NON-LINEAR VIBRATION ANALYSIS OF A ROBOT ARM

Michael Feldman
Faculty of Mechanical Engineering
Technion — Israel Institute of Technology
Haifa, 32000, Israel

Simon Brann
Faculty of Mechanical Engineering
Technion — Israel Institute of Technology
Haifa, 32000, Israel

ABSTRACT

The possibility of fault detection in structures from changes in their non-linear parameters is one of the actual questions of dynamic system monitoring. This study concentrates on signal processing techniques for non-linear system investigation, based on Hilbert transform and envelope and instantaneous frequency, which enables us to estimate system instantaneous dynamic parameters (natural frequencies, damping characteristics and their dependencies on a vibration amplitude and frequency) during free vibration analysis and different kinds of excitation of the system. The direct time domain techniques allow a direct extraction of the linear and non-linear systems parameters from the measured time signal of input and output. Experimental results of two robot arms testing according to the vibration signal analysis technique have been outlined in this paper. Using an impulse and also a sweep harmonic excitation from the motor (below-after each resonance) we get an input torque signal which together with output vibration signal was used for the modal system analysis. The obtained natural frequency and logarithmic decrement are dependent on the input torque level, so the robot system includes non-linearity both in the spring and damping.

1. INTRODUCTION

Recently a new technique of vibration analysis was presented in [1], [2], which enables us to estimate non-linear system spring and damping characteristics by using different kinds of input excitation. Unlike existing modal analysis methods, it uses an input (force) data and an output vibration of the system, together with the Hilbert transform technique in time domain. The method estimates a type and a value of non-linear spring and non-linear damping characteristics by extraction backbones and damping curves of vibration systems, so we are able to analyze obtained backbone and damping parameter vs amplitude dependency for each mode of vibration. This results to identification of existing non-linearity (backlash, dry friction etc.). The proposed method of force vibration analysis determines instantaneous modal parameters even if an input signal is a high sweep frequency quasi-harmonic or random signal. The proposed method opens the way for extraction of elastic and damping characteristics of machines running at nonstationary, including very high speed, and also for reduction of a non-linear system testing duration in several times. The technique is demonstrated both experimentally and by using non-linear vibration modeling.

2. NON-LINEAR FEATURES CALCULATION

The present method can be conveniently subdivided into a free vibration procedures FREEVIB and a force vibration procedures FORCEVIB.

2.1. FREEVIB PROCEDURES

In the FREEVIB technique [2], the measured output vibration signal is integrated or differentiated by an integrator or differentiator to three signals: acceleration, velocity and displacement. Thus each of these three signals is transformed by the Hilbert transform into conjugate signal. All of these six signals are used for direct algebraic calculation of instantaneous modal parameters of the test mechanical system in time domain. Obtained instantaneous modal parameters (natural frequency, damping coefficient, logarithmic decrement) together with vibration amplitude (envelope) and vibration frequency are recorded on two-axes recorders so to plot mechanical system backbone (so called skeleton curve, which is a dependency of vibration amplitude vs natural frequency) and damping dependencies (damping coefficient vs amplitude and log. decrement vs amplitude).

Instantaneous natural frequency and damping parameters are functions of time and can be determined at any point of the damped process. The total number of these points which map the free vibration is much greater than that of the peak points of the process. It opens the way for establishing non-linear relations between instantaneous functions and for using statistical processing procedures, making the analysis more precise. The proposed method for analysis of the machine vibration offers a way to direct plotting of the system skeleton curve, which includes modal frequency and non-linearity in spring characteristics, as well as dependencies between damping parameters and the amplitude, which contains modal damping together with non-linearity in friction. The method is suitable for efficient oscillatory system testing avoiding time-eating forced response analysis.

There are two ways of implementation of a free vibration process in the system tested: either switch off suddenly the exciter during vibration resonant sinusoidal testing, or use the shock excitation and a narrow band filter to get a response in the area around the natural frequency. The free vibration analysis means, that we deal only with vibration
when the excitation signal already ceased.

