Distributed Vibration Control Using Electro rheological Fluids

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Abstract

Recent developments in the smart materials technology have led to the introduction of intelligent structures that use composite members filled with electrorheological (ER) fluids. This paper outlines the scope of the research and the role of the viscoelastic characteristics of ER materials. The shortcomings of the present models in prediction of mechanical properties of these composites are discussed. A methodology to extract the complex modulus of ER layer from modal parameters of the beam is presented. A modal modification scheme is proposed based on the discrete dynamic model of the composite beam.

Nomenclature

<table>
<thead>
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<th>Symbol</th>
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<tr>
<td>$E$</td>
<td>electric field intensity</td>
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<tr>
<td>$E_i$</td>
<td>elastic Young's modulus of $i^{th}$ layer of sandwich beam</td>
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<tr>
<td>$H_i$</td>
<td>thickness of the $i^{th}$ layer</td>
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<tr>
<td>$A_i$</td>
<td>cross-section area of the $i^{th}$ layer</td>
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<td>$L$</td>
<td>length of beam</td>
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<tr>
<td>$b$</td>
<td>width of beam</td>
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<tr>
<td>$\eta_i$</td>
<td>loss factor of the ER material (2nd. layer)</td>
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<tr>
<td>$G$</td>
<td>material complex shear modulus = $G(1+i\eta)$</td>
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<tr>
<td>$x$</td>
<td>longitudinal coordinate</td>
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<tr>
<td>$t$</td>
<td>time variable</td>
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<tr>
<td>$m$</td>
<td>mass per unit length</td>
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<tr>
<td>$p$</td>
<td>modal coordinate</td>
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<tr>
<td>$\omega$</td>
<td>natural frequency</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>damping ratio</td>
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<tr>
<td>$[M]$</td>
<td>mass matrix</td>
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<tr>
<td>$[K]$</td>
<td>stiffness matrix</td>
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<tr>
<td>$[C]$</td>
<td>damping matrix</td>
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<tr>
<td>$[U]$</td>
<td>displacement vector</td>
</tr>
<tr>
<td>$[F]$</td>
<td>vector of applied force</td>
</tr>
<tr>
<td>$[I]$</td>
<td>identity matrix</td>
</tr>
<tr>
<td>$[\sigma]$</td>
<td>mass normalized modal matrix</td>
</tr>
<tr>
<td>$(\cdot)^T$</td>
<td>matrix transpose operation</td>
</tr>
<tr>
<td>$(\cdot)^n$</td>
<td>$n^{th}$ derivative w.r.t. $x$, ($n$ in Roman numbers)</td>
</tr>
<tr>
<td>$(\cdot)'$</td>
<td>time derivative</td>
</tr>
<tr>
<td>$(\cdot)'^*$</td>
<td>non-dimensional variable (as is defined)</td>
</tr>
<tr>
<td>$(\cdot)_{i}$</td>
<td>complex variable with real part $(\cdot)$</td>
</tr>
<tr>
<td>$i$</td>
<td>subscript referring to the $i^{th}$ mode</td>
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</table>

Introduction

The introduction of "smart" composites which incorporate ER fluids, has spawned an emerging field of research that could revolutionize vibration and noise control technologies in the near future. The unique property of an ER fluid is its ability to turn into a solid gel with controllable shear modulus, when subjected to a voltage. Using this substance in a sandwiched beam or panel would give the assembly the ability to adjust its stiffness and damping parameters. Thus, incorporating this within layers of traditional materials forms a composite, which can actively change its dynamic characteristics to modify the dynamic response of the entire structure in various conditions, when controlled using the appropriate electronics and software.

The traditional viscoelastic layer solutions, are replaced by new high performance "smart" composites with adjustable stiffness and damping factors that will provide the designer with the ability of employing active control schemes successfully to reduce or eliminate vibration and noise problems.

