SELECTION OF CONTROL TOPOLOGY FOR DECENTRALISED MULTI-SENSOR MULTI-ACTUATOR ACTIVE CONTROL

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ABSTRACT. Active vibration control is using increasingly large numbers of sensors and actuators to achieve ever-improving results in the control of distributed systems. As the number of actuators and sensors increases, computational effort for control purposes increases. With full use of modal filtering, and assuming that no adaption will take place, computation effort is proportional to the square of the numbers of sensors and actuators. As the number of actuators and sensors grows, so too does the frequency-range over which it is sensible to attempt active control and the time available for control calculations is therefore shrinking. Notwithstanding the remarkable rate at which processor speeds continue to increase, it is evident that full multi-input multi-output control cannot continue to be applied for increasing numbers of sensors and actuators. This paper addresses the issue of how best to constrain vibration controllers so that fast control is implemented in several small multi-input multi-output groups. The Block Relative Gain is used as an interaction measure to try to determine which sensors and actuators should belong to which groups.

1. INTRODUCTION
In the design of a smart structure the placement of the actuators and sensors is vital for good controllability and observability, and therefore good performance of the control system. The measures of controllability and observability are well understood [1, 2], in structural dynamics techniques for choosing actuator and sensor locations are also well established [3, 4, 5]. Control actuator placement using optimisation techniques is possible [for example, 6], although for systems with a large number of degrees of freedom this may be impractical.

In systems with a large number of sensors and actuators it is difficult to implement a single multi-input multi-output (MIMO) controller, sometimes because of communication problems and often because of the limited computation power available. If the system has defined sub-systems, often spatially separated, then decentralised control may be applied. In decentralised control the local actuator signals are derived from the local sensor measurements, thus reducing the order of the local control problem. The two principal methods of achieving the required decentralisation are to use high gain local controllers with subsystem interactions treated as disturbances, or to use a state observer to estimate the required external states [7, 8]. The success of these methods is often improved if the system states are grouped so that subsystem interaction is minimised. Interaction measures, such as the block relative gain, may be help in this choice of control system topology [9, 10]. West-Vukovich et al. [11] considered the decentralised control of large flexible space structures, but did not explicitly consider the topology of the controller. This paper considers the use of interaction measures in smart structures.

2. THE BLOCK RELATIVE GAIN
The Block Relative Gain (BRG)[9, 10] is a device to determine the level of interaction between a given set of actuators and sensors and the remainder of the actuators and sensors present in a system. The method extends the concept of the Relative Gain Array (RGA) [12] which is valid for single input single output loops. The BRG has the advantages that it is only dependent on the open loop transfer function of the plant and is independent of the scaling of the sensors and actuators. Consider a system whose inputs and
outputs are partitioned into two groups, so that the system is modelled in the standard open loop transfer function, $G$, may be defined by

$$ y = Gu $$

or

$$ \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} $$

(2)

where $u$ is the input and $y$ is the output. We would like to introduce a decentralised controller that takes the output $y_1$ and produces a control force $u_1$, with no knowledge of $y_2$, and similarly for the second subsystem. The left and right BRGs, $BRG_L$ and $BRG_R$, are then defined as

$$ BRG_L = G_{11} \left[ G^{-1} \right]_{11} $$

(3)

and

$$ BRG_R = \left[ G^{-1} \right]_{11} G_{11} $$

(4)

$$ = \left[ I - G_{12} G_{22}^{-1} G_{21} G_{11}^{-1} \right]^{-1} $$

where $[ ]_{11}$ denotes that part of the matrix corresponding to $u_1$ and $y_1$. Obviously if there is no interaction then either $G_{12}$ or $G_{21}$ are zero, and the BRG matrices are the identity matrix. As the level of interaction increases then the BRG matrices become very much different from the identity matrix.

Manousiouthakis et al. [10], recommended looking at the diagonal elements or the eigenvalues of the BRG matrices, to determine how far these matrices are from the identity. These measures should be 1 for systems with no interaction, and have the great advantage that they are independent of the scaling of the inputs and outputs.

### 3. CONTROLLER DESIGN USING LQR

The linear quadratic regulator (LQR) approach will be used to obtain the optimal controller for our system. Although this is not necessarily the best controller for this application it will serve to demonstrate the controller topology selection. Luzzato [13] described an optimal controller that was very similar to the LQR controller described here. The LQR controller also has the advantage that the cost function may be easily split into separate parts concerned with the sub-systems, as shown in the next section. This makes the comparison between a full MIMO controller and the decentralised controllers easier.

