Time Effects in Rock Mechanics

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ABSTRACT
A short presentation of the time dependent constitutive equation for rocks is presented. One is starting from the experiments to show how the constitutive equation if formulated. After that one is presenting the results of application of the constitutive equation for many rocks, concrete and particulate materials.

INTRODUCTION
The time dependent constitutive equation for rocks was already written in several papers and in two books about rocks. Since in the time elapsed it was somehow improved and developed, it is presented here again in the last version (Cristescu [1989]; [1998]).

EXPERIMENTAL FACTS
The whole time dependent rock mechanics is time dependent. Let us show some time effects. First in uniaxial test the time effects is evident, see Fig.1 for Schist. One can see that the loading rates are changing the whole stress-strain curve, from the beginning. Also the failure is time-dependent since the star at the end of each curve is showing failure. The figure is also showing two creep curves. The constant stress is maintained constant 20 days, 20 days, 21 days, and some additional days. Failure is at a much lower stress, and the strain is different than in the other stress-strain curves. Thus the whole stress-strain curve is time-dependent. That is a uniaxial stress-strain curve. Everything is time dependent.

Fig.1. Uniaxial stress-strain curves for schist for various loading rates, showing time influence on the entire stress-strain curves, including failure. The thick line is showing a creep curve.

Then, let us show the triaxial tests. These are the devices in which a cylindrical specimen is subjected to an axial loading and to a lateral pressure loading. First both loadings are increased together, but in the second part of the test the lateral pressure is maintained constant and only the axial loading is increased. For the three-axial curves one obtains the results shown in Fig.2 for granite (Maranini and Yamaguchi [2001]).
$\bar{\sigma} - \varepsilon_1$, $\bar{\sigma} - \varepsilon_2$ and $\bar{\sigma} - \varepsilon_v$ curves obtained for 100 kN/min. In order to find the correct values of the elastic modulus the tests have been stopped at various levels of stresses for a few minutes. A significant creep is observed. After this period, a small unloading and reloading have given the correct elastic parameters. The elastic parameters are also not really constant. They are increasing with the confining pressure. From the last curve one can see that initially the volume is compacting and afterwards dilating.

Fig. 2. The stress-strain curves for granite obtained in three axial tests. The curve is obtained for a confining pressure of 20 MPa.

During lodging the two velocity of propagation $v_p$ and $v_s$ are also variable, they first increase and then decrease. See Fig. 3 showing an increase in the compaction domain and a decrease in the dilatant's domain.

Fig. 3. The variation of the longitudinal and transverse velocities in a triaxial test. They increase in the compressible domain and decrease in the dilatant.

The two domains are shown in the Fig. 4. All the curves shown are time dependent. For very large pressures they are unknown, but one can guess that the dilatant curve is approaching the hydrostatic axes. The failure curve is also strongly time dependent, rising very much at fast raising stresses.

Fig. 4. The domains of dilatancy and compressibility shown as unique in time. They are certainly not unique in time.
There are many time effects to be followed. First the elastic "constants" are not fixed, but variable. To find it we propose a method to find them exactly. At any stage of loading the stress is kept constant for a short period of time, 10 or 30 minutes. During this time a creep takes place. We register the creep by representing $\dot{\varepsilon}_1$ or $\dot{\varepsilon}_2$ as function of time. When their magnitude is already small enough, a small unloading following a loading is taking place as shown in Fig.5. That is giving a linear behavior and exact elastic modules.

![Fig.5. Static procedure to determine elastic parameters in unloading processes following short creep periods (Cristescu [1989])](image)

The time effect is to be seen everywhere in the rock mechanics. See for instance the stress-time curve shown below. It is giving the stress variation during the period of creep for porous chalk.

![Fig.6. Comparison between the model prediction (continuous line) and the experimental data for a relaxation test for porous chalk (Dahu et al. [1995]).](image)
Fig. 7. (a) Stress-strain curves including unloading cycles during triaxial test without relaxation phase. (b) Stress-strain curves including unloading cycles during triaxial test with relaxation phase (Tournemiere shale by Niandou et al. [1997]).

The strain rate is relaxing during creep very fast. An example is given for instance, for concrete is the Fig. 8 (Schmidt et al. [2009]). That is a variation during a creep period, when the stress is constant.