The ER effect refers to the abrupt dramatic change in the rheological properties of certain suspensions upon application of an electric field. When subjected to a voltage potential, the ER suspension converts to a solid gel almost instantaneously. The solidified fluid, then, resists shear stresses up to a yield point. Winslow [1] first observed this phenomenon and attributed it to the specific microstructural formation of the fluid particles. Fig. 1 shows the microscopic photographs of an ER fluid. The random pattern of the particles turns into a chain like structure as soon as the electric field is applied. Comprehensive reviews on ER fluids can be found in [2,3,4].

These properties, in particular, quick response and full reversibility of phase transformation in ER fluids coupled with the capability to interface integrated electronics and advanced computers have provided a great technological promise. These unique features, offer decided advantages when applied to a variety of mechanical systems such as high speed valves, clutches, active dampers, modern flexible structures and robot manipulators. Furthermore, noise attenuation and vibration control in intelligent
systems can be achieved through applying real time control schemes. The use of ER Fluids as discrete or distributed elements in vibration damping has been discussed in [5,6].

The change in the rheological characteristics of ER fluid contained in typical domains of intelligent structure due to the imposition of a specified electric field, alters the stiffness and damping characteristics of the structure. The supporting computer software and/or hardware can control the magnitude and shape of the input voltage in order to achieve the desired dynamic response. Fig. 2 illustrates such a notion as it is applied to a flexible arm robot manipulator. The Frequency response functions illustrated in Fig. 3 show a remarkable increase in the damping capacity and stiffness of the beam by applying the indicated voltage.

Scope of the research

There have been several investigations on ER fluid performance in its liquid state, but relatively little on the rheological properties of solidified ER fluids [7,8,9,10]. There is no comprehensive image of its performance as a viscoelastic solid (i.e. pre-yield behavior).

However, the very recent application of solidified ER fluid emerged due to the works by Gandhi, Thompson, and co-workers [11]. They simulated the ER material incorporated in a multilayer composite as viscoelastic medium. Then the equations governing the viscoelastic continuum were used to make a finite element model for analysing the mechanisms built in these smart composites. Coulter et al [12] and Choi [13] made further efforts in investigating the mechanical properties of the ERF-based sandwich beams.

With the new role of ER composites as a means of tuning the stiffness and the damping factor of structural components, it would be quite appropriate to establish a link between the classical concept of constrained layer damping and this new technology by treating the ER layers as viscoelastic ones with the capability of having a changeable storage modulus and loss factor.

Since the 1950s, several types of sandwich plates/panels have been developed and successfully utilized as high performance mechanisms for vibration suppression and noise reduction in modern structures [14,15]. The so-called Unconstrained/Constrained Layer Damping (UCLD/CLD) treatment basically builds up a multilayer plate which dissipates vibrational energy in periodic extensional or shear deformation of the viscoelastic layer. Now, using appropriately the concept of complex modulus for CLD which has been fully developed after the pioneering works by Oberst [16] and Kerwin [17], the analysis and design of the new smart composites can be handled more effectively.

With this link, thus, built up between traditional CLD treatment and the modern smart ER-fluid-based sandwich elements, a field of research and development is going to be opened to investigations seeking new solutions to the vibration and noise problems. The investigations would range from feasibility studies and evaluation of these composites to resolve noise and vibration problems to introducing innovative elements and techniques such as advanced smart anti-vibration tapes, to control them practically. The high performance dynamically tunable damping wavelets fabricated in a layer of ER material could be easily applied to any vibrating surface (Fig. 2).

However, the very desperate need in the analysis of the systems incorporating ER fluids is to identify the material property in a desired range of variables. It has become a common approach to assume that ER materials behave as linear viscoelastic when activated with external voltage. In contrast, some experimental evidences [13] promote the belief that the complex modulus of EH fluid is highly strain dependent. Therefore, the linearity is not still confirmed. The classical RKL (Ross-Kerwin-Ungar) formulations and the resultant standard ASTM E 756-83 have been used to verify the modal factors of ER-based composite beams [12,13]. So far, the results show considerable discrepancies compared with simple shear tests.