Suppose our system is modelled in the standard control notation

$$ \dot{x} = Ax + Bu $$

$$ y = Cx $$

(5)

where $x$ is the state vector, $y$ is the output or measurement vector and $u$ is the vector of force inputs. The LQR approach produces a controller

$$ u = -Kx $$

(6)

such that the cost function

$$ J = \int (y^T Q_r y + u^T R_r u) \, dt $$

(7)

is minimised, where $Q_r$ and $R_r$ are weighting matrices that define the relative importance of minimising particular outputs or inputs. $Q_r$ must be positive semi-definite and $R_r$ must be positive definite, and usually these matrices are taken to be diagonal. The derivation of the gain matrix $K$ is standard [1, 14] and a function exists in the MATLAB Control Toolbox to calculate it. Notice that the controller requires the state, $x$, to calculate the input vector, $u$, in Equation (6). Generally this is not available as we only measure the outputs, $y$. We therefore have to estimate the state from the output measurements. This is implemented using the following state estimator

$$ \dot{x} = A\dot{x} + Bu + L(y - C\hat{x}) $$

(8)

where $\dot{x}$ is the estimate of $x$ and $L$ is Kalman gain matrix, obtained by optimisation based on the process and measurement noise covariance matrices $Q_e$ and $R_e$. Once again the derivation of the gain matrix $L$ is standard [1, 14] and a function exists in the MATLAB Control Toolbox to calculate it. The feedback control law, Equation (7), may now be correctly stated as

$$ u = -K\hat{x} $$

(9)

The dynamics of the system and the state estimator may be combined to give the following equations incorporating both the controller and the state estimator.

$$ \begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A & -BK \\ LC & A - LC - BK \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} $$

(10)

$$ y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} $$

(11)
4. DECENTRALISED CONTROL

We now partition the controller into $n_c$ parts. Each controller measures the local outputs, $y_i$, and produces a local force vector, $u_i$, based only on the local outputs. The system equations of motion, Equation (5), may now be written as

$$\dot{x} = \begin{bmatrix} A & -B_1 K_1 \\ L_1 C_1 & A - L_1 C_1 - B_1 K_1 & \vdots & \vdots \\ L_n C_n & 0 & \cdots & A - L_n C_n - B_n K_n \end{bmatrix} \begin{bmatrix} x \\ \hat{x}_1 \\ \vdots \\ \hat{x}_{n_c} \end{bmatrix}$$

(11)

$$y_i = C_i x$$ \text{ for } i = 1, \ldots, n_c

(12)

An LQR controller may be designed for each input/output pair $(u_i, y_i)$ assuming that the other force inputs are zero. If the weighting matrices are block diagonal, so that for example

$$Q_i = \text{block diag} \left( Q_{ri} \right)$$

and similarly for the other weighting matrices, then the cost function given by Equation (7) may be split,

$$J = \sum_{i=1}^{n_c} J_i$$

(13)

where $J_i = \int y_i^T Q_{ri} y_i + u_i^T R_{ri} u_i \, dt$. We may then obtain the control law, equivalent to Equation (9), as

$$u_i = -K_i \hat{x}_i$$

(14)

where $\hat{x}_i$ is the state estimator based on the local measurements $y_i$. This state estimator is given by the equivalent of Equation (8),

$$\hat{x}_i = A \hat{x}_i + B_i u_i + L_i (y_i - C_i \hat{x}_i)$$

(15)

Combining all the controllers and estimators gives the combined controlled system and state estimator as Equation (15).
assume that the velocity may be measured, although in practice a Luenberger observer or Kalman filter would be used [14].

