ABSTRACT

A lot of advanced signal processing and analysis methods are available for digitized vibration signals with a fixed sampling frequency. On the other hand, specialized hard- and firmware has been designed and commercialized to synchronize the sampling frequency during a measurement with a second but related event. A well known mechanical application is the extraction of “order-related” phenomena of engine vibrations, based on the measurement of the rotation speed of one of its components.

This paper describes the background and use of well-established digital signal processing techniques like digital resampling and filtering to convert signals from one domain into another, using a known or measured relation between both domains (e.g. between the time domain and the engine cycle domain). It particularly illustrates how phenomena are difficult to analyze or interpret in one domain, but become clear and obvious in the other domain.

Theoretical background and principle

The key component in the transformation procedure of measured data from one domain into the other is the accurate relation between both domains. Although the technique discussed in this paper applies to any combination of two domains, this section focuses by way of example and for clarity on the transformation from the time domain into the engine angle domain. Figure 1 shows the steps involved.

In this case, the starting point is the evolution of the crank shaft angle in time. This function can easily be measured in practical circumstances by mounting a tachometer on the shaft. The measured tacho pulse train can then be converted directly to the angular position, or indirectly to an RPM/time function, to be integrated to obtain the angle as a function of time.

The transformation process is in principle very straightforward. When one needs the value of the measured signal at a particular angle position \( \alpha_i \), the \( a(t) \) function is consulted to find out the corresponding time instant. The value of the measured time signal at that instant is then taken over as the value for the desired angle position. This process is repeated for every value in the angle domain.

Depending on the resolution of the original signal and the relation between both domains, interpolation may be required for both the time and the angle axes. This is the phase where advanced digital signal processing techniques play a crucial role. In order to maintain the dynamic nature of the signal, the spectral contents must absolutely be preserved as much as possible during the transformation. This means that appropriate digital filters must guard the spectral contents of the signal during the up-sampling, interpolation, and down-sampling steps indicated in Figure 1. An adequate implementation in a standard software routine requires correct design and application of low-pass finite impulse response (FIR) filters with enough suppression in the stop-band, a low ripple in the pass-band, and yet optimal speed performance for acceptable computing times.

Remark that one and the same relation between both domains allows forth and back transformation of the data. This is particularly useful when manipulation is easier in the derived domain than in the original domain. The section on “Order domain processing” will illustrate this.

The presented approach does no longer require dedicated hardware devices to track some phenomenon and adapt the sampling frequency synchronously to avoid digital signal processing errors such as leakage and aliasing. Instead, standard hardware tools can be used, and a software resampling is performed off-line, based on a reliably measured tracking signal to drive the conversion.

Using this 2-step approach, a measurement set-up can be designed for general purposes. During the data acquisition phase, tracking transducers (such as tacho-meters or velocity probes) need to be mounted
on the relevant components defining the domains for further analysis afterwards. This makes it easier to set up the experiment, and offers the possibility to extract more information from a given data set, even if the measurement set-up itself does no longer exist. As such, it considerably shortens the experiment.

**Quasi-stationary data analysis**

A first application is the analysis of measurement signals under quasi-stationary operating conditions.

A tachometer is mounted on the spindle of a grinding stone, and the tacho pulse train is measured together with an acceleration signal on the work piece. Figure 2 shows the RPM evolution. During the first part, the RPM oscillates between 833 and 835 RPM. As the grinding stone looses contact with the work piece, the rotation speed first increases to 842 RPM, and then drops to around 829 RPM. The tacho signal is sample at 20 kHz.

To illustrate the effect of slight RPM variations, the tacho signal itself is analyzed during the first second of the process. This is a quasi-stationary period with a change of less than 2 RPM on an average of 834 RPM.

The discrete Fourier transform (DFT) of this signal is shown in Figure 3. The order related spectral components are sharp for the first orders, but become smeared out for higher frequencies. This is caused by the small RPM variations, leading to leakage errors in the frequency domain. For applications that need to investigate higher order phenomena (such as gear box analysis for example), such smearing makes it very difficult to discriminate order from resonance components.

