Elimination of singularities in parallel robotic manipulators

Manoj Kumar
SASTRA University
Reg.no-010612007, 4TH year, B.Tech, Mechatronics
Plot no.11, Somasundaram Nagar, R.S College Post, Thanjavur, Tamilnadu, India-613005
E-mail id: manojksahi@gmail.com

Asst. Prof. Anjan Kumar Dash
SASTRA University
School of Mechanical Engineering, SASTRA University, Tirumalaisamudaram, Thanjavur,
Tamilnadu, India-613402
E-mail id: anjandash2001@yahoo.com.sg

ABSTRACT

Parallel robotic manipulators have received considerable attention from researchers over the past decade motivated by the fact that they can provide high stiffness and accuracy. But the biggest problem that the parallel manipulators face is complex kinematics singularities in the workspace. These singularities, during their operation, are potentially dangerous and frequently cause the manipulators inoperable and unstable in certain configurations. In this paper, we introduced a new approach to eliminate the singularities, though not all. It is known that the Jacobians of a parallel manipulator changes with change in the placement of actuators at different joint locations. This change in Jacobians affects the singularity loci within the workspace of the parallel manipulator. This technique is adopted in reducing the number of singularities from the workspace for a particular configuration of a parallel manipulator. As a case study, a five bar planar parallel manipulator is chosen. Singularity points are determined for different combinations of positions of actuators at four joint locations. The best combination of positions of actuators are determined considering minimum number of singularities.

1. Introduction

A manipulator consists of a series of rigid members, called links, connected by joints. It ends with a link on which a tool can be mounted. Manipulators can be classified into two types: serial manipulators and parallel manipulator. A serial manipulator is one in which the links and joints are arranged in alternate fashion, with the additional constraint that closed loops are not formed, while a parallel manipulator has a closed loop. A parallel manipulator is one of the most active areas of current mechatronics and robotics research since it is promising to have supplementary features such as high rigidity, high stiffness, high accuracy, high speed, high acceleration and high payload that a serial manipulator is in lack of. However, they are much more complicated than the serial manipulator due to the presence of closed loop constraints. In general, the problems associated with the kinematics and geometry of parallel manipulator is more complex than those arising in analysis of serial manipulator. The existing singular conditions during operation are potentially dangerous and frequently cause the manipulator inoperable in certain configuration.

In this paper, research has been directed towards singularity analysis of a 2-DOF parallel manipulator and elimination of the singular points by shifting of the position of revolute actuators while keeping the workspace area constant. However, this approach can be extended to all other configurations namely, 3 DOF and 6 DOF parallel manipulators.

2. Singularity analysis of parallel manipulators

The velocity equations of a parallel manipulator can be written as

\[ [J1] [\dot{x}] = [J2] [\dot{\theta}] \]

(1)

where \( [\dot{\theta}] = [\dot{\theta}_1, \dot{\theta}_2, ..., \dot{\theta}_n]^T \)

\( [\dot{x}] = [\dot{x}_1, \dot{x}_2, ..., \dot{x}_n]^T \)

\([J1]\) and \([J2]\) are square matrix of size \( n \times n \) where \( n \) = degrees of freedom. \([J1]\) and \([J2]\) are also called as Jacobian matrices.

Here vector \( \theta \) represents the actuated joint co-ordinates of the planar parallel manipulator and is defined as \( \theta = [\theta_1, \theta_2, ..., \theta_n]^T \). In other words, it is the vector of kinematics input. Vector \( x \) denotes the Cartesian co-ordinates of the manipulator gripper representing the kinematics output.

Gosselin and Angeles [1] have defined three types of singularities that can occur in parallel manipulators.

1] First type of singularity occurs when determinant \([J2] = 0\). In such a case, the gripper of manipulator loses one or more degrees of freedom and lies in a dead point position. In other words, the gripper can resist one or more forces or moments without exerting any torque or force at actuated joints. These configurations correspond to a set of points defining the outer and internal boundaries of the workspace of the manipulator. These points are known as inverse singularities.

2] The second type of singularity occurs when determinant \([J1] = 0\). As opposed to the first case, the gripper of the manipulator gains one or more degrees of freedom, as it cannot resist the forces or moments from one or more direction even when all actuated joints are locked. The actuated joints are at a dead point. This kind of singularity corresponds to set of points within the workspace of the manipulator. These points are known as direct/forward singularities.