2.2. FORCEVIB PROCEDURES

In FORCEVIB technique an excitation signal (force or torque) is also used for vibration system modal parameters estimation [2]. It opens the way for modal analysis in the case of forced vibration and for a mechanical system mass value estimation. The measured input force signal from a transducer is transformed by the Hilbert transform into a conjugate signal. Then this complex force signal together with six components of a complex vibration signal is used for modal mass value algebraic calculation. The value of the mass is given by tangent of the slope angle of straight line for each linear single-degree-of-freedom system. Every mass of multi-degree-of-freedom system has its own value, and a form of a plot for mass calculation of such a complicated system would look like a broken line. In the case of multi-degree-of-freedom system it is necessary to use FORCEVIB procedures for each mode of the system separately.

Then the mass value is combined with the six component of the vibration signal for direct algebraic calculation of instantaneous modal parameters. Also obtained modal parameters are recorded on two-axes recorders so to extract the tested system backbones and damping dependencies. Thus using all of the components of the vibration signal, excitation signal, and their Hilbert transforms we can determine instantaneous modal parameters of mechanical system. The identification technique should be of value in many areas of mechanical oscillatory systems having various features of their non-linear behavior.

3. NON-LINEAR FEATURES CLASSIFICATION

In the particular case of a linear vibration system the instantaneous natural frequency and instantaneous damping coefficient or decrement do not vary in time. In general case of investigation of non-linear systems the instantaneous natural frequency and damping parameters become functions of the amplitude. In most cases if the system to be tested has non-linear elastic forces, the natural frequency will be decisively depend on the amplitude of vibrations. We present this departure from synchronism of the vibration in the form of a regression curve of the instantaneous amplitude on the instantaneous frequency which represents a kind of skeleton curve (or backbone) of the system under the test. Every typical non-linearity in spring (Duffing system, backlash or pretensioned system, bi-linear system, impact system etc.) has its unique form of skeleton curve. Several possible forms of \( A(\omega) \) were shown on [1].

Suggested analysis of the topography of the skeleton curve is essential for evaluation of the properties of the particular vibrating system, e.g., through reconstruction of any characteristics of the elastic forces. For example, a skeleton curve of the system with backlash is a monotonic increasing curve which has a trivial vertical line of linear system as an asymptote on the right side and cuts off a clearance value on the amplitude axis on the left.

3.1. SMALL AMPLITUDE NON-LINEAR BEHAVIOR

There are cases where vibration systems show their specific non-linear behavior only in a small amplitude range of vibration. Dynamic system with backlash is a typical example of such mechanical non-linear systems, because for large amplitude values it just operates like a linear one with a constant natural frequency. Only for small vibration amplitudes commensurable to a clearance value the system can display its non-linear properties when natural frequency decreases with decreasing of amplitude [1]. Mechanical system with preloaded (precompressed) restoring force is another example of non-linearity in small amplitude range. Really, around large vibration amplitudes natural frequency practically does not depend on vibration amplitude. Only for small amplitudes of vibration motion commensurable to a precompressed deformation natural frequency increases extremely [1].

Among vibration systems with non-linear damping characteristics also could be structures, which show its non-linearities only in a small amplitude range, for example, a system with Coulomb or dry friction, whose plot of logarithmic decrement vs vibration amplitude is a monotonic hyperbola (or backbone). Presence of dry friction together with viscous one means that only for small vibration amplitudes logarithmic decrement increases extremely.

3.2. LARGE AMPLITUDE NON-LINEAR BEHAVIOR

Most of all are known cases where non-linearity occurs in large-amplitude oscillations of elastic systems, for instance, non-linear spring element with hard or soft restoring force, or non-linear friction quadratic force. Whereas the amplitudes of vibrations are large that the occurrence of these spring or damping non-linearities cannot be ignored.