Fig. 8. Volumetric strain rate vs. time from the second cycle (unload at 100 MPa) of a hydrostatic test up to 500 MPa confining pressure.

**CONSTITUTIVE EQUATION**

We start from the starting point that all rocks are creeping under applied stress. That is why the constitutive equation is, from the beginning, written in a differential form. For the initial elastic part a differential form of the constitutive equation is written as:

$$
\dot{\varepsilon}^E = \frac{\sigma}{2G} + \left( \frac{1}{3K} - \frac{1}{2G} \right) \dot{\sigma} I
$$

(1)

with the elastic parameters variables and $I$ is the unit tensor. From dynamic point of view, the two velocities of propagation, the longitudinal, transverse wave, and bar velocity are propagating with the velocities.
\[ v_p^2 = \frac{3K(1-v)}{\rho(1+\nu)}, \quad v_s^2 = \frac{G}{\rho}, \quad v_b^2 = \frac{E}{\rho} \]  

These are used in order to determine dynamic the two elastic parameters, from

\[ K = \rho \left( v_p^2 - \frac{4}{3} v_s^2 \right), \quad G = \rho v_s^2 \]

In order to develop the constitutive equation, we have to describe both the steady state creep and the transient creep. For the irreversible part of the rate of deformation due to transient creep, we can use the formula

\[ \dot{\varepsilon}^i_t = k_T \left( 1 - \frac{W(t)}{H(\sigma)} \right) \frac{\partial F}{\partial \sigma} \]  

or if one is not able to determined the viscoplastic potential \( F(\sigma) \) one can use

\[ \dot{\varepsilon}^i_t = k_T \left( 1 - \frac{W(t)}{H(\sigma)} \right) N(\sigma) \]

where \( H(\sigma) \) is the yield function, with

\[ H(\sigma(t)) = W(t) \]

the equation of the stabilization boundary and

\[ W(T) = \int_0^T \sigma(t) \cdot \dot{\varepsilon}(t) dt = \int_0^T \sigma(t) \dot{\varepsilon}^i(t) dt + \int_0^T \sigma'(t) \cdot \dot{\varepsilon}'(t) dt = W_r(T) + \dot{W}_p(t) \]

In order to describe the steady-state creep on can adapt accordingly either the function \( H \), or one can add to (3) an additional term as for instance

\[ \dot{\varepsilon}^j_s = k_s \frac{\partial S}{\partial \sigma} \]

where \( S(\sigma) \) is a viscoplastic potential for steady-state creep and \( k_s \) is a viscosity coefficient for steady-state creep.

The constitutive equation for transient creep will be written in the form

\[ \dot{\varepsilon} = \dot{\sigma} + \left( \frac{1}{2G} - \frac{1}{3K} \right) \dot{\sigma} \cdot I + k_T \left( 1 - \frac{W(t)}{H(\sigma)} \right) \frac{\partial F}{\partial \sigma} \]

A stress variation from \( \sigma(t_\theta) \) to \( \sigma(t) \neq \sigma(t_\theta) \) with \( t > t_\theta \), will be called loading if

\[ H(\sigma(t)) > H(\sigma(t_\theta)) \]

\[ \frac{\partial F}{\partial \sigma} \cdot I > 0 \text{ or } \frac{\partial F}{\partial \sigma} > 0 \text{ compressibility} \]

\[ \frac{\partial F}{\partial \sigma} \cdot I = 0 \text{ or } \frac{\partial F}{\partial \sigma} = 0 \text{ compressibility/dilatancy boundary} \]

\[ \frac{\partial F}{\partial \sigma} \cdot I < 0 \text{ or } \frac{\partial F}{\partial \sigma} < 0 \text{ dilatancy} \]

We also use also the formulae: for the mean stress \( \sigma = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \),

and for the equivalent stress \( \bar{\sigma}^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1 \),

and for the octahedral shear stress \( \tau = \sqrt{\frac{2}{3}} \bar{\sigma} = \left( \frac{2}{3} \bar{\sigma}^3 \right)^{1/2} \)

For the second invariant \( I_{\sigma} = (1/2) \sigma' \cdot \sigma' \)

and for the stress deviator we use \( \sigma' = \sigma - \sigma I \).
RESULTS

The model was applied first to rocks as granite, schist, mortar, concrete, shale, coal, rock salt, porous chalk etc., to wet and dry sand, to a lot of powders as alumina of various powder magnitudes, silica, microcrystalline cellulose, and others.