3] The third kind of singularity occurs when the positioning equation degenerates i.e. when both determinants \([J1]\) and \([J2]\) simultaneously become zero. This type of singularity is referred to as architecture singularity. This kind of singularity can occur only when the parameters of a manipulator satisfy certain special conditions. It corresponds to a set of configurations where a finite motion of the gripper of the manipulator is possible even if the actuated joints are locked or where a finite motion of the actuated joints produces no motion of the gripper. The singularity types classified above can be applicable to any 2-DOF parallel manipulator.
3. Case study of a 2 DOF planar parallel manipulator

A planar 2 DOF parallel manipulator is represented in Fig. 1. All its joints are revolute joints. The 2-DOF manipulator can be used to position a point on a platform noted P (Xp, Yp). It is based on 5 bar mechanism, i.e. it has four movable links and five revolute joints (A, B, O, D, and P). It consists of two actuators whose axes pass through joints O and D. This produces the input angular motion $\theta_1$ and $\theta_2$. Therefore $\theta_1$ and $\theta_2$ are the driving angles for $\beta$ and $\gamma$ which are passive joint angles at A and B. It should be noted that we restrict our study to a special case of the symmetric planar 5 bar manipulator, with assumption made that the links lie in parallel planes and hence do not interfere with each other. In the above configuration, the lengths of the links are l and m where $l = OA = BD$ and $m = AP = PB$

The following steps can be used to analyze the singularity of a 2 DOF planar manipulator –

Step 1) Formulation of output positions in terms of two input active joint angles for different positions of the actuators.

Step 2) Establishing relationship between input and output velocities.

Step 3) Drawing the singularity manifolds by finding the zero determinant values of Jacobian matrices in each case.

3.1 Formulation of Jacobians and singularity analysis

Following cases are considered in relation to position of actuators at different joint locations.

Case 1) Actuators at points O and D

Using the geometrical relationships between the gripper position P (Xp, Yp) and the joint angles from two sides we get the following equations

Considering the links OA and AP (Fig. 1)

\[ X_p = l\cos\theta_1 + m\cos\beta \]  \hspace{1cm} (2)
\[ Y_p = l\sin\theta_1 + m\sin\beta \]  \hspace{1cm} (3)

Considering the links PB and BD (Fig. 1)

\[ X_p = d + l\cos\theta_2 + m\cos\gamma \]  \hspace{1cm} (4)
\[ Y_p = l\sin\theta_2 + m\sin\gamma \]  \hspace{1cm} (5)

Differentiating equations (2), (3), (4) and (5) and solving for velocity of the gripper, we get
Equations (6) and (7) can be rewritten as below,

\[ X' = (m1^*k1(\theta1) - m2^*k2(\theta2))(m1^*m4 - m2^*m3) \]  
\[ Y' = (-m3^*k1(\theta1) + m1^*k2(\theta2))(m1^*m4 - m2^*m3) \]  

where

- \( m1 = \cos\beta \) and \( k1 = l^*\sin(\beta - \theta1) \)
- \( m2 = \sin\beta \) and \( k2 = l^*\sin(\gamma - \theta2) \)
- \( m3 = \cos\gamma \)
- \( m4 = \sin\gamma \)

Further, equations (6) and (7) can now be written in the form of equation (1) as below,

\[ [J1] [\dot{X}] = [J2] [\dot{\theta}] \]  

where

\[ J1 = \begin{bmatrix} \cos\beta & \sin\beta \\ \cos\gamma & \sin\gamma \end{bmatrix} \quad J2 = \begin{bmatrix} l^*\sin(\beta - \theta1) & 0 \\ 0 & l^*\sin(\gamma - \theta2) \end{bmatrix} \]

as \( J1 = \begin{bmatrix} m1 & m2 \\ m3 & m4 \end{bmatrix} \) and \( J2 = \begin{bmatrix} k1 & 0 \\ 0 & k2 \end{bmatrix} \)

However, these expressions of Jacobians contain passive joint angles which are eliminated as follows.

