ABSTRACT
Since mechanical behaviors of polymeric materials are viscoelastic under load a certain part of energy given from external input force and/or displacement is consumed as loss energy in the material. The consumed loss energy could be changed into thermal energy which leads to heat generation and temperature rise. It is important to evaluate the correlation between temperature change and the amount of loss energy accumulated by mechanical loading. In this study, the authors discuss on temperature rise and its distribution in a soft epoxy resin due to periodic tensile load at several temperatures and make an approach to the evaluation of stress accompanied with heat generation. Firstly, mechanical properties of a soft epoxy resin were measured by dynamic test. Secondly, taking heat generation and temperature rise into account, deformation behavior in rubbery elastic condition is compared with that in viscoelastic one using data obtained by digital image correlation. As the results, remarkable difference of dynamic deformation behaviors under viscoelastic and rubbery conditions was confirmed. Under viscoelastic condition, spatially nonuniform heat generation and conduction lead to nonuniform temperature distribution in the center part of the specimen.

Key words: Viscoelasticity, Loss Energy, Heat Generation, Thermal Conduction and Convection
1. Introduction
The mechanical properties of most polymeric materials are remarkably viscoelastic under load. Namely, the response to input delays, then a certain part of external work given by input force and/or displacement is converted to loss energy. The loss energy consumed could be changed into thermal energy, which leads to temperature rise of the material. In addition, since the mechanical properties of the material are dependent on both time and temperature as well known, it is important to evaluate the relation between temperature change and the amount of loss energy accumulated under cyclic loading.
In this study, the authors discuss the coupled thermal and mechanical analyses for quantitative evaluation of thermo viscoelastic displacement distribution in a soft epoxy resin accompanied by heat generation under cyclic load. Firstly the authors measured mechanical properties of a soft epoxy resin by using dynamic test. Secondly deformation behavior in rubbery region is compared with that in viscoelastic region including remarkable heat generation using digital image correlation technique.

2. Measurement of mechanical properties\(^{[1],[2]}\)
Let us show the fundamental of measurement of storage modulus and loss modulus by dynamic viscoelasticity.
The complex modulus \(E^*\) can be shown by the following expression when the maximum value of periodically changing load and displacement is expressed as \(P_{\text{max}}\) and \(u_{\text{max}}\), phase delay as \(\delta\).

\[
E^*(i\omega) = \frac{P_{\text{max}}}{u_{\text{max}}} \left( \cos \delta + i \sin \delta \right)
\]

Then,

\[
E' = \frac{P_{\text{max}}}{u_{\text{max}}} \cos \delta
\]

\[
E'' = \frac{P_{\text{max}}}{u_{\text{max}}} \sin \delta
\]

Thus, loss angle \(\tan \delta\) can be calculated from eq.(4)

In most polymeric materials, temperature dependence of mechanical behavior is remarkable. The reduction or enlargement of frequency (or time) scale in an arbitrary temperature can be given by the time and temperature shift factor such as W.L.F equations, i.e., eq.(4) to eq.(6).

\[
\log a_T = \frac{-C_1(T - T_0)}{C_2 + (T - T_0)}
\]

(4)

\[
T_g \equiv T_0 + 50[K], C_1 = 8.86, C_2 = 101.6
\]

(5)

\[
\alpha_T = t / t' = \omega' / \omega
\]

(6)

where, \(T_g\) is the reference temperature and \(T_0\) the glass transition temperature of the material.

A specimen of soft epoxy made of Epikote 871 resin (Shell Chem.Co.) and TETA hardener was mixed and cured for 24 hours at 353K. The geometries of specimen are 35mm in length, 10mm in wide, 1mm in thickness. The glass transition temperature of the material is measured as 260.5K.

Mechanical properties are measured under several angular frequencies and various temperatures using a dynamic tester made of Rheometric Co. The experimental procedure is shown as follows: temperature is lowered to \(T=243K\) by liquid nitrogen and is kept constant at least for 30 minutes. Then, sinusoidal loading is applied to the specimen changing temperature with every 5K from 243K to 313K. Strain amplitude is given constant as 0.1% and frequency is varied from 1 to 100 rad/s.

Using W.L.F equation, a master curve of storage modulus, loss modulus, loss angle against the reduced time is shown in logarithmic scales in Fig.1,2.

In Fig.2, when the reduced time comes to \(\log t' = -5\), the loss angle shows the maximum value which corresponds to the biggest delay of response to input displacement. In the same figure, at the bottom values of loss angle, the material behaves glassy or rubbery elastic response.
3. Dynamic properties in linear viscoelasticity\textsuperscript{[1],[2]}

When a specimen made of viscoelastic material is loaded by uniaxial cyclic displacement $\varepsilon^*(t)$ with amplitude $\varepsilon_0$ and angular velocity $\omega$,

$$\varepsilon^*(t) = \varepsilon_0 e^{i \omega t}$$

Standing on the theory of linear viscoelasticity, the output stress $\sigma^*$ is given as follows,

$$\sigma^*(t) = E^*(\omega)\varepsilon^*(t)$$

$E^*(\omega)$ is the complex modulus is given as

$$E^*(\omega) = E'(\omega) + iE''(\omega)$$

where, $E'(\omega)$ is the storage modulus and $E'(\omega)$ is the loss modulus.