The same data section was transformed to the angle domain with a resolution of 2 degrees. The DFT spectrum is shown in Figure 4. It now shows amplitude versus order instead of frequency. Orders show up needle sharp until the 80th order! Of course, any present resonance phenomenon would have been smeared out in this domain. This shows that the frequency and order domain representations are complementary to one another. Depending on the need, data can be interpreted in the domain most suited for analysis.

**RPM fluctuation removal**

The previous data set illustrates how slight RPM variations introduce analysis errors in the frequency domain. The transformation to the angle domain can artificially correct for such errors under the assumption that the small RPM fluctuations do not introduce additional dynamic behavior to the measured signals.

Let us assume one measures a signal at a rotation speed of 6000 ± 60 RPM (100 ± 1 Hz) with a sampling frequency of 100 kHz (At = 1e-5 sec). This means that 1000 data samples are observed per rotation. The angle/time relation will then appear as an almost perfectly straight line, with a small ripple due to the RPM fluctuation. This relation is then used to transform the signal into the angle domain with a resolution of 0.36 degrees without loss of information. If one then artificially modifies the sampling resolution from 0.36 degrees back to 1e-5 seconds, the new data series will appear to be measured at a perfectly stationary rotation speed of 6000 RPM, the RPM fluctuations being accounted for during the transformation. Note that this back-substitution is in fact only a trick, and only allowed if the dynamic behavior of the signal does not vary significantly between 5090 and 6010 RPM. Nevertheless, the “correction” may prove to be useful for statistical analysis, as discussed in the following section.

**Synchronous averaging**

For synchronous averaging, it is essential that repetitive phenomena occur at the very same instant for the different signal sections that are averaged. Using the synchronous resampling technique, the data can be transformed into that particular domain in which the phenomena are indeed repetitive.

An important practical application for synchronous averaging is the analysis of the dynamics inside a combustion engine cylinder. Because of the influence of small RPM variations as discussed above, it becomes very difficult to analyze such signals in the time domain. However, when the data are transformed into the crank shaft angle domain, the same phenomena will appear at approximately the same angle position, e.g. the spark ignition, or the opening and closing of a valve. Such phenomena are best analyzed after averaging over a number of combustion cycles to remove random errors or signal components not correlated with that cycle. Synchronous averaging is illustrated on the tacho signal from which the RPM of Figure 2 is computed. One period in the time domain at an average RPM of 834 and a sampling frequency of 20 kHz corresponds to 1439 data samples. With a resolution of 2 degrees, this corresponds with 180 data samples in the angle domain.

Figure 5 shows in both domains the original signals, overlaid with the averaged signal over the first second. Further experiments have shown that the angle domain average over the entire 2.5 seconds is not even influenced by the RPM-transient introduced when the grinding stone looses contact.
Engine cycle domain editing

Once transformed into the angle domain, measurements on a" engine cylinder are readily available for interpretation and analysis.

Statistical analysis on a sufficient number of engine cycles is useful to determine for example the average angle position of ignition and valve opening or closing. The average duration of the expansion and compression cycle, or between openings of different valves, or between the valve closing and ignition points, etc...

Next to these analysis procedures, it becomes feasible to actively influence the signals by editing. To investigate the influence of removing the impact of a closing valve for example, becomes so much easier and more feasible because one can automate the editing process once that particular event occurs in every cycle at a fixed position.

To complete the circle, the modified data can be transformed back again from the engine cycle into the time domain. This allows the user for example to evaluate the effect of such editing operations on the engine’s sound quality by listening to the modified signals. Back transformation is also discussed in the following section.

Order domain analysis

In the same way as the Fourier transform presents the contents of time domain data in the frequency domain, its converts angle domain data to the order domain. Just as something that happens twice every second has a frequency of 2 Hz, something that occurs twice every cycle is related to order 2.

Figure 6 depicts an" acceleration signal measured at 20 kHz on a" engine during a steady and slow run-up from 1500 to 5000 RPM in about one and a half minute. Figure 7 shows a map with the evolution of the frequency spectra with RPM. The order related phenomena become clearly visible as straight lines, where order 4 is dominant.