Position of point P, calculated using OAP chain can be given as below.

\[ \{ X' - ( d + l^*\cos\theta2) \}^2 + \{ Y' - l^*\cos\theta2 \}^2 = m^2 \]  

Position of point P, calculated using BAP chain can be given as below.

\[ \{ X' - l^*\cos\theta1 \}^2 + \{ Y' - l^*\sin\theta1 \}^2 = m^2 \]  

Equating (11) and (12), we get

\[ X' = F + G^* (Y') \]
\[ Y' = \{- I + \sqrt{(I^2 - 4*H*J)} / (2*H). \]

where

\[ F = \{ 2*(l^2 - m^2) + d^2 + 2*d*l^*\cos\theta2 \} / \{ 2*(d + l^*\cos\theta2 - m^*\cos\theta1) \} \]
\[ G = - (l^*\sin\theta2 - m^*\sin\theta1) / (d + l^*\cos\theta2 - m^*\cos\theta1) \]
\[ H = 1 + G^2 \]
\[ I = 2*F^*G - 2*G*m^*\cos\theta1 - 2*m^*\sin\theta1 \]
\[ J = F^2 + m^2 - I^2 - 2*F^*m^*\cos\theta1 \]

From equations (2), (3), (4) and (5), the following equations can be derived.

\[ \cos\beta = (X' - l^*\cos\theta1) / m, \]  
\[ \sin\beta = (Y' - l^*\sin\theta1) / m \]
\[ \cos\gamma = ((X' - (d + l^*\cos\theta2)) / m \]
\[ \sin\gamma = (Y' - l^*\sin\theta2) / m \]

Substituting these expressions of \( \cos\beta \), \( \sin\beta \), \( \cos\gamma \) and \( \sin\gamma \) from equations (14), (15), (16) and (17) into the expression of \( J1 \) and \( J2 \) in equation (10), the relationship is now established in terms of active joint angles and end-effector point.
Case 2) Actuators at points A and D

The Jacobian matrices for this case are

\[
J_1 = \begin{bmatrix} \cos \beta & \sin \beta \\ \cos \gamma & \sin \gamma \end{bmatrix} \quad J_2 = \begin{bmatrix} l^* \sin(\beta - \beta_1) & 0 \\ 0 & m^* \sin(\gamma - \gamma_2) \end{bmatrix}
\]

Same procedure as shown in case (1) is adopted here to get the final Jacobian matrices in terms of active joint angles.

Similar procedures are also adopted in case 3 and 4 below.

Case 3) Actuators at points A and B

\[
J_1 = \begin{bmatrix} \cos \beta & \sin \beta \\ \cos \gamma & \sin \gamma \end{bmatrix} \quad J_2 = \begin{bmatrix} m^* \sin(\beta - \beta_1) & 0 \\ 0 & l^* \sin(\gamma - \gamma_2) \end{bmatrix}
\]

Case 4) Actuators at points O and B

\[
J_1 = \begin{bmatrix} \cos \beta & \sin \beta \\ \cos \gamma & \sin \gamma \end{bmatrix} \quad J_2 = \begin{bmatrix} m^* \sin(\beta - \beta_1) & 0 \\ 0 & m^* \sin(\gamma - \gamma_2) \end{bmatrix}
\]

3.2 Results

For simulation purpose, a 5 bar parallel manipulator having following links lengths are chosen:
l=2 units, m= 3 units, d= 2 units, \( \theta_1 \) and \( \theta_1 \) is varied from 0 to 360 deg.

For each set of input angles, two configurations of the manipulator are identified. They are as shown in Fig. 2 and Fig. 3. In Fig 2 and 3, links AP and BP are upward and downward respectively. Thus for each of these above cases, singularities are identified.

Case 1: For the above mentioned set of link lengths and the joint angles, point P (Xp, Yp) is plotted to determine the singularity points and workspace for both the configurations keeping the condition

\[ \det[J_1] = 0 \quad \text{and} \quad \det[J_2] = 0 \]

respectively.

Note that the singularities shown in figures (4), (5), (6) and (7) consider both the above configurations.

Plot specifications:
- Green points: workspace (inverse singular points)
- Red and blue points: direct singular points
  - where Red points: upward configuration and
  - Blue points: downward configuration

Plots have been created using MATLAB 7.0 software.
Considering both the configurations, using an increment of 1 degree in the joint angles, the total number of singularities, in this case, is determined to be 424.

Case 2: total number of direct singular points is 1076

Case 3: Total number of direct singular point is 1114
The arrangement of direct singular points in case 2 and case 3 is the mirror image of each other.

Case 4: Total Number of direct singular points 368
This case gives the minimum number of direct singular points compared to previous cases.
CONCLUSIONS:

Change in the placement of the actuators in a parallel manipulator changes the singularity loci because there is corresponding change in the Jacobian of the manipulator. By suitable placement of the actuators, minimum numbers of singularities are encountered while keeping the workspace area constant. In case of 5 bar planar manipulator system, minimum numbers of singularities occur when the actuators are at A and B (See fig.7).

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