6. AN EXAMPLE PROBLEM

We will now consider the bending vibration of an aluminum cantilever plate which has six piezo-ceramic patches bonded to the surface. The motivation for this example arises from the NASA experiments on flutter suppression which use a tapered plate containing many actuators [16]. The piezo-ceramic patches are used as both sensors and actuators, the so called self sensing actuator [17]. The use of collocated sensors and actuators in this way reduces the possibility of instabilities due to feedback [18]. Figure 1 shows the plate which measures 0.8 m long, 0.3 m wide and is 3 mm thick. The plate is modelled using 24 square plate elements and each piezo-ceramic patch is assumed to completely cover a single element. The excitation of the plate by the patches is modelled very simply, namely that a voltage applied to the piezo-ceramic is assumed to produce moments at each node of the element. This assumption is motivated by the comments of Crawley and de Luis [19] for a perfectly bonded actuator. These moments are assumed to be proportional to the applied voltage and for the purposes of illustration are taken to be 1 Nm/V. Similarly rotations at the nodes are assumed to produce outputs when the patches are used as sensors, and the sensitivity is taken to be 1 V/rad. Although a number of different actuator and sensor positions could be considered, elements 1, 2, 3, 16, 17 and 18 are assumed to include the piezo-ceramic patches for the purposes of illustration. The positions of these elements are highlighted in Figure 1. This choice of actuator locations consists of those elements that apply the largest force to the first six modes. Proportional damping is assumed, with the coefficients of mass and stiffness being 0.5 and 0.0001 respectively. For the design and analysis of the control system the model is reduced by retaining only the lower 10 modes of the system, leading to a 20 state plant model.

The natural frequencies and damping ratios of the plate are given in Table 1, and Figure 2 shows the FRFs for the uncontrolled case, where the force input is taken to be at the corner of the plate, and the outputs are the piezo-ceramic sensor output at element 1.

To gauge the success of our controllers we will assume that the plate is disturbed by an unit impulse, or impact, at one of the free corners, shown in Figure 1. The response at the 6 piezo-ceramic sensors to the unit impulse decays slowly because of the light damping in the structure. Now suppose that we had sufficient computational power to be able to apply a full MIMO controller utilising all six inputs and six outputs. The weighting matrices in the LQR design problem, Q, and R, and the weighting matrices in the state estimator design Qe, and Re, are all taken to be multiples of the identity matrix. The Q matrices are a multiple of 1000, and the R matrices the identity.

The resulting controller produces a response due the unit impulse at the free corner of the plate that now decays more rapidly. Figure 2 shows the FRF for the controlled system with force input at the free corner and the response being the sensor output at element 1, and highlights the increase in damping that the controller has produced. A measure of the effectiveness of the controller is the sum of squares of the responses for this particular excitation. Table 1 shows the values of this sum for no controller and the full 6 input / 6 output controller.

Suppose that we wish to group the actuators into two groups of three actuators and sensors, and to apply control to each set independently. Thus it might be better to choose the topology so that the interactions between the two groups are minimised. Figure 3 shows the maximum eigenvalues of the BRG matrices for two possible partitions of the sensors/actuators that produce the 'most' and the 'least' interactions. Figure 4 shows the maximum diagonal elements for the same actuator / sensor groups. There are a total of 10 possible partitions of the 6 sensor/actuators into 2 groups of 3. Even the best partition still contains considerable interactions, as might be expected from our system. For each partition, a pair of 3 input / 3 output LQR controllers and state estimators were designed using the equivalent weighting matrices to the full 6 input / 6 output case (the only difference is that the matrices are 3x3 rather than 6x6). The time responses to the unit impulse are almost indistinguishible from the full 6 input / 6 output controller case, although there is some degradation in the response, as shown in Table 2. Table 2 also shows the control effort required. Notice that there is little difference between the two partitions of the sensors and actuators, and indeed little difference between the decentralised control and the full 6 input / 6 output controller.

Table 2 shows the results for the modal filter controller. In the case of the full 6 input / 6 output controller, 5% damping has been added to modes 1 through 5 and 7. Mode 6 is missed because all of the first 3 torsional modes are unobservable. The controller is designed using a reduced 6 degree of freedom model, and then tested on the 10 degree of
freedom model. The pair of 3 input / 3 output controllers are designed in a similar way, except that the 5% damping is added to the first 3 modes only, for both decentralised controllers. The resulting controller shows that the topology with the least interaction, involving actuator groups (1, 2, 3) and (16, 17, 18), produces a larger response, and also uses more actuator effort. Both decentralised controllers produce a lower response, but this is mainly because they are providing control over a more restricted frequency range.

