Image Matching Error Assessment in Digital Image Correlation

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Abstract
Using concepts from statistics and probability theory and a least squares matching metric, the effects of various parameters on image plane motion measurements are demonstrated. Specifically, theoretical formulae defining the effect of noise in the intensity pattern, image contrast and interpolation method on both bias and variability in one-dimensional motion measurements are presented. Image correlation simulations (through subset matching) confirm the theoretical results for the case of linear intensity pattern interpolation.

I. Background
Reconstruction of object motions from image-based measurements remains an area of active interest in both the vision community as well as among experimental mechanicians. Since image plane errors during pattern matching will affect both single camera and stereovision reconstruction accuracy, there is a long-standing need to develop an understanding of the key parameters affecting motion measurements.

In this regard, there is some existing literature regarding aspects of image based error analysis. Assuming isotropy in image-based matching, investigators [1, 2, 3] have developed estimates for positional errors, suggesting that variability in the intensity pattern is directly related to the accuracy of the image motions. Schreier et al [4] showed that sub-pixel matching errors are a function of the interpolation process used to reconstruct the deformed intensity pattern, even when intensity noise is not considered.

Assuming Gaussian intensity pattern noise and a least squares pattern matching metric, the authors provide theoretical equations that clearly demonstrate the importance of various parameters (e.g., interpolation method, intensity noise) when reconstructing one-dimensional motions. Both numerical simulations and theoretical results are presented to demonstrate the effect of specific parameters.

II. Basic Formulation and Definitions
A least squares matching metric for subset-based intensity-pattern comparisons is used to define the optimal image-plane motion. Letting $I(x, y)$ be the noiseless intensity pattern in the reference image at pixel location $(i, j)$, we initially assume that digitization error (quantization) are small and do not affect the optimally estimated motions.

Additive intensity pattern noise is defined to be a Gaussian distribution that is uncorrelated from pixel to pixel. Here, we let $\varepsilon_1$ and $\varepsilon_2$ denote the additive noise term at each pixel location in the reference and translated images, respectively, with zero mean and standard deviation of $\sigma$ (gray levels).

The least squares intensity matching metric can be written in the following form;
SSD = \sum_{i=1}^{N} (\bar{T}(\xi_{i},y) - \hat{T}(x_{i},y))^{2}

\bar{T}(\xi_{i},y) = T(\xi_{i},y) + \varepsilon_{x}(\xi_{i},y) = \text{intensity in the translated image, with additive white noise}
\hat{T}(x_{i},y) = I(x_{i},y) + \varepsilon_{x}(x_{i},y) = \text{intensity in the reference image, with additive white noise} \quad (1)

x_{i} = \text{integer position along the x-direction in reference image.}
\xi_{i} = x_{i} + \hat{u} = x_{i} + u_{b} + u_{e}

where \( u_{0} \) is the exact translation, \( u' \) is the optimally estimated translation and \( u_{e} = u' - u_{0} \) is the measurement error.

III. Linear Interpolation and One-Dimensional Translation

If one assumes the use of linear interpolation to reconstruct the translated pattern, the intensity at a non-integer location is estimated by the intensity values at two integer locations,
\[ T(\beta + \tau, y) = T(\beta, y) + [T(\beta + 1, y) - T(\beta, y)] \cdot \tau \]
where
\( \beta \) = integer pixel location
\( \tau \) = fractional distance between two pixel location, \( 0 \leq \tau \leq 1 \)

Expanding the translated pattern in a linear Taylor’s series about the exact position, we can write an approximate expression in the form;
\[ T(\xi_{i},y_{j}) \approx T(x_{i} + u_{0}, y_{j}) + \nabla T_{x} (x_{i}, y_{j}) \cdot u_{e} \]
where
\( \xi_{i} = x_{i} + u' + u_{e} \)

Combining Eqs (1-3) and minimizing Eq (1) with respect to \( u_{e} \), the optimal error can be determined. Using basic probability concepts, the expectation and variance in the optimal error expression, \( E(u_{e}) \) and \( \text{Var}(u_{e}) \), respectively, can be readily determined. The term \( E(u_{e}) \) represents translation measurement bias and \( \text{Var}(u_{e}) \) represents variability in the displacement measurement obtained by intensity pattern matching. It can be shown that the expectation and variance in the error have the following form [5];
\[ E(u_{e}) = A + B \]
\[ \text{Var}(u_{e}) = C \]
\[ A = \frac{-\sum_{i=1}^{N} \sum_{j=1}^{N} \{h(x_{i}, y_{j}) \cdot \nabla T_{x}(\xi_{i},y_{j})\}}{\sum_{i=1}^{N} \sum_{j=1}^{N} [\nabla T_{x}(\xi_{i},y_{j})]^{2}} \]
\[ B = f(\tau) \cdot \frac{N^{2} \sigma^{2}}{\sum_{i=1}^{N} \sum_{j=1}^{N} [\nabla T_{x}(\xi_{i},y_{j})]^{2}} \]
\[ f(\tau) = 1 - 2\tau \]
\[ \text{Var}[(\varepsilon_{x})^{2}] = \text{Var}[(\varepsilon_{y})^{2}] = \sigma^{2} \]
III.a. Discussion