4. ROBOT ARM TEST FACILITY

In order to justify the proposed method, two experimental specimens of a robot system having non-linear transmissions were used. The first one consists of a robot arm, constructed at Technion using an industrial quality single degree of freedom arm of "Scorbot-ER VII" (wrist roll axis) with a harmonic drive transmission (flexible spline drive), powered by a 12V DC motor, and a shaft-encoder counter board for monitoring joint position. The test rig includes a special base for the robot arm supporting in the horizontal plane, coupled to a bed plate, a cantilever beam as the arm, and also a special mass, with changeable position along the arm. The second experimental specimen consists of another robot construction with planetary gear, also driven by DC motors.

The experimental program for the first stage concentrates on vibration analysis of a single joint of the robot [3]. It is implemented on the robot gear mechanism equipped with computer control of applied torque. Experimental measurements of signals from the robot system are based on a dynamic micro-excitation derived only from the robot motor itself. The robot motor is good for the dynamic system micro-excitation in the frequency range 1 - 50 Hz for its identification without an external artificial vibration exciter. A robot DC motor can be operated in the open loop mode by using of an analog output from the computer card. Precision excitation provides a measurable input of mechanical force into the robot structure which sets it into vibratory motion. Resulting structural vibration is typically registered by the accelerometer. The measurement process is carried out by the computer.
5. IMPULSE EXCITATION

Attempts to get a response from these two robots under impulse excitation have shown that the structure being under the test is a heavy damping system. In the range of first low natural frequencies the free motion practically loses its vibratory character. Only in the frequency range 30-50 Hz the system with harmonic drive presents free vibration and a proper graph of the motion is shown in Fig. 1. This acceleration-time curve was used according FREEVIB procedures [2] when a first and a last curve point corresponds to the beginning and to the end of each free vibration process due to its own impulse. The resultant backbone and decrement dependencies for four different impulses with different amplitudes are shown in Fig. 2. The figure shows that these backbones, which are practically coincided, contain information about system modal parameters. The third natural frequency of the system (without an additional mass) is equal to 35 Hz for high level of the vibration and it increases up to 40 Hz for low vibration amplitude. The logarithmic decrement value 0.6-0.8 aren’t changed with the excitation level changing. It should be noted that every mode has very large damping value so the traditional Fourier analysis doesn’t display natural frequency values precisely enough. The high damping in the robot system makes the FREEVIB procedures also not very effective. For such kind of a system better use a FORCEVIB technique [2].

6. SWEPT FREQUENCY EXCITATION

Swept frequency excitation in the range of frequency including several different natural frequencies of the robot system, create conditions where the forced frequency and each natural frequency is very close to each other. In these conditions input and output signals were used for modal parameters estimation of a corresponding normal mode shape on the base of FORCEVIB procedures. Fig. 3 shows the typical plot (1000 points in time) of swept frequency torque excitation (a) and of vibration response (acceleration) (b) around the first natural frequency of the robot system with the harmonic drive.

Fig. 4 shows an auxiliary plot for modal moment of inertia (mass) calculating. The moment of inertia of each vibration mode of the system is equal to tan of the slope angle of straight line for single-degree-of-freedom system. Thus using the excitation and vibration signals, their instantaneous characteristics according FORCEVIB procedures we have determined modal parameters of the robot system in the required frequency range. For example, Fig. 5, (a) shows the presence of the first natural frequency equal to 5 Hz. These results of the robot system identification obtained for a constant position of the robot, when modal parameters didn’t vary in time, are represented in the form of backbone and logarithmic decrement vs amplitude dependency. But in the case of system control motion we will represent modal parameters as function of time.

6.1. MODAL PARAMETERS MODULATION

By setting a constant voltage value to the motor it is possible to control robot in the open loop mode to achieve a constant speed rotation of the motor. Some experiments of
uniform velocity show that constant speed rotation of the robot with harmonic drive is accompanied by small natural pulsation of the motion. The frequency of the pulsation is directly proportional to the input motor voltage. This frequency of the harmonic pulsation can be associated with the rotational speed of a robot sub-element. Using signal from the encoder, the angular phase of pulsation could be presented as a sinusoid with an angle depending on the input shaft angular displacement.

