FREQUENCY-DOMAIN BASED APPROACHES FOR DAMAGE DETECTION AND LOCALIZATION IN AERONAUTICAL STRUCTURES

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ABSTRACT. The modern maintenance criteria for the aeronautical structures are based upon to consider the fatigue life of structural components without landing them (“on-conditioning” criteria); indeed, damage detection and localization for aeronautical structures by the usual non destructive techniques (X-rays, eddy currents, ultrasound) are sometimes impossible on boarding or in-flight condition. For the last decade several methods based on the correlation between the existence and propagation of a crack with variations of the vibrational characteristics of the structures have been developed. The errors introduced by the numerical and experimental approaches and the relevance of the damping which is difficult to model, would indicate that the direct frequency-domain based methods (e.g., natural-frequency variations, FRF variations, ...) are promising approaches for damage analysis: indeed, recently it has been shown that the presence of the noise in the measurement can easily affect the capability of these methods. In this work comparisons and further developments on frequency-domain based methods to detect, localize, and quantify a structural damage, are presented. Natural-frequency variations and FRF variations approaches have been merged in one method in order to improve the robustness of the inverse-sensitivity algorithm with respect to the presence of the noise. A numerical simulation of the methods for a FE beam model and a FE plate model have been included in the analysis.

1. INTRODUCTION

Methods using variations of vibrational characteristics of the structures for the damage location and also for its quantitative evaluation, have been developed for the last decade and almost often correlated with the updating techniques of the analytical model: some reviews and classifications of these methods are available in Refs. [1, 2]. Most of the methods are based upon the error-matrix method (see, e.g., Ref. [3]), i.e., on the measurement of modal characteristics to evaluate the spatial modification (see e.g., Ref. [4] for some applications). Other methods are based on the minimization of the residual forces in the frequency-domain model due to the variation of the spatial model (Ref. [5]) or in the evaluation of the residual forces in the free-response balance equation (Ref. [6]): close to the previous methods are those based on the variation of the curvatures of the shape-mode functions (Refs. [7, 10]) although the measurements of these are particularly difficult to realize in practice. Furthermore, statistic approaches in using the available experimental data (Ref. [11]) of the use of the minimum rank perturbation theory (Refs. [12, 14]) are promising methods in the perspective of no use of a reference analytical model in the damage analysis.

Modern maintenance criteria for the aeronautical structures are actually based upon to monitorize the life of structural components without landing them (“on-conditioning” criteria) i.e., on boarding: then, the use of a portable frequency domain analyzer with attached devices (sensors and actuators) could more easily satisfy the above requests. In this paper two frequency-domain based approaches to locate, and quantify the damage in a structure are considered and
a further development of them is also proposed. These methods belong to the group of the correlation methods (Ref. [2]), for the applications of interest here, the error is represented by a system variation given by a damage due to a lack in stiffness.1 Probably the simplest approach of the two is that introduced by Stnbbs, Refs. [14, 18], since it can be reduced on considering only the variations of the natural frequencies of the system in the solution of the inverse problem. As it will be shown in the next Section, this method may be not satisfactory for the accuracy as it supposes a first order approximation for the evaluation of the damage: moreover, frequency-change information only may not be sufficient to locate the damage position since similar defects at different positions may cause the same amount of frequency changes. On the other hand, this approach may be more robust with respect to the presence of the noise in the available data. The second method considered here is that proposed in Refs. [19, 20], which is based on an exact formulation for the sensitivity equation (other exact frequency-domain approaches are recently introduced in Ref. [21]); however, also this approach can exhibit some difficulties whenever some errors (given, e.g., by the presence of the noise) are present in real FRF data (see for instance, Refs. [22, 23]).

The objective of this paper is to analyze the performance of the previous two approaches emphasizing the similarities and then introducing a possibility of a mixed method: in the next Section the two formulations of Refs. [14] and [19, 20] will be presented with the objective to emphasize the similarity of the procedure: then, it also allows to compact the two methods in order to obtain a more robust algorithm for the damage detection and quantification. In the Section 3 numerical results and comparisons between the methods are shown. Some considerations and conclusions are finally presented in Section 4.