As far as the applications are concerned, the sandwich beam test procedure: more closely simulates the existing structural elements. In addition it is simpler and more convenient. Therefore, more elaborated theory should be employed to express the relations between the viscoelastic characteristics of the ERF layer and the dynamical properties of the composite beams.

In this study, modal testing and analysis is used to obtain the information about dynamic characteristics of ER layers. The results, expressing the stiffness and damping ratio in different modes, would offer more insight into the pre-yield behavior of ER materials. A series of impact tests run on a cantilevered sandwich beam with an ER layer, would give the modal parameters of the composite beam, in relation to the effect of the applied electric field intensity. Based on the experimental results, with the aid of a solution methodology for the governing equation of multilayered beam with a viscoelastic layer, the complex modulus of the ER layer will be obtained and an analysis of the results will be given. Considering the discrete model of the vibrating smart structure a modal control strategy has been proposed.

Governing Equations; Continuous Model

The smart composite beam/plate incorporating an ER layer is simulated as the classical sandwich one with viscoelastic core. An evidence of such a behaviour is the shape of the hysteresis loop of ER material [16]. The equations governing the transverse vibration follow the derivation by Mead and Markus [19] or by Rao [20]. The assumptions made are:
(a) the elastic face-plates carry only longitudinal stresses, 
(b) the ER core carries only shear stresses and is modeled as a linear viscoelastic material, 
(c) transverse strains are neglected in both core and facets, 
(d) there is no slippage at the core-face interfaces 
(e) longitudinal and rotary inertia effects are ignorable.

The equation for free vibration of the sandwich beam is

$$\frac{\partial^2 w}{\partial x^2} - g^* (1 + \gamma) \frac{\partial^4 w}{\partial x^4} - g^* \frac{\partial^2 w}{\partial t^2} = 0$$  \hspace{1cm} (1)$$

where

$$\bar{x} = x/L, \quad \bar{t} = t/l_0; \quad [l_0 = (mL^4/D_t)^{1/4}]$$  \hspace{1cm} (2)$$

$g^*$ is a shear parameter and $\gamma$ is a geometric parameter where $d = H_t + (H_s + H_f)/2$ and

$$g^* = G_s (1 - t_s) \frac{L^3 b (E_t A_t + E_s A_s)}{H_t} \frac{E_t A_t + E_s A_s}{E_t A_s}$$  \hspace{1cm} (3)$$

$$\gamma = \frac{(d^2/D_t) E_t A_t E_s A_s}{(E_t A_t + E_s A_s)}$$  \hspace{1cm} (4)$$

$$D_t = b (E_t H_t^2 - E_s H_s^2)/12$$  \hspace{1cm} (5)$$

The modal parameters are obtained through a standard eigensolution

$$w(x,t) = W_i(\bar{x}) \ T(\bar{t})$$  \hspace{1cm} (6)$$

substituting (6) into (1) leads to the time and space equations

$$\bar{\tau} \omega_i^2 \ T = 0$$  \hspace{1cm} (7)$$

and

$$W_i^{(0)} - g^* (1 + \gamma) W_i^{(0)} - \omega_i^2 (W_i^{(0)} - g^* W_i^{(0)}) = 0$$  \hspace{1cm} (8)$$

where $\omega_i = \omega_0 (1 + i \eta_0)^{1/4}$, $\omega_0$ being the normalized angular frequency ($\omega_0 = \omega_0 l_0$).

Examining the general solution of equation (8) as

$$W_i^{(0)} = \sum_{k=1}^{6} A_{ik} e^{i \xi_i x}$$  \hspace{1cm} (9)$$

results in the auxiliary equation

$$\lambda_{ik}^* \xi_i^8 - g^* (1 + \gamma) \lambda_{ik}^* ^4 - \omega_i^2 (\lambda_{ik}^* ^2 - g^*) = 0$$  \hspace{1cm} (10)$$

the roots of which are readily found by solving the cubic equation in $\lambda_{ik}^*$ for three pairs of conjugate poles.