The phase shift angle $\delta$ between input strain and output stress is explained as,

$$\tan \delta = E''/E'$$

Then, the energy $\Delta W$ consumed during one cycle ($\tau = 2\pi/\omega$) is calculated. The real part of input strain of eq.(7) is expressed as

$$\varepsilon(t) = \text{Re}[\varepsilon^*(t)] = \varepsilon_0 \cos \omega t$$

Thus, output stress which corresponds to strain is represented as.

$$\sigma(t) = \text{Re}[E^*(\omega)\varepsilon(\omega)] = \varepsilon_0 (E' \cos \omega t - E' \sin \omega t)$$

Consequently, loss energy $\Delta W$ consumed during a cycle of load is calculated as below;

$$\Delta W = \int_0^\tau \sigma(t) \frac{d\varepsilon}{dt} dt = \pi E' \varepsilon_0^2$$

When displacement is recovered, $\Delta W$ could be lost and a certain part of it could be converted to heat.
4. One dimensional heat conduction [1],[2]

Since the loss energy is considered to be converted into heat, temperature of the body rises somewhat up. The quantity of heat generation per unit volume and unit time, \( q \), is given as

\[
q = \Delta W/\tau
\]  

(14)

Where \( \tau \) is the period of strain. Also, the equation of unsteady heat conduction is represented by

\[
\nabla^2 T + \frac{H}{\lambda} = \frac{1}{a} \frac{\partial T}{\partial t}
\]

(15)

where \( \lambda \) is heat conductivity, and \( \alpha \) is thermal coefficient. \( H \) is the quantity of heat used for temperature rise per unit volume and unit time, and is expressed as

\[
H = q - \frac{2q_0}{d}, \quad q_0 = h(T - T_i)
\]

(16)

where \( T_i \) is the initial temperature of specimen and \( T_c \) is circumferential temperature and \( h \) is heat convection.

Now assuming that strain in the specimen is uniform, and that heat coming in or going out from surroundings is negligible, the temperature field in the specimen becomes uniform, thus eq.(15) is rewritten as follows,

\[
\frac{dT_i}{dt} = \frac{H}{c\rho}
\]

(17)

where \( c \) is specific heat and \( \rho \) is mass density.

5. Cyclic Loading tests [3],[4],[5],[6]

In this section, a brief explanation of cyclic loading test is presented. Viscoelastic materials under periodic loading are accompanied with heat generation caused by loss energy. Since the mechanical properties depend on time and temperature and heat conduction and convection, temperature rise in the material will also be dependent on circumferential temperature. At lower circumferential temperature, non-uniform temperature distribution is apt to show up in the center part of specimen. Therefore, displacement distribution of specimen builds up different temperature distribution. In this experiment, displacement distributions at several temperatures were measured by the digital image correlation (DIC) technique just after unloading.

A specimen of soft epoxy made of Epikote 871 resin (Shell Chem. Co.) and TETA hardener was mixed and cured for 24 hours at 353K. The geometries of specimen are shown in Fig.3. Then, monochrome random pattern is sprayed on the surface of specimen to facilitate the analysis of displacement using digital image correlation (DIC) technique.

The specimen is gripped by a pair of air chucks in a constant temperature chamber shown in Fig.4. Hysteresis curves are measured under constant strain amplitude under various temperatures using a cyclic-loading tester made of Shimadzu Co. The maximum number of cycle was set as 6000 cycles and frequency as 10Hz.

At the same time, an infrared thermometer made of NEC Sanei Co. is set through a window on one side of temperature chamber to take thermal data on the surface of specimen. A digital camera with a white light source is set on the opposite side of the infrared thermometer. The center part of specimen is photographed with a digital camera just after unloading. The resolution of image is 2.00×10^{-2} mm/pixel. Just after loading during 10 min period, images are taken at instants of 0s, 60s, 120s, 180s, 240s, 300s under the glassy(263K), viscoelastic(273K) and rubbery(293K) temperatures of the material. Comparing every two images, DIC technique is applied.
Fig. 5 shows the comparison of hysteresis loops in one cycle of loading obtained under two different temperatures. The hysteresis loop at 278K, namely the loss energy in a cyclic loading, is bigger than that at 293K. This comes from the fact that the delay of response in viscoelastic region is more remarkable than that in elastic region. Temperature rise by heat generation is dependent on the size of hysteresis loop. The variation of average temperature rise in the center part of specimen against time obtained under several circumferential temperatures is compared in Fig. 6. This comes from the fact the different size of loss energy. Therefore, in viscoelastic region, where is bigger delay of response to input displacement, temperature rise on specimen surface is higher. The variation of temperature distribution by heat generation with time is shown in Figs. 7 and 8. It is obviously see that temperature distribution changes due to the difference of circumferential temperature. Here, taking loss energy evaluated from hysteresis loop into account, eq. (13) can give necessary data for the master curves of storage modulus, loss modulus and loss angle. In Fig. 9, the master curve of storage or loss modulus obtained from loss energy data in cyclic test is compared with those by Rheometric tester. The reduced time in logarithm scale is calculated by the well-known W.L.F. equation. The results of mechanical characteristics are obviously dependent on test method and procedure. Particularly in short time period, the moduli obtained from loss energy is much lower than those by dynamic test, because of the difference in geometries, heat conduction and convection due to temperature difference between specimen surface and temperature chamber. However, the loss tangent shows almost same with the results by dynamic test.
6. Conclusion

- In this paper, mechanical properties of a soft epoxy resin were measured by using dynamic test.
- The difference of temperature rise caused by heat generation due to loss energy under cyclic loading in a soft epoxy resin is investigated for rubbery and viscoelastic region by thermographic measurement. As the results, in viscoelastic region, heat conduction and heat convection accompanied with heat generation show more remarkable temperature rise and uneven distribution in the center part of specimen under cyclic loading test.
- the master curve of storage or loss modulus obtained from loss energy data from loss energy in cyclic test is compared with those by Rheometric tester. The results of mechanical characteristics are obviously dependent on test method and procedure.
- In near future, the authors will do quantitative evaluation about the nonuniformity of temperature distribution and displacement in the viscoelastic region both from theoretical and experimental approaches.

7. Reference