2. FRF FORMULATIONS FOR DAMAGE DETECTION

Let us consider the spatial model of the free vibrations of an undamped dynamical system

\[ [M] \{\ddot{z}\} + [K] \{z\} = 0 \]  

(1)

We shall suppose in the following that the effects of the damage are essentially a local lack of stiffness of the system: this variation produces a change in the FRF characterization, i.e., either in the natural frequencies (the poles of the FRF matrix, Ref. [14, 15]), or in the values of this matrix function for each value \( \omega \) of the frequency (Ref. [22, 20]). The inverse-sensitivity approaches based on these two philosophies are briefly presented in the next two subsections with some variations in the original notation in order to show, in the subsequent subsection, the possibility to merge the two approaches to get the maximum information from either the variation in abscissae (natural frequencies) or in ordinates (FRF values) of the system response function.

2.1 Natural Frequency variation approach

Following the results given, for example, in Refs. [14] and [15], the stiffness variation given by a damage yields

\[ \{\phi^{\text{und}}(r)\} [AK] \{\phi^{\text{und}}(r)\} = D_r \quad r = 1, 2, \ldots, m \]  

(2)

where the superscript ‘und’ (dam) will mean in the following undamaged (damaged), \( \phi^{\text{und}} \) is the \( r \)th mode of the undamaged structure and

\[ D_r := \Delta \omega^2_n \]  

(3)

in the hypothesis of undamped system and neglecting the higher-order contribution of the mode-shape variations: the latter hypothesis is considered here since, as mentioned in Section 1, only variations in the frequency-domain data are supposed available in the damage analysis. Consequently, the present inverse-sensitivity formulation could be considered less accurate with respect to that presented in the next subsection.

In order to allow the spatial localization of the damage, a FE reference model is to be typically introduced at this stage of the procedure: indicating with \( \{T(j)\} \) the FE locator matrix such as for the \( j \)th element-d.o.f. vector \( \{x(j)\} = [T(j)] \{x\} \), one has

\[ [AK] = \sum_{j} N_e \{T(j)^T\} \left[ \Delta K(j) \right] \{T(j)\} \]  

(4)

where \( N_e \) is the number of the finite elements, and \( \Delta K(j) \) is the stiffness-matrix variation due to the damage for the \( j \)th element. Moreover, it is assumed that each element matrix is function of a damage parameter,\(^2\) i.e., \( [K(j)] = [K(j)(\alpha_j)] \), and then, for

\(^1\)In both the original formulations also a mass variation can be taken into account but it would represent a less realistic simulation of the damage.

\(^2\)An arbitrary number of damage parameter is assumed on the original formulation, Refs. [14], but for the sake of clarity only one damage parameter is considered here.
a first order approximation, it may be also assumed
\[
\begin{bmatrix} \Delta K^{(j)} \end{bmatrix} = \frac{\partial \begin{bmatrix} K^{(j)} \end{bmatrix}}{\partial \alpha_j} \bigg|_{\alpha_j=0} \Delta \alpha_j
\] (5)

Furthermore, if the damage is supposed due to a (negative) variation of the Young modulus, for a classical FE model of an isotropic beam or plate element, one has
\[
\begin{bmatrix} \Delta K^{(j)} \end{bmatrix} = \frac{\partial \begin{bmatrix} K^{(j)} \end{bmatrix}}{\partial E_j} \bigg|_{E_j=0} \Delta E_j = \begin{bmatrix} K^{(j)} \end{bmatrix} p_j
\] (6)
where \( p_j := \Delta E_j/E_j \) is the dimensionless damage parameter. Finally, using the Eqs. 2, 4, and 6, one obtains
\[
\begin{bmatrix} S_1 \end{bmatrix} \{ p \} = \{ D \}
\] (7)
with
\[
(\begin{bmatrix} S_1 \end{bmatrix})_{ij} := \begin{bmatrix} \phi^{(i)\text{und}} \end{bmatrix} \begin{bmatrix} T^{(j)} \end{bmatrix} \begin{bmatrix} K^{(j)} \end{bmatrix} \begin{bmatrix} T^{(i)} \end{bmatrix} \begin{bmatrix} \phi^{(i)\text{und}} \end{bmatrix}
\] (8)

where a transposition matrix has been considered. Let us consider the \( i \)th column (or row) of the previous equation
\[
\{ \Delta \alpha(\omega;_{(i)}) \} = -\begin{bmatrix} \alpha(\omega)\text{und} \end{bmatrix} \begin{bmatrix} \Delta Z(\omega) \end{bmatrix} \begin{bmatrix} \alpha(\omega)_{\text{dam}} \end{bmatrix}
\] (11)
Supposing that the damage is due to a variation in stiffness, \( \Delta Z = \begin{bmatrix} \Delta K \end{bmatrix} \), and the same kind of finite element reference model given by Eqs. 4 and 6, Eq. 11 yields
\[
\begin{bmatrix} \alpha(\omega)\text{und} \end{bmatrix} \sum_j^{N_\omega} \begin{bmatrix} T^{(j)} \end{bmatrix} \begin{bmatrix} K^{(j)} \end{bmatrix} \begin{bmatrix} \alpha(\omega)_{\text{dam}} \end{bmatrix} p_j = \{ \Delta \alpha(\omega;_{(i)}) \}
\] (12)