Applying the boundary conditions of a certain beam in the general solution (9) leads to a set of six homogeneous complex equations in $A_{ik}$ with coefficients that are functions of $\omega_i$. Rewriting the system of equations in a matrix form, yields

$$[ B_{ij} ] (A_{ik}) = (0), \quad i,j,k=1,2,3,\ldots,6$$  \hspace{1cm} (11)$$

for which a non trivial solution requires det$[B]$=0. Solving this equation will give the desired eigenfrequencies $\omega_i$.

Fore a cantilever beam the boundary conditions are

$$W(0) = 0, \quad W(0) - W_0' = 0$$  \hspace{1cm} (12)$$

$W(0) - g^* (1 + \gamma) W(0) - W_0 \omega_i^2 = 0$

At the free end and

$$W = 0, \quad W(0) = 0, \quad W(0) - g^* Y W(0) = 0$$  \hspace{1cm} (13)$$

at the clamped end.

Experimental Procedure

The test arrangement is shown in Fig.4. The sandwich beam, is attached rigidly to a heavy base resting on isolating interfaces. To obtain the modal parameters, impact testing is performed. The equipment layout is presented and the specifications of the designed specimens are shown (Fig.5). Prior to any dynamic test, static tests has been performed to find the stiffness of the beam in a range of varying field intensities (Fig.6).

The test specimens are being built in simple three-layered sandwiches. Each beam is made of aluminium sheets as face electrodes, separated with silicon sealant which provides the necessary room for ER fluid between the electrodes as well as insulating them electrically. The dimensions indicated by $H_t$, $H_s$, $H_f$, $L$, $L_0$, $b$ and $b_s$ are 1, 1, 1, 365, 100, 30.5 and 5 millimetres respectively. The fluid has been supplied by Lord Corporation.

A serious difficulty arises when making a 3-layered beam with an ERF core. That is, one should provide the room for ER fluid by means of some additional sealant/adhesive, which also keeps the face plates apart. The addition of this sealant disrupts the specimen and adversely impacts the whole derived predictive model. To eliminate this effect, the tests have been run for 3 specimens with different widths of sealant strips ($b_s = 5.3, 2$ mm). The virtual modal data for the sealant-free beam, are estimated by an extrapolation of the measured sets to the values corresponding to $b_s=0$.

Parameter Identification

The frequency response functions (FRF) of the test specimen were obtained (Fig.3). The modal parameters of the beam in different voltages were derived by curve fitting of the FRFs.
Solving the eigenproblem corresponding to the boundary conditions of the cantilever beam results in a six-order characteristic determinant, that must vanish for non-trivial solutions. A direct process that seeks the modal values for a beam with known parameters is complicated. It deals with solving equations (11) to get a set of infinite number of eigenfrequencies. The results of the numerical solution have been curve fitted, giving formulas for frequency and loss factor in each mode. A good correlation was obtained in the following forms

\[ r = \sum_{n=0}^{\infty} a_{nn} x^n \]

\[ \omega = \sum_{n=0}^{\infty} b_{nn} x^n \]

where \( a_{nn} \) and \( b_{nn} \) are known constants [20].

This study introduces the inverse procedure of parameter estimation which is preferred. It involves solving the frequency equation with known eigenvalues provided from the modal tests. Thus, it ends up solving a second order equation in \( g \). A solution procedure has been adapted from the available symbolic computer algebra package; MAPLE [21]. A long closed form solution is also achieved. The complex modulus \( G_0 \) of the smart viscoelastic ER material, can readily be evaluated through the definition (3) for \( g \).

To examine the method, a verification of the ERF complex modulus for a known beam was performed. The data of reference [13] for the EFR-based beams was used. Storage modulus and loss factor of the beam were then evaluated at the first three modes. The results are graphically presented in Fig.7. A better correlation has been obtained in contrast to the RKU based results. The solid line represents the ER fluid modulus calculated from RKU formulation. The results of the simple shear test has been obtained from a Couette cell type oscillating rheometer.

**Bases of Active Control; Discrete Model**

By introducing the ERF-based smart Composites, it is possible to control both the damping Characteristics and natural frequencies using dynamically-tunable ER materials as distributed parameter actuators. This objective could typically be achieved by tuning the rheological properties of ERF domains associated with particular sensors.