Rotation of the motor results in both undamped natural frequency and logarithmic decrement modal parameters modulation. The most modulation takes place for the second mode of the vibration system with the harmonic drive. Figure 6 gives an overview of the modal parameters modulation during a fast speed of robot motion. Modal parameter modulation under different speed is a coherent function to the sinusoid pulsation function. This sinusoid pulsation (dash line in Fig. 6), presented by means of the encoder signal, also has additional small micro vibration (with a very small amplitude value $2 \times 10^{-4}$ m) exciting the system to extract the parameters in the frequency range around 24 Hz.

Instantaneous natural frequency and instantaneous logarithmic decrement change $180^\circ$ out of phase with each other. Since the natural frequency (depending on spring) reaches its maximum, the logarithmic decrement (depending on friction) is about its minimum. It means that during one half of the period of pulsation spring increases and friction decreases and during the second one on the contrary spring decreases and friction increases. A double amplitude of the natural frequency modulation is approximately 15%, that corresponds to one third of the spring constant changing. A double amplitude of logarithmic decrement modulation is about of 50%.

As far as modulation of modal parameters of the second mode, which is based on the harmonic drive elements, corresponds to its shaft rotation, it is naturally to decide that the harmonic drive causes the modulation of motion. It could be noted also, that harmonic drive has from 25 to 30% of the total number of teeth in contact at the same time. This is why modulation effect is not an influence of its teeth meshing.

In order to explain the reason of modulation better consider
the flexible spline deflection. Really, the rotating wave generator deforms and forces the flexible spline to move relative to rigid spline. As it takes place, the teeth of the flexible spline successively fall into the spaces between those of the rigid spline. But if the flexible spline has an irregular stiffness and/or friction contact with the wave generator, or if the wave generator has an eccentricity, the harmonic drive will produce a modulation during rotation. The modulation frequency depends on the speed of motor rotation and can be very close to some resonance frequencies of the system. Therefore flexible spline could be a potential source of errors of robot system.

7. DIAGNOSTICS OF NON-LINEARITIES

During the experimental investigation of two dynamic robot systems different sources of non-linearities were found.

7.1. PRELOADED SPRING

It is clear from Fig. 2 that the system with harmonic drive is subjected to a combination of preloaded spring and also of dry friction (not mention linear part of the system and viscous damping). The obtained undamped natural frequency is dependent on the input voltage level, i.e. backbone of the third mode (Fig. 7, a) shows that the natural frequency of the harmonic drive subsystem skew to the right for small amplitude of vibration motion. It indicates a presence of preloaded force and deformation in the system [1]. When the wave generator is removed the coaxial splines (rigid circular and a flexible spline) have a uniform clearance between them. As soon as it is put inside the flexible spline, the wave generator deforms the flexible spline radically, thereby giving it the shape of an ellipse. As a result, the teeth lying in the direction of the major axis of the ellipse mesh through the whole working depth with the teeth of the rigid spline, whereas a radial clearance is formed between the gears along the minor axis. Because of the natural preloading and the many teeth in contact there is no backlash in a harmonic drive, but there is a preloaded deflection. Obtained backbone (Fig. 7, a) could be used for a preload value estimation. For this purpose let's take an equation of backbone of a preloaded system [1]:

\[ A(\omega) = \frac{F_0 \omega_0}{k} \tan \frac{\pi \omega_0}{2 \omega_n} \]

where \( A \) is an amplitude of vibration, \( \omega_n \) - undamped natural frequency, \( F_0 \) - preloaded force value, \( k \) - linear spring constant, \( \omega_n \) - natural frequency of linear system when \( F_0 = 0 \).

