ABSTRACT

Damage in a structure causes changes in the dynamic properties of the structure. Given a relationship between the dynamic property changes and the damage, measurement of the changes can thus be used to detect and quantify, to some extent, the damage. Use of modal testing as a means of damage detection in large structures is attractive since it is not location dependent, thus eliminating the need of access to hidden locations. Many methods have been proposed to detect and quantify damage using experimental modal analysis. This paper briefly reviews four methods and makes a comparison of their effectiveness. One of the methods exploits the changes in the eigenparameters while the other three utilize system identification/model updating procedures. These three methods were originally developed to locate errors in mathematical models. For the comparison studies, response data were obtained from simulation analysis of a simply supported steel beam. Relative merits and limitations of the methods are discussed.

NOMENCLATURE

[K] stiffness matrix
[K'] diagonal modal stiffness matrix
[M] mass matrix
[M'] diagonal modal mass matrix
{U*} eigenparameter vector
{AK} stiffness error matrix
{A} general error matrix
{λr} eigenvalue for mode r
{Φr} rth eigenvector
{w} mode shape matrix
[.]' transpose of a matrix
[.]' inverse of a matrix
[.]' pseudo inverse of a matrix

Subscripts
0 undamaged structure
D damaged structure
r rth mode

INTRODUCTION

To ensure continued serviceability of civil engineering structures, periodic structural integrity assessments are necessary to monitor the condition of the structure. Many assessment engineers prefer Non-Destructive Test (NDT) methods since no damage, or
limited surface damage is induced on the structure as a result of the testing. NDT methods that are not location dependent and involve the participation of the complete structure under investigation are more suitable for large engineering structures. One of such methods is vibration monitoring and relies on the fact that dynamic performance is sensitive to structural integrity.

Damage in a structure causes changes in the vibration response of the structure as characterised by natural frequencies, mode shapes and modal damping values. This subsequently leads to changes in the system parameters - mass \([M]\), stiffness \([K]\) and damping \([C]\). If the changes in the system parameters are known, it should be possible to detect, locate and quantify, to some extent, the damage. In practice, the dynamic response of the structure is measured at a few easily accessible locations. The time histories are processed to yield response spectra from which modal and system parameters can be obtained. Since the dynamic properties are global, the measurement points can be chosen to suit the test situation.

Many experimental modal analysis based methods have been proposed to detect and locate damage in structures. Some of these utilise shifts in the eigenvalues and modal damping, modification of mode shapes or a combination of changes in all the modal parameters [1-3]. Similarly, many model updating methods using experimental modal analysis have been suggested to validate predictions from theoretical models [4]. As a first step, most updating schemes locate regions of errors in the finite element model before the updating proper. If it is assumed that the (analytical) system matrices to be updated were obtained from a prior vibration measurement, then the error localisation stage can be utilised to detect, locate and quantify damage in a structure from any two measurements (made at different times but not necessarily consecutively) of the vibration response.

In this paper, the performance of four damage detection schemes is investigated. One of the methods uses the changes in natural frequencies and mode shapes to locate the damaged area. The other three methods were originally formulated to correct/update theoretical models using measured data. In adapting these three methods, the original (analytical) system matrices were obtained using system identification techniques on 'measured' data obtained from the undamaged structure.

'EXPERIMENTAL' ARRANGEMENT

The structure used in the studies is a 4.9m simply supported beam. The beam was modelled, as is shown in Figure 1, with 7 uniform beam elements having two degrees of freedom - one translation and one rotation - at each node. The vertical displacements at nodes 0 and 7 and the rotational displacement at node 0 were constrained to zero. The computer program used automatically reduces the finite element results to prescribed master coordinates using dynamic condensation [5] and computes the response spectra at these master coordinates using the first six modal frequencies and mode shapes at the selected locations. Hence, the output from the program consists of 'measured' frequency response functions at designated (measurement) locations. Nodes 1-6 were used as the 'measurement' points. The frequency response functions were then curve-fitted to obtain the modal
parameters. A typical curve-fit is shown in Figure 2.

The analysis was conducted for the undamaged beam to give the initial 'measured' response. Subsequent analysis were conducted to give the response spectra of the 'damaged' structure after inducing 'damage' in the beam. The damage in each case was simulated by reducing the EI value of a chosen element by 50%. Four different cases were considered: case 1 - no damage; case 2 - damage in element 4; case 3 - damage in element 2 and case 4 - damage in elements 2 and 4. Table 1 shows the natural frequencies, first six modes, for the different cases.