or \( \{ F(\omega;i) \} \{ p \} = \{ \Delta \alpha(\omega;_{(i)}) \} \). The previous equation may be written for \( N_\omega \) different values of the frequency \( \omega \) and also for \( N_\omega \) rows of the FRF matrix: then it may be rewritten for the general case as
\[
\begin{bmatrix} S_2 \end{bmatrix} \{ p \} = \{ \Delta \Lambda(\omega) \}
\] (13)
where
\[
\begin{bmatrix} \begin{bmatrix} F(\omega_1)_{(1)} \end{bmatrix} \\
\begin{bmatrix} F(\omega_2)_{(1)} \end{bmatrix} \\
\vdots \\
\begin{bmatrix} F(\omega_{N_\omega})_{(1)} \end{bmatrix} \\
\end{bmatrix} \begin{bmatrix} \alpha(\omega)_{\text{und}} \end{bmatrix} = \begin{bmatrix} \Delta \alpha(\omega)_{(1)} \\
\Delta \alpha(\omega)_{(2)} \\
\vdots \\
\Delta \alpha(\omega)_{(N_\omega)} \\
\end{bmatrix}
\] (14)
Also the above equation can be solved, for example, in a least-square sense using a SVD.

2.2 Frequency Response Function variation approach

Considering the matricial identity \( [A] + [B] = [A]^{-1} - [A][B][A]^{-1} \) and setting \( [A] = \begin{bmatrix} Z^{\text{und}}(\omega) \end{bmatrix} \) and \( [A] + [B] = \begin{bmatrix} Z^{\text{dam}}(\omega) \end{bmatrix} \) (the dynamic stiffness matrix of undamaged and damaged system respectively), one has, Refs. [19, 22],
\[
\begin{bmatrix} Z^{\text{dam}}(\omega) \end{bmatrix}^{-1} = \begin{bmatrix} Z^{\text{und}}(\omega) \end{bmatrix}^{-1} - [Z^{\text{und}}(\omega)]^{-1} [Z^{\text{und}}(\omega)] - [Z^{\text{und}}(\omega)]^{-1} \begin{bmatrix} Z^{\text{und}}(\omega) \end{bmatrix}^{-1}
\] (9)
or, in term of the dynamic compliance \( \alpha(\omega) := [Z(\omega)]^{-1} \),
\[
\begin{bmatrix} \Delta \alpha(\omega) \end{bmatrix} := \begin{bmatrix} \alpha(\omega)_{\text{dam}} \end{bmatrix} - \begin{bmatrix} \alpha(\omega)\text{und} \end{bmatrix} = \begin{bmatrix} \alpha(\omega)\text{und} \end{bmatrix} [AZ(\omega)] \begin{bmatrix} \alpha(\omega)_{\text{dam}} \end{bmatrix}
\] (10)

Also the above equation can be solved, for example, in a least-square sense using a SVD.

2.3 A mixed approach

The mixed approach proposed here consists on merging the Eqs. 7 and 13, i.e., on using both the informations of the variations in abscissae and ordinates of
\[^3\]The general formulation, Refs. [19, 20], is given also in terms of variations in mass.
the system-response function. Then, the final resolving equation is

\[
\begin{bmatrix}
S_1 \\
S_2
\end{bmatrix}
\{p\} = \begin{bmatrix}
D \\
\Delta A(\omega)
\end{bmatrix}
\]

(14)

As it will be shown in the numerical results section, solving the linear Eq. 14 (in a least square sense) would make a merging of the characteristics of the two approaches and then an improvement of the algorithm robustness. An iterative procedure identical to that presented at the end of subsection 2.1 is also used for the convergence of the algorithm.

In the next Section numerical results and comments in using the approach presented above will be shown.