Let us mention first the result shown is for sand. It has given a long time ago, in 1991. As one can see from Fig.9 the two surfaces $H=\text{constant}$ and $F=\text{constant}$ do not coincide. The dilatancy domain is also shown.

Fig.9. The characteristic plane for saturated sand. The surfaces $H=\text{const.}$ and $F=\text{const.}$ are quite different.

Also the stress-strain curves are quite O.K. See Fig.10 for example. It is for a three axial state, the experimental points are compared with the theory.

Fig.10. Stress-strain curves for saturated sand for confining pressure of 14.71 kPa.

Many other materials have been considered meantime. For the three axial tests $\sigma_2 = \sigma_3$ one has to use the formulae

\[
\frac{\partial F}{\partial \sigma} = \frac{\dot{\varepsilon}_i^I}{k \left( 1 - \frac{W(t)}{H(\sigma)} \right)}, \quad \frac{\partial F}{\partial \sigma} = \frac{2}{3} \frac{\dot{\varepsilon}_i^I - \dot{\varepsilon}_i^I}{k \left( 1 - \frac{W(t)}{H(\sigma)} \right)}
\]

And for the creep during the stationary stress state:

\[
\dot{\varepsilon}_i^I = \left( \frac{1}{3G} + \frac{1}{9K} \right) \sigma_i^I + \left( \frac{1}{H(\sigma)} \frac{\partial F}{\partial \sigma} \right) \left\{ 1 - \exp \left[ k \left( \frac{\partial F}{\partial \sigma} + \frac{\partial F}{\partial \sigma} \right) (t_0 - t) \right] \right\}
\]
For instance for Microcrystalline Cellulose PH-105 we use $K = 35$ MPa, $G = 30$ MPa and

$$
\varepsilon' = \left(-\frac{1}{6G} + \frac{1}{9K}\right) \sigma' + \left\{1 - \frac{W(t)}{H(\sigma)} \frac{\partial F}{\partial \sigma}ight\} \left\{1 - \exp \left[ \frac{k}{H} \left( \frac{\partial F}{\partial \sigma} + \frac{\partial F}{\partial \sigma^2} \right) (t - t_0) \right] \right\}
$$

Fig. 11. Comparison with the test of $W_D^{I}$ for Microcrystalline cellulose.

$$
H(\sigma', \sigma) = a \sigma' + b \frac{\sigma'}{\sigma' - \frac{3}{7}}
$$

$a = 0.00043$ kPa, $b = 0.017$ kPa

$$
F(\sigma, \sigma) = -0.0000025 (\sigma')^{12} (\sigma)^{10} + \frac{0.000003}{1.29} (\sigma')^{10} + \frac{0.000013}{1.2} (\sigma')^{12}
$$

Fig. 12. Constitutive domain shows: stabilization boundaries (gray full lines), viscoplastic potential surfaces (black full lines), compressibility/dilatancy boundary (dot lines), and failure surface (dash line). The dilatancy domain is wary small.
Fig. 13. Stress-strain curves, theoretical and experimental for Microcrystalline cellulose, for confining pressure 20 psi.

Fig. 14. Stress-strain curves for granite; test (dotted) compared with theory (full) (Maranini and Yamaguchi [2001]).

Let us consider now the application to concrete (Schmidt et al [2009]). The tests were done for very high pressures. In the figure which follows is showing the variation of the bulk modulus $K$. The variation of the Young’s modulus is given by $E(p)=E_\infty-b \exp(-p/c)$ where $E_\infty$ is the limit value of $E$ when the pressure increases very much ($p \to \infty$), that is the value of Young modulus corresponding to fully compacted state (Cristescu [1989]).
Fig. 15. Comparison between theoretical and experimental variations of the bulk modulus $K$ with the mean stress.

As one can see, all the examples given here, besides many other examples, are time dependent and satisfying the constitutive law given above.

REFERENCES