METHOD 1: EIGENPARAMETER METHOD

The eigenparameter method uses the changes in the eigenvectors to locate the damage in beam like structures. The theory of the method is given in reference [6]. To provide a common base of reference for the damaged and undamaged cases, the eigenvalues are used as normalisation factors for the eigenvectors. The eigenparameter for mode r is defined by Yuen [7] as the vector difference between the damaged and undamaged case of the mass orthonormalised eigenvector divided by the corresponding eigenvalue and is mathematically expressed as:

\[ (U^*)_r = \left( \frac{\Phi_{oe} - \Phi_{ue}}{\lambda_{oe}} \right) \times 10^6 \]  

(1)

The $10^6$ factor is merely to increase the values of the elements of the eigenparameter vector since normalisation by the eigenvalues leads to very small values. For the present study, $(U^*)_r$ in equation (1) is taken to be the displacement eigenparameter. Characteristics of the plots of $(U^*)_r$ against the measurement points are used to infer the damage position. For the simply supported beam, change in slope of the plots at the damage position is expected. Only the fundamental modes are used in this method.

METHOD 2: DIRECT ERROR MATRIX METHOD

For two matrices $[K_d]$ and $[K_u]$, the error matrix between them is defined as:

$$\Delta K = [K_d] - [K_u]$$  

(2)

In model updating procedures, $[K_d]$ represents the experimentally derived stiffness matrix while $[K_u]$ is the analytical matrix. In the current implementation, both matrices are 'experimentally' derived ($[K_d]$ - damaged stiffness matrix; $[K_u]$ - undamaged stiffness matrix) from the 'measured' response using pseudo inverse of the mode shape matrix and the orthogonality properties. The system matrices are given by [8]:

\[ [M] = [\Psi']^T [\Psi] [\Psi]^T \]  
\[ [K] = [\Psi']^T [\Phi] [\Psi]^T \]  
\[ [C] = [\Psi']^T [\Sigma \Psi] [\Psi]^T \]  

(3a)

(3b)

(3c)

This system identification procedure has been shown to be very stable and to yield system matrices from which the original modal parameters can be obtained [8,9]. This procedure was used to identify the damaged and undamaged system matrices in this and subsequent methods.

METHOD 3: MODIFIED STIFFNESS ERROR MATRIX METHOD

Many modified versions of the direct error matrix have been suggested. The version implemented here is that
proposed in [10] from which details of the method can be found. The relation \( \lim_{n \to \infty} [\Delta K]^n = 0 \) as \( n \to \infty \) is assumed and equation (2) becomes

\[
[\Delta K] \equiv [K_o][[K_o]^{-1} - [K_p][K_o]^{-1}[K_o]^{-1} [K_o]^{-1} \tag{4}
\]

Here, \([K_o]\) is obtained from equation (3a) while the pseudo-flexibility matrices \([K_o]^{-1}\) and \([K_p]^{-1}\) are obtained from the eigenvalues and eigenvectors of the undamaged and 'damaged' structures respectively. Thus,

\[ [K_o]^{-1} = [\Psi_o][\lambda_o^{-1}][\Psi_o] \tag{5a} \]
\[ [K_p]^{-1} = [\Psi_o][\lambda_o^{-1}][\Psi_o] \tag{5b} \]

**METHOD 4 : MATRIX CURSOR METHOD**

The matrix cursor method [11] is based on vector space theory and attempts to locate the region of error by identifying non-zero rows and columns of the matrix \([\Delta]\) defined as

\[
[\Delta] = 0.5(2[M_o][\Psi_o][\lambda_o^{-1}][\Psi_o][\Psi_o][\Psi_o][M_o] - [M_o][\Psi_o][\Psi_o][\Psi_o][\Psi_o][K_o]) \tag{6}
\]

In practice, the rows and columns of \([\Delta]\) associated with the damage region will contain elements much greater than other elements in the matrix. Equation (6) is a general error expression which identifies either (or both) mass or (and) stiffness errors without distinguishing between the two. Since the aim of the present investigation is to locate 'damage' in 'real' structures, it is not particularly relevant to distinguish between mass and stiffness errors as long as the damage area is adequately identified.