Often it is desirable to minimise the interactions between subsystems in a decentralised controller. In this case, little is gained from choosing the controller partition in this way. There appears to be two reasons for this. First, the interactions for all partitions are very high, leading to the conclusion that all partitions will have a similar performance. Second, the plate modes are fully controllable and observable using any three elements as sensors and actuators, leading to a considerable redundancy in the controller. Thus the performance of the decentralised controller is very similar to that of the full 6 input / 6 output controller. This is highlighted in Table 3 which shows the condition number of the controllability grammian for a number of sets of actuators. The results for the observability grammians will be the same since the sensors and actuators are collocated. In our example, choosing the control topology that gives the least interaction is not the optimum choice from a controllability and observability considerations.

7. CONCLUSIONS

This paper has considered the grouping of actuators in the control of continuous structures, and considered the merits of using interaction measures to help in the choice of controller topology. It appears that typical smart structures have actuators that interact strongly. Control is successful, in that the response is considerably reduced, with a full controller and also any pair of decentralised controllers. The controller topology does matter to some extent, although the interaction between actuator groups is of little use in determining a optimal topology. Considerations of controllability and observability appear more robust. More actuators, controlled via decentralised controllers are very useful in applications involving piezo-ceramic actuators, as they are able to exert more force on the structure.

Perhaps it should be no surprise that the sub-system interaction is not important. If we recall the two approaches to decentralised control from the introduction, then either the controller should have a high gain so that the other sub-systems appear as disturbances, or the states for the other sub-systems should be estimated. In our application the states for all the sub-systems are the same, and most conveniently given as the response at the lower modes. Therefore we automatically estimate the states for all the sub-systems based on the local output. The minimisation of sub-system interaction is more applicable to the first case, where ‘disturbances’ from the other sub-systems must be rejected. But our application is more like the second case, where all the states are estimated.

8. REFERENCES

13. Luzzato, E., Active Protection of Domains of a Vibrating Structure by Using Optimal Control Theory:


<table>
<thead>
<tr>
<th>Natural Frequency (Hz)</th>
<th>Damping Ratio (%)</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>3.938</td>
</tr>
<tr>
<td>2</td>
<td>21.87</td>
</tr>
<tr>
<td>3</td>
<td>24.70</td>
</tr>
<tr>
<td>4</td>
<td>69.36</td>
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<tr>
<td>5</td>
<td>69.78</td>
</tr>
<tr>
<td>6</td>
<td>127.2</td>
</tr>
<tr>
<td>7</td>
<td>137.7</td>
</tr>
<tr>
<td>8</td>
<td>186.7</td>
</tr>
<tr>
<td>9</td>
<td>200.2</td>
</tr>
<tr>
<td>10</td>
<td>220.7</td>
</tr>
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Table 1. Natural Frequencies and Damping Ratios for the Plate Example

<table>
<thead>
<tr>
<th>Actuator Group</th>
<th>Norm of the Controllability Gramian</th>
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<tbody>
<tr>
<td>1, 2, 3, 16, 18</td>
<td>3.39x10^7</td>
</tr>
<tr>
<td>1, 2, 3</td>
<td>3.39x10^9</td>
</tr>
<tr>
<td>16, 17, 18</td>
<td>1.16x10^8</td>
</tr>
<tr>
<td>1, 3, 17</td>
<td>2.92x10^7</td>
</tr>
<tr>
<td>2, 16, 18</td>
<td>8.54x10^7</td>
</tr>
</tbody>
</table>

Table 3. Controllability using Different Actuator Groups

<table>
<thead>
<tr>
<th>Case</th>
<th>LQR Controller</th>
<th>Modal Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>no control</td>
<td>(\int u^T u , dt)</td>
<td>max(u)</td>
</tr>
<tr>
<td>6x6 controller</td>
<td>0.590</td>
<td>0.904</td>
</tr>
<tr>
<td>actuator groups</td>
<td>0.597</td>
<td>0.936</td>
</tr>
<tr>
<td>(1,2,3) and (16,17,18)</td>
<td>0.500</td>
<td>0.840</td>
</tr>
</tbody>
</table>

Table 2. Performance of the Full and Decentralised Controllers

Figure 1. Plate Example. The Shaded Elements Contain Actuators/Sensors
Figure 2. FRF for Plate Example: Force is at a Free Corner, Response is for a Sensor at Element 1

Figure 3. The BRG Interaction Measures for 2 Partitions of the Actuator/Sensor Set

Maximum Eigenvalues

Figure 4. The BRG Interaction Measures for 2 Partitions of the Actuator/Sensor Set

Maximum Diagonal Elements