( \psi(\vec{x}) \) is the mother wavelet, \( \psi^* \) denotes the complex conjugate of \( \psi \), \( n \) is the normalization parameter, and \( W(\vec{b}, a, \theta) \) is the wavelet transform coefficient.

In general signal and image processing using wavelet transform, one typical mother wavelet employed to detect oriented features is Morlet wavelet. In the advanced wavelet transform technique, the 2-D Morlet wavelet is chosen and modified for the interferogram processing. The modified wavelet is expressed as:

\[ \psi(\vec{x}) = \exp \left[ -m|x|^2 + i2\pi v \right] \]  

(3)

where \( m \) is a shape parameter of the wavelet. A traditional Morlet wavelet function uses \( m = 0.5 \); however, for photomechanics interferograms, \( m = 2.0 \) is a better selection.

Although the parameter \( n \) in conventional image and signal processing is chosen to be 0.5 and 1.0 for 1-D and 2-D cases respectively [2], derivation shows that setting \( n \) to 2 can normalize the wavelet transform coefficients for interferograms. It is also shown that the maximum normalized wavelet transform magnitude can be obtained when \( a = A \) and \( \theta = \Theta \), where \( A (A > 0) \), and \( \Theta \) are the local fringe period and orientation at arbitrary point \( \vec{b} \), respectively. In this case, consequently, the phase can be calculated from:

\[ \phi(\vec{b}) = \Phi = \tan^{-1} \left( \frac{\text{Im}[W(\vec{b}, a, \theta)]}{\text{Re}[W(\vec{b}, a, \theta)] - I_0 \frac{\pi}{m} \exp \left( -\frac{\pi^2}{m} \right)} \right) \]  

(4)

where \( \text{Im} \) and \( \text{Re} \) denote imaginary and real parts of a complex value, respectively. \( I_0 \) can be set to the mean intensity of the interferogram; actually, in Eq. (4) the term that contains \( I_0 \) is negligible.

A direct numerical calculation of Eq. (2) is extremely time-consuming. To reduce the computation time, a fast algorithm introduced below can be employed:

\[ W(\vec{b}, a, \theta) = a^{-n} \int_{-\infty}^{\infty} \xi(\vec{b} - \vec{x}) d^2 \vec{x} = a^{-n} I(\vec{x}) * \xi(\vec{x}) = a^{-n} \text{IFFT}[\text{FFT}[I(\vec{x})]\text{FFT}[\xi(\vec{x})]] \]  

(5)

where

\[ \xi(\vec{x}) = \exp \left[ -ma^{-2}(x^2 + y^2) + i2\pi a^{-1}(x\cos\theta + y\sin\theta) \right] \]  

(6)

where “*” denotes convolution and “*” denotes dot product, \( \text{FFT} \) and \( \text{IFFT} \) denote the forward and inverse fast Fourier transforms, respectively. Eq. (5) can significantly reduce the computation time.
It is noteworthy that the phase information, fringe frequency and orientation can be directly employed for further interferogram processing such as determination of fringe order gradients

\[ g_x = a^{-1} \cos \theta \text{ for horizontal direction} \]
\[ g_y = a^{-1} \sin \theta \text{ for vertical direction} \] (7)

and interferogram filtering based on fringe reconstruction from phase distributions

\[ I'(\tilde{x}) = 127.5[1 + \cos(\phi(\tilde{x}))] \text{ for 256 colors image} \] (8)

EXPERIMENTS

The advanced wavelet transform technique was applied to a series of photomechanics experiments. Fig. 1 (a) represents a vertical displacement field of a notched specimen subjected to a cyclic loading, obtained by moiré interferometry. Fig. 1 (b) is the smoothed image after applying the wavelet transform filtering. It is evident that the advanced wavelet transform technique can accurately detect the whole-field fringe information even when the noise level is high.

![Fig. 1. (a) initial interferogram, (b) reconstructed interferogram using wavelet transform filtering.](image)

The advanced wavelet transform technique was then applied to the warpage measurement of a tape-automated bonding (TAB) electronic package using Twyman-Green interferometry. Fig. 2 (a) is the initial interferogram obtained from the experiment, (b) is the reconstructed interferogram after filtering, (c) is the wrapped phase map, (d) is the unwrapped phase map, and (e) is the 3-D warpage.

![Warpage Measurement](image)
Fig. 2. (a) initial interferogram, (b) filtered interferogram, (c) wrapped phase map, (d) unwrapped phase map, (e) 3-D warpage.

Fig. 3 shows another application. Fig. 3 (a) represents the thermally induced vertical displacement fringe pattern of a solder ball interconnection in an electronic packaging component; the experiment was implemented by using microscopic moiré interferometry. Figs. 3 (b) – (d) show the results where (b) is the wrapped phase map, (c) is the map of fringe order gradients, and (d) is the filtered interferogram after reconstruction.

Fig. 3. (a) initial interferogram, (b) wrapped phase map, (c) map of fringe order gradients (strains), (d) filtered interferogram.

The above experiments proved the validation of the applications of the advanced wavelet transform technique.

CONCLUSION

The advanced 2-D continuous wavelet transform technique is capable of detecting local fringe frequencies and orientations for photomechanics interferograms. It is suitable for the analysis and processing of general photomechanics interferograms, such as interferogram filtering, whole-field displacement and strain determinations. The technique is quite simple and fast, and it has been successfully applied to various photomechanics applications.
REFERENCES