3. NUMERICAL RESULTS

The first test case considered is a cantilever aluminium beam (\(E = 70 \text{ GPa}, \rho = 2700 \text{ kg/m}^3\)) with length \(l = 0.48 \text{ m}\) and rectangular section with dimensions \(a = 0.015 \text{ m}\) and \(h = 0.003 \text{ m}\); the beam has been discretized using a standard FE method (see, e.g., [24]). 10 finite elements are considered for the following results and the damage is simulated by a lack of stiffness (Young modulus) of the 10% in the 6th element. Figure 1 shows the variation in the element \(\alpha(\omega)_{11,11}\) of the FRF matrix due to the assumed damage.

First, let us consider the application of the FRF-variation method. Figure 2 depicts the location and the quantitative evaluation of the damage using 512 frequency samples, with constant step, considering a maximum frequency \(f_{\text{max}} = 1000 \text{ Hz}\) (i.e., 6 modes), and only one row of the FRF matrix (see Eq. 13): it has to be emphasized that identical figures can be obtained i) considering 128, 256, 512, and 1024 frequency samples (other parameters fixed); ii) \(f_{\text{max}} = 30, 300, 500, 1100, 2000 \text{ Hz}\), respectively 1, 3, 4, 6, 8 modes assumed (other parameters fixed); iii) number of rows of the FRF matrix equal to 1, 2, and 3 (other parameters fixed). It could be argued that the FRF-variation method is not particularly sensitive to these parameters: then, in the following results \(f_{\text{max}} = 1000 \text{ Hz}\); 512 frequency samples, and only one row of the FRF matrix are considered. Next, the presence of the noise in the data has been studied considering \(\alpha(\omega)_{\text{noise}}\) with noise given by the expression

\[
\alpha_{\text{noise}} = \alpha (1 + R_{\text{rand}}f_{\text{rnd}}) + R_{\text{rand}}f_{\text{rnd}} f_{\text{bg}}
\]

(15)

where \(R_{\text{rand}}\) is a random number generator between -0.5 and 0.5. \(f_{\text{rnd}}\) is a factor representing a percent of noise depending on the amplitude of the signal, and \(f_{\text{bg}}\) is a factor representing a percent of background noise (in all the applications \(f_{\text{bg}} = 0.0001\)); note that this data with noise affect either the sensitivity matrix or the right-hand side of Eq. 13. Figure 3 depicts the same entry of the FRF matrix for \(f_{\text{rnd}} = 0.01\), i.e., a noise level of 1%; Figure 4 shows the same results of Fig. 2 in presence of noise for \(f_{\text{rnd}} = 0.001, 0.01, 0.1, 0.2\), i.e., a noise level of 0.1%, 1%, 10%, and 20% respectively. Furthermore, Fig. 5 shows that taking several averages of the data, the results cannot be improved. Next, the same analysis has been done considering the frequency-variation method. First, in Fig. 6 the sensitivity of the method to the number of frequency-variations or modes (equations) used in the analysis is presented: note that the number of the assumed frequency variations can affect the damage location, i.e., the modal information is more influential here with respect to the previous approach, as the FRF information take into account all the modal information even if a very limited frequency window is considered. Moreover, note also that here the number of equations cannot be less than the number of the damage parameters (10) in order to obtain a unique solution by solving Eq. 7 (in a least square sense). In the Fig. 7 the presence of the noise is studied considering for the natural frequency variation data a noise expression similar to that given by Eq. 15: note that for the natural frequencies (noise in abscissa variable of the FRF) the noise percent is reasonable assumed to be equal to one tenth of that present in the corresponding ordinate value, Eq. 15 (FRF value); note also that the method gives acceptable results up to a 0.1% level of noise in the abscissa natural frequency data, and, correspondingly (see Fig. 4), the FRF-variation method gives from 1% of noise (ordinates) on, completely chaotic results. Furthermore, also in this case the presence of the average in the data can not improve the results. Finally, in Figs. 8-9 the proposed mixed method is considered: Figure 8 depicts the same type of results considering 10 natural-frequency variations, \(S_1\)-portion of Eq. 14, and 512 sampled frequencies for the FRF matrix, \(S_2\)-portion of Eq. 14 \((f_{\text{max}} = 1000 \text{ Hz}, 1 \text{ row of the FRF matrix assumed})\): again, acceptable results can be obtained up to a noise level of 1% in ordinates (0.1% in abscissae) but, as shown in Fig. 9, the same result can be obtained considerably reducing the modal information, i.e., only using 128 frequency samples for the FRF and 5 frequency variations.