This signal was transformed to the angle domain with a resolution of 0.5 degrees. The Fourier spectra evolution with RPM is depicted in Figure 8. This map shows orders as vertical lines (just as fixed resonances would show up as vertical lines in Figure 7). To show the wealth of possibilities of analyzing the signal in the order domain, the angle domain series is run through a regular 10th order IIR Butterworth filter. This filter is positioned around order 4, and has a width of ±0.1 order with a suppression of at least 90 dB. Figure 9 shows the frequency domain map (cf. Figure 7) of the filtered signal, where the 4th order has disappeared.

By way of comparison, the 4th order was also removed by running a sharp Kalman filter [2] on the original time domain data (Figure 10). This process removes harmonic components with a frequency that can be driven by a" external event. In this case, the center frequency of the Kalman filter equals 4 times the fundamental rotation frequency, derived from the RPM/time function. The Kalman filter runs considerably faster than the transformation back and forth to the angle domain, but the latter approach offers all capabilities and flexibilities of digital filtering in the order domain. One could for example easily attenuate the lower orders with a certain controllable factor, “sing a” IIR or FIR low-pass filter.

Figure 10 shows that the 4th order has not completely disappeared from the signal for higher RPMs. This may be due to inaccuracies in the RPM/time function that causes a sharp filter to miss the order component partly.

Removing Doppler effects

As stated in the introduction, the transformation process is valid between any 2 domains that are related in a known or measurable way. To illustrate this, the synchronous resampling technique is used to remove the well-known Doppler effect from sound data recorded at a fixed position while a passenger car passes by. This results in a compression of the sound waves (resulting in higher frequencies) while the car approaches the receiver, and their expansion (resulting in lower frequencies) once the car has passed the receiver. This situation is sketched in Figure 11.

Two time scales are involved in the measurements. The first time scale $T_e$ is the one of the vehicle that emits the sound. The second time scale $T_r$ is the one of the fixed receiver. The relation between both time scales is derived from Figure 11:

$$T_r = T_e + \frac{\mid d(T_e) \mid}{c_o}$$

where $c_o$ is the speed of sound in air (343 m/s), and $d(T_e)$ is the instantaneous distance between the receiver and the emitter, composed as follows ($v$ is the vehicle’s velocity):

$$d(T_e) = \sqrt{D^2 + (a - \int_e^e v) \cdot c_o}$$

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Relation (1) is shown in Figure 12 for 2 different values of the distance D between the receiver and the vehicle’s trajectory.

Measurements were carried out according to the SAE standard J986 [3]. In this case, the distance D equals 15 m, and the Doppler effect is reasonably small. Figure 13 shows a frequency/RPM map of the received signal, where the 4th order frequencies do not line up as expected. Before the pass-by point, the frequencies are higher. Once passed, the frequencies are lower.

Using Eq. (1), the receiver signal is transformed from the $T_r$ domain to the $T_e$ domain, and a similar map is drawn in Figure 14. The orders are now aligned.

**Conclusion**

A technique for off-line synchronous resampling was described and applied for quasi-stationary signal analysis, RPM fluctuation correction, synchronous averaging, engine cycle domain editing, order domain analysis, and Doppler effect removal.

Although the described technique is not a fundamental breakthrough in terms of new technology, its value lies in the integrated use of advanced digital signal processing techniques (interpolation, resampling, filtering) each tuned to the application and the circumstances. This approach opens up new applications for measurement data acquired with standard and thus relatively cheap data acquisition hardware systems, and brings the flexibility of well established DSP techniques to areas where these could not be applied successfully before.

**References**


Figure 3: Spectral Analysis of Tacho Signal shows Smearing

Figure 4: Spectral Analysis of Angle-Domain Tacho Signal is Leakage Free

Figure 5: Original (_) and Averaged (... or ++++) Tacho Signals in Time Domain (top) and Angle Domain (bottom)
Figure 6: Vibration Signal during Engine Run-up

Figure 7: Campbell Diagram shows Dominant 4th Order

Figure 8: Order Map of Angle-Domain Data
Figure 9: Campbell Diagram after Removal of 4th Order in the Angle Domain

Figure 10: Campbell Diagram after Kalman Filtering of 4th Order in the Time Domain

Figure 11: Pass-by Noise Measurement Set-up
Figure 12: Relation between Emitter Time ($T_e$) and Receiver Time ($T_r$) Scales

Figure 13: Contour Map shows Doppler Effect

Figure 14: Contour Map after Compensation for Doppler Effect