An active tuning scheme can be developed based on the discretized model of the system. The method suggested in [22] is based on the assumption of Kelvin model for viscoelastic behaviour of all layers. The discretized equations of motion for a beam-like structure fabricated in smart ERF-based material could be expressed as

\[ [M] \ddot{w} + [C(E)] \dot{w} + [K(E)] w = f(t) \]

or equivalently

\[ [M] \ddot{w} + [\delta C(E)] \dot{w} + [\delta K(E)] w = 0 \]

(for \( f(0) = 0 \)) where the global mass, stiffness, and damping matrices are denoted by \( [M] \), \( [K(E)] \) and \( [C(E)] \) respectively. The stiffness and damping characteristics are functions of the intensity of the external electric field \( E \), applied across the fluid domains within the composite beam. \( [M] \), \( [K] \) and \( [C] \) are the spatial matrices of the reference structure when \( E=0 \), and \( [\delta K(E)] \) and \( [\delta C(E)] \) are the incremental matrices due to the change in voltage. By introducing the modal transformation \( \{w\} = \{\phi\} \{\psi\ \} \) associated with \( [K] \) and \( [M] \), pre and post multiplication of equation (17) by \( \{\psi\} \) and \( \{\psi\} \), the governing equation is modified to

\[ [\bar{M}] \ddot{\phi} + [\bar{C}] \dot{\phi} + [\bar{K}] \phi = \{0\} \]

\[ [\bar{M}] = [I] \]

\[ [\bar{C}] = [C] + [\delta C(E)] \]

\[ [\bar{K}] = [K] + [\delta K(E)] \]

(assuming proportional damping) where \( k_0 = \omega_0^2 \) and \( c_0 = 2\zeta_0 \omega_0 \), \( \zeta_0 \) and \( \omega_0 \) being the modal frequency and damping ratio.

A perturbation analysis was conducted to investigate the magnitude of the changes that occurs in the modal matrix entries due to the changes in \( M \) and \( K \). It indicates that the off-diagonal terms of the increased damping and stiffness matrices associated with modal matrix \( \{\phi\} \) are negligible in comparison to the diagonal ones, hence

\[ [\bar{\phi}] [\delta C(E)] [\phi] = [c_0(E)] \]

\[ [\bar{\phi}] [\delta K(E)] [\phi] = [k_0(E)] \]

\[ [k_0(E)] = 2\zeta_0(\omega_0) \omega_0(E) \]

\[ [k(E)] = \omega_0^2(E) \]

where \( \omega_0(E) \) and \( c_0(E) \) are the increased modal frequency and damping ratio respectively. Thus, equation (18) will reduces to

\[ \{f\} \{\dot{\phi}\} + ([C] + [\delta C(E)]) \{\dot{\phi}\} + ([K] + [\delta K(E)]) \{\phi\} = \{0\} \]

The matrices in equation (18) represent structural matrices which are symmetric and positive definite, therefore the system modeled by this equation is asymptotically stable, provided that there is no zero frequency in the original system described by \( [K] \), \( [M] \) and \( [C] \).
However, the previous result for each mode implies

\[ \ddot{\rho}_r + 2 \zeta_1 \omega_1 \dot{\rho}_r + \omega_1^2 \rho_r = -2 \zeta_1(E) \omega_1(E) \rho_r - \omega_1^2(E) \rho_r \]  

(21)

where subscript \( r \) refers to the \( r \)th mode. This equation clearly implies the classical proportional-derivative state feedback scheme.