If linear interpolation is used to reconstruct the intensity pattern, Equation (4) indicates that the translation measurement is biased due to a combination of two effects. The first term is designated interpolation bias and is due solely to the use of intensity interpolation. The term can be reduced by using optimized interpolation functions. The second term is the noise-induced bias and is due to a combination of Gaussian intensity pattern noise and the interpolation method used to reconstruct the pattern. It is important to note that a similar form is obtained for cubic and quintic interpolation; changing the interpolation method results in different \( f(\tau) \) functions and hence different interpolation bias.

Variability in the translation measurement is directly proportional to the magnitude of the pixel-level intensity noise and hence can be reduced through appropriate improvement in the imaging system. Both measurement bias and measurement variability can be reduced by increasing spatial gradients in the intensity pattern. This is achievable by increasing contrast in the image. For example, if the initial contrast is 0 to 50 gray levels and this is improved to give a range from 0 to 100 gray levels, then the potential for a 4-fold increase in the denominator of Eq (4) exists.

IV. Image Correlation Simulations and Measurement Bias

To demonstrate the accuracy of Eq (4), synthetic images were developed and image correlation simulations were performed. Accurate image synthesis was performed using the procedures outlined previously [4, 5, 6], and Figure 2 shows the two-dimensional random pattern for the undeformed image. Each pixel value is assumed to be recorded with a value from 0 to 65535 gray level range (16 bits), with the minimum and maximum intensity values of 24,084 and 42,434 in the reference image. Gaussian intensity pattern noise with standard deviation \( \sigma \) and percent noise is defined by the following equation.

\[
\Gamma_N = \frac{\sigma}{(I_{\text{max}} - I_{\text{min}})} \cdot 100\%.
\]  

(5)

Gaussian noise given in Eq (5) was added to both the reference image and the translated image. In this work, noise levels \( \Gamma_N = 0\%, 3\%, 5\% \) and \( 7\% \) are used.

For each Gaussian noise level, images were synthetically constructed to incorporate exact translations of 0.001, 0.1, 0.2... 0.999 pixels, respectively. After translation, the Gaussian noise for each pixel is determined separately and added to the pixel values in the translated image and the value is truncated to the nearest integer to obtain the noise-corrupted image intensity field. For each \( \Gamma_N \), a total of 1500 deformed and undeformed subsets are constructed and image correlation performed using an SSD matching metric to compute the translation. After performing image correlation for all 1500 images using a 41x41 subset, the experimental mean and variance for the translation are computed. Figure 1 presents a direct comparison between the simulation predictions and the theoretical predictions for the translation bias, \( u_e \).

V. Discussion

As shown in Figure 1, a linearly interpolated, noiseless intensity pattern has bias with amplitude of \( \approx 0.01 \) pixels. The functional form for linear interpolation bias in the measured image plane displacement is \( 0.01 \sin(2\pi(\tau - \frac{1}{2})) \) pixels, where \( \tau \) is the sub-pixel portion of the translation, \( 0 \leq \tau < 1 \).
τ ≤ 1 pixel. As a simple example, if one assumes that uniform strain of 1x10^{-3} is applied to a specimen that is imaged onto a 1000 pixel array, then the bias shown in Fig 1 would introduce a maximum error in the measured strain of ≈ 6.3% (i.e., 6.3x10^{-5}).

Figure 1 also shows that increasing Gaussian noise in the intensity pattern introduces measurement error that is maxima near the integer locations, with errors up to 0.04 pixels near the integer pixel locations. The trend shown in Fig. 1 was quantitatively confirmed experimentally in a recent publication [5] using cubic interpolation to reconstruct the intensity, providing clear evidence of the potential for bias to affect experimental measurements.

VI. Acknowledgements
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VII. References
Figure 1: Comparison of image correlation simulation results for measurement bias to theoretical predictions using Eq (4), with Gaussian intensity pattern noise from 0 to 7%.

Figure 2: Synthetic speckle pattern with 41x41 subsets shown. Average speckle size is 4 pixels in both directions.