Thus the preload deformation from Eq.(1) is \( \Delta = 0.36A_0 = 0.0113 \) [mm]. It is clear that the preloaded backbone is non-linear mainly at low amplitude of vibration.

7.2. BACKLASH

Fig. 8 shows the typical plot (1000 points in time) of swept frequency input voltage excitation (a) and of vibration response (acceleration) (b) around the first natural frequency (1.9 Hz) of the robot system with planetary gears. Thus exciting the robot by using input signal with decreased amplitude function (Fig. 8) shows that the obtained backbone around of a small vibration amplitude is non-linear and corresponds to the typical backlash backbone [1]. Note that backbone of the system with backlash is a monotonic increasing curve which has a trivial vertical line as an asymptote on the right (because of a constant natural frequency of a proper linear system without backlash). In this case around very small natural frequency obtained backbone shows the clearance value equal to \( 0.5 \times 10^{-3} \) m (Fig. 9, a). The damping characteristic of the first mode getting a typical hyperbole form shows that there is a dry friction in the system together with the viscous damping (Fig. 9, b). Linear viscous damping is operated in the system at high amplitude where decrement value is a constant (\( \delta \approx 1.5 \)). In the small amplitude range decrement decreases with the increasing amplitude.

7.3. DRY FRICTION

The obtained logarithmic decrement is also dependent on the input voltage level. The damping characteristic of the first mode getting a hyperbole form (Fig. 9, b) shows that there is a dry friction in the system together with the viscous
damping.
Linear viscous damping is operated in the system at high amplitude \( A > 7 \times 10^{-4}, [m] \) where decrement value is a constant \( \delta \approx 1.5, \text{Fig. 5, b} \). In the small amplitude range decrement decreases with the increasing amplitude. Considering the total frictional force as a sum of typical frictional elements (viscous and dry) we can determine partial logarithmic decrement value, concerning only dry friction (Fig. 5, b):

\[ \delta = 2.5 \text{ and } \delta = 1.5 \text{ at the vibration amplitude 2 and 3} \times 10^{-4} [m] \text{ correspondingly. Obtained damping dependence could be used for a dry friction force value estimation. For this purpose let's take a formula of instantaneous decrement as a function of amplitude of a system with dry friction [1]:} \]

\[ \delta = \frac{2\eta}{\omega^2 A_\phi} \]

where \( \eta \) is a friction coefficient, \( A_\phi \) - an angular amplitude of vibration, \( \omega = 2\pi 7.9 \text{ [rad/s]} \) - undamped natural frequency. There is a certain relation between friction coefficient \( \eta \) and friction force (torque) \( M \) value [1]:

\[ M = I\eta \]

where \( I \) is a moment of inertia. After calculation of the expression

\[ M = I\delta\omega^2 A_\phi/2 = \frac{0.28 \times 2.5 \times 49.6^2 \times 2 \times 10^{-4}}{2 \times 0.46} = 0.37 \]

we received the dry friction torque value \( M \approx 0.4 \text{ [Nm]} \), which is very close to the one, obtained by a special static test (Fig. 10).

8. CONCLUSION

The principles of a new vibration signal analysis technique based on the Hilbert transform have been outlined, and results of robot arm testing reported and discussed. The robot motor itself is good for the dynamic system micro excitation in the frequency range 1 - 50 Hz for it identification without an external artificial vibration exciter. Using a sweep harmonic excitation from the motor (below-after each resonance) we get an input torque signal which together with output vibration signal was used for the system non-linear modal analysis. The obtained natural frequency and logarithmic decrement are dependent on the input torque level, that the robot system includes non-linearity in the spring and damping. There is a large modal parameters' modulation \((30 \div 60\%)\) during the motor rotation with a constant moment of inertia. The robot system with planetary gears is characterized by backlash and dry friction. The harmonic drive is also a non-linear system with preloaded spring and dry friction elements through its flexible spline deflection due to an irregular stiffness and/or friction contact with the wave generator, or to an eccentricity of the wave generator.

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References