A complete model, obviously, needs the information between the stiffness and energy dissipation increments and the intensity of applied field. Further studies are needed to identify the effect of applied field on the viscoelastic characteristics of ER materials. This information virtually provides a model of actuator dynamics. However, an experimental investigation performed on a beam-like specimen has shown [22] that

\[ \zeta_1(E) \omega_1(E) / (\zeta_1 \omega_1) = \alpha_1 E \]  

(22)

\[ \omega_1^2(E) / \omega_1^2 = \alpha_2 E \]

where, \( \alpha_1 \) and \( \alpha_2 \) are known constants. Therefore, once the same rule applies to other modes, equation (21) may be rewritten in a generalized PD (Proportional-Derivative) state feedback form as

\[ \ddot{\rho}_r + 2 \zeta_1 \omega_1 \dot{\rho}_r + \omega_1^2 \rho_r = -\alpha_1 \omega_1(E) \rho_r \]

(23)

A parameter estimation, here, is conducted to calculate \( \alpha_1 \) and \( \alpha_2 \) in each mode. A set of experimental data including the applied voltage, modal damping and modal frequency were introduced to a least square routine that evaluates the constants associated with that mode.

For active tuning objectives the applied voltage could be altered. Consequently, the resulting gain \( k \) in each mode is adjusted to reach a desired modification of the required modes. The easiest way for practical implementation is an on-off control of the DC output of the power supply at prescribed times [13,22].

This approach could be followed to extend a state feedback over the required modes of vibrations. The schematic diagram of Fig. 6 demonstrates the experimental set-up in order to examine the active tuning model. The face layers of the test specimen are electrically non-conductive, coated in patches of conductive paint. This pattern, eventually, offers the possibility of controlling different domains of the continuous beam by applying the voltage separately to them, according to the particular control scheme. For instance, if the design is directed toward a higher energy dissipation, the intervals which are experiencing higher shear deformations in the specific mode-shape would be activated.

Discussion and Conclusion

The notion of smart shear layer has stemmed from a basic but intuitive understanding of ER phenomenon. This will be clear once one observes the microstructural formation of ER fluid in an electric field. Therefore, the significant enhancement which occurs in the damping factors of the composite beam, is greatly attributed to the energy dissipated in shear deformation of ER layer. Hence, the assumptions made to analyze such a composite as a constrained layer would come to be more realistic.

The classical RKU analysis and the ASTM standard E 756-83 do not adequately predict the modal parameters of the composite beams [12,13] (or inversely the complex modulus of ER layer). Therefore, more elaborated equations were employed to derive the complex modulus of ER layer from the known modal parameters of the ER-based sandwich beam.

The current discrete formulation obviously restricts the form of the induced ER damping mechanism to the conventional proportional damping which is neither a realistic assumption nor a consistent one. Yet, the proportionality could not be verified even if it exists. Other damping models have been developed to model viscoelastic properties beginning with the classical Kelvin-Voigt and Maxwell models and continuing with modern ones such as GH method [23], the Fractional Calculus method (Bagley, Torvic [24]) and the Lesieutre method [25]. Finite element codes that consider the frequency dependence properties have also been developed. Since these Codes need the complex modulus of the viscoelastic material this information on the particular ER material has to be already acquired from experiments. However, with the lack of a viable model, it would be justified to make the spatial model of structure inversely, from its modal parameters. A reliable complete set of modal test data, thus, can be employed to construct the spatial matrices representing an approximate dynamic behaviour of the structure [26], although the modal space representation itself, readily available from test, contains sufficient information to run a modal control scheme appropriately [27].

While the positive definiteness of spatial matrices of the smart ERF-based system guarantees the stability of the controlled structure, it implies that this sort of control is passive in nature. It makes more sense when one notices that there is not any source of energy pumped in, causing possible instabilities, especially in comparison to other methods such as the ones which use piezoelectric actuators. Hence, the control techniques using ER materials are to be classified as virtually passive techniques.

References


Fig. 1: Microscopic structure of ER fluid
(a) Zero Electric Field (b) Non-zero Electric Field
Fig. 2: Active Control of Flexible Arm with ER-based tapes

Fig. 3: Frequency Response Functions

Fig. 4: Schematic of Test Equipments and Instrumentation

Fig. 5: Test Specimen

Fig. 6: Static Stiffness of Smart Beam
Tip Load/Tip Deflection

Fig. 7: ERF Complex Modulus vs. Frequency

Fig. 8: Schematic for Active Control