Next test case considered is a cantilever aluminium flexural plate \((E = 70 \text{ GPa}, \rho = 2700 \text{ kg/m}^3)\) with rectangular plant with edges \(a = 2 \text{ m}\) (built-in at \(y = 0\),
free at \( y = b \) and \( b = 4 \) m (free-free), and thickness \( \tau = 0.01 \) m: the plate has been discretized using a standard FE method (MZC quadrilateral element, 3 grid d.o.f., see e.g., [24]) with \( 4 \times 10 \) elements in the \( a \times b \) domain (150 global d.o.f.). The damage is simulated by a lack of stiffness (Young modulus) of the 10% in the 27th element, i.e., the 3rd rows (x direction) and the 7th column (y direction) of the FE mesh. Figure 10 shows the variation in the element \( a(\omega)_{15,16} \) of the FRF matrix due to the assumed damage. Note that in the present case the range of the frequencies of interest is lower with respect to the beam case. Figure 11 depicts the location and the quantitative evaluation of the damage using the FRF-variation method with 128 frequency samples (constant step), maximum frequency \( f_{\text{max}} = 20 \) Hz (9 modes), and only one row of the FRF matrix\(^4\) (i.e., \( N_{\text{dof}} \times N_{\text{freq}} = 150 \times 128 = 19200 \) equations): specifically, Figs. 11-a, 11-b, 11-c, and 11-d, correspond to a noise level \( f_{\text{noise}} = 0, 0.001, 0.01, 0.1 \) respectively; the same results are presented in Fig. 12 considering, in the same range of frequencies, only 40 frequency samples (constant steps).

Next, the same analysis has been done with the frequency-variation method. The first case in Figs. 13-14 shows the damage location and evaluation for 40 and 90 assumed modes respectively with the same levels of noise considered above: note that also in this case, 40 frequency variations (i.e., equations) is the minimum number of data in order to solve Eq. 13.

Finally, the mixed approach is applied: Figure 15 shows the damage location and evaluation obtained considering only 5 frequency variations and the variation of the first row of the FRF matrix sampled in 128 values of the frequencies (constant frequencies step) up to a maximum value of the frequency \( f_{\text{max}} = 20 \) Hz (5 \( + 19200 \) equations, Eq. 14). It has to be emphasized that similar results are obtained, Fig. 16, considering only 20 frequency variation and 40 sampled data for the first row of the FRF matrix i.e., \( 20 + 40 \times 150 = 6020 \) equations for system 14. As shown, this procedure do not require so much information for the higher natural frequencies as the frequency-variation method and, moreover, it seems more robust with respect to the FRF-variation method considering the used amount of information.

4 This parameters choice has the same motivations given above for the beam case.

4. CONCLUDING REMARKS

In this work comparisons and further developments on frequency-domain based methods to detect, localize, and quantify a structural damage, are presented: these are methods that could satisfy the requests of the modern maintenance criteria for the aeronautical structures which are based on monitoring the life of structural components without landimg them. The presented applications and comparisons between the method of Ref. [14], that of Ref. [19], and that presented here, suggest some conclusions and recommendations for future works.

\( i \). Natural-frequency variation method (abscissa variation) may require a too high number of equations to detect a damage also for a simple structure discretized with a coarse mesh and it can sometimes not locate the damage position since similar defects at different positions may cause the same amount of frequency change.

\( ii \). FRF variation method (ordinate variation) approach typically needs only one row of the FRF matrix and around one hundred of frequency samples to obtain precise damage localization and quantitative evaluation but it is particularly sensitive to the presence of the noise.

\( iii \). The results of both approaches in presence of noise seem not to be improved using averaging of the available data.

\( iv \). In the mixed approach the number of the frequency-variation equations can be also less than the number of damage parameters and a consistent modal information (i.e., in abscissa and ordinate) can be taken into account.

\( v \). The method might be applied also as a first step of an updating procedure.

\( vi \). The meaning of the mixed approach (as the FRF can be composed by an arbitrary number of equations) should be physically explained.

\( vii \). The behavior of the method on real system or simulated damped system should be investigated.

\( viii \). No original analytical model would be used in the future: the models considered here are deeply dependent on a reference FE model.

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6. REFERENCES


Figure 11: FRF-variation method. 128 fr samples

Figure 12: FRF-variation method. 40 fr samples
Figure 13: natural-fr. variation method 40 fr. assumed

Figure 14: natural-fr. variation method 90 fr. assumed

Figure 15: mixed method 5 fr. variation/l28 fr. samples

Figure 16: mixed method 20 fr. variation/40 fr. samples