**RESULTS AND DISCUSSIONS**

To evaluate the effectiveness of the four methods, three damage situations were considered: (1) damage in element 4; (2) damage in element 2 and; (3) damage in element 4 after damage in element 2. Situations one and two respectively simulate damage in high and low stress regions of the beam while situation three is designed to test the ability of the methods to detect progressive deterioration. In a typical modal test, the number of modes measured will be less than the number of measurement locations. Hence, only the first three modes were used (in conjunction with the six 'measurement' points) in the error localisation schemes. Due to space limitation, only selected plots are presented.

In Figure 3, the eigenparameter plot correctly identifies the damage area to be around point 4 although it is not clear if the damage is in element 4 or 5 (see Figure 1). Figure 4 shows that the eigenparameter method could not locate the damage in element 2, which is in a region of low bending stress. However, for the case of damage in element 4 after damage in element 2, the method accurately locates the damage in element 4 (Figure 5).

The system matrices identified using equation (3) will depend on the particular response curve (hence measurement point) from which the modal matrices were obtained. This would be expected to affect the error matrices and the effect was noticed to an extent, on methods 2-4 although to a lesser extent for method 2. In Figures 6-12, the modal matrices were obtained from point 1 (see figure 1). In all damage cases, the direct error matrix method was unable to locate the damaged areas and ascribed large error values to points which are not physically realisable (Figures 6 and 7). Mannan and Richardson [9] have suggested that use of higher vibration modes could lead to improvement in the error localisation using this method. Since these higher modes are generally not
available from experimental results, it is inappropriate to justify the effectiveness of the method on them (higher modes).

Method 3 was equally unable to locate the 'damaged' regions. Figure 8 (damage situation 1) shows widespread error and the damaged area to be indistinguishable. For the third damage situation, Figure 9 shows that Points 1 and 2 were somehow identified as the damaged areas with other spurious locations. The matrix cursor method also identified spurious locations as the damaged areas (Figures 10 and 11). However, this method performed better than methods 2 and 3 in that the general damaged area could be seen. The results obtained when the response at points other than point 1 were used to identify the system matrices show similar trends to what has been described.

One curious observation with methods 2-4 is that if the connectivity of the measurements points is upheld, the damage localisation is improved. For example, since point 2 can only be physically connected to points 1 and 3, any entry in column 2 and row 2 for other points (4-6) would be ascribed the minimum value in the matrix. The result of such an exercise would be that the elements of the error (or any similar) matrix would only contain their true (as given by the formulation of the method used) values at positions that can be physically connected on the real structure. When this procedure was applied to the methods, the best results were obtained from method 4. Figure 12 is a repeat of Figure 10 but with the connectivity of the points enforced as described above. The improvement is obvious from a comparison of the two figures.

CONCLUSIONS

Four methods of error localisation, one using changes in the eigenparameters and the others utilising system identification and model updating procedures have been implemented. The effectiveness of the methods was tested by using data obtained from simulation of damage events in a simply supported beam. The results show the eigenparameter method to be the best for the type of structure considered although it was incapable of locating the damage in a lightly stressed zone. The matrix cursor method appeared to be the best of the three model updating methods and its performance was enhanced when the physical connectivity of the measurement points was enforced. The effectiveness of these three methods could have been affected by the system identification method used and this would be a subject for further investigations. The procedure adopted in this paper is quite attractive because no analytical model of the structure is required to locate the damage regions. Experimental results from modal testing of the structure are all that is needed.

ACKNOWLEDGEMENT

The first author is supported by a grant awarded by the University of Plymouth, England.

REFERENCES

2. Cawley, P. and Adams, R.D., "The Location of Defects in Structures from Measurements of


### Table 1. Natural Frequencies

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
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<td>Mode 1</td>
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</tr>
<tr>
<td>Mode 2</td>
<td>148.127</td>
</tr>
<tr>
<td>Mode 3</td>
<td>289.285</td>
</tr>
<tr>
<td>Mode 4</td>
<td>512.625</td>
</tr>
<tr>
<td>Mode 5</td>
<td>792.245</td>
</tr>
<tr>
<td>Mode 6</td>
<td>1184.970</td>
</tr>
</tbody>
</table>

**Figure 1**

**Figure 2**