MODEL REDUCTION USING GUYAN, IRS, AND DYNAMIC METHODS

Christopher C. Flanigan
Manager, Advanced Test and Analysis
SDRC Operations, Inc.
11995 El Camino Real, Suite 200
San Diego, California 92130 USA

ABSTRACT

Many methods exist to reduce mass and stiffness matrices of a finite element model to a test-analysis model (TAM) whose degrees of freedom correspond to modal survey accelerometer locations. This paper reviews the Guyan, Improved Reduced System (IRS), and Dynamic reduction methods. Strengths and weaknesses of each method are discussed.

NOMENCLATURE

Abbreviations

DOF	degree of freedom
FEA	finite element analysis
FEM	finite element model
IRS	improved reduced system
TAM	test-analysis model

Matrices

I	identity matrix
K	stiffness matrix
M	mass matrix
P	applied force vector
T	transformation matrix
U	displacements
λ	eigenvalues
ϕ	mode shapes

Subscripts

a	accelerometer set
f	free set (a + o)
o	omitted set

INTRODUCTION

A basic objective of a modal survey is to verify that the finite element model (FEM) of a structure is sufficiently accurate to predict the structure's response to operating environments. A modal survey measures the natural frequencies and mode shapes of the structure for use in model verification. In general, the FEM will have many more degrees of freedom (DOF) than the test configuration will have accelerometers. In order to compare the FEM with the test results directly, a reduced representation called a Test-Analysis Model (TAM) must be generated. The degrees of freedom of the TAM will correspond one-for-one with accelerometers in the modal survey test configuration.

The development of a TAM serves several major functions. The selection of TAM DOF optimizes the test measurements and excitation locations. The reduced mass matrix provides an ability to calculate on-site orthogonality checks of the test modes. Finally, the TAM enables a quantitative comparison of the accuracy of the FEM during posttest correlation activities in the form of orthogonality and cross-orthogonality checks. All of these tasks require an accurate reduction of the FEM mass and stiffness matrices to the TAM DOF.

Many methods for reducing FEM matrices are currently in use. This paper examines three reduction approaches: Guyan [1], Improved Reduction System [2], and Dynamic [3]. These reduction methods differ in accuracy, ease of use, and computation resources required. This paper reviews the theory for these three

reduction methods and the use of these methods for mode shape expansion.

MATRIX REDUCTION METHODS

Matrix reduction procedures for test-analysis models are based on transformation methods of the form:

$$B = T^T A T \tag{1}$$

where A = original matrix
B = new matrix
T = transformation matrix

The major challenge for TAM matrix reduction methods is the large matrix size difference between the FEM and the TAM. The number of DOF in a finite element model is usually very large (10,000 to 1,000,000 DOF). However, the number of accelerometers in a modal survey test is usually only 10 to 500. The TAM reduction method must be able to accurately estimate or "guess" the motion of the FEM DOF using a limited set of "known" values at the accelerometer locations. Since the ratio between "guessed" versus "known" DOF is so large, the "guessing" procedure must be very precise in order to accurately reduce the FEM matrices to the TAM degrees of freedom. The interpolation shapes used to "guess" the motion of the noninstrumented DOF are the key to the accuracy of TAM matrix reduction methods examined in this paper.

GUYAN REDUCTION

The simplest TAM procedure uses the Guyan reduction method [1]. This approach assumes that the interpolation shapes can be calculated using the FEM stiffness matrix to solve a static solution of the form:

$$\begin{bmatrix} K_{oo} & K_{oa} \\ K_{ao} & \overline{K}_{aa} \end{bmatrix} \begin{Bmatrix} U_o \\ U_a \end{Bmatrix} = \begin{Bmatrix} P_o \\ P_a \end{Bmatrix} \tag{2}$$

The overbar notation for K_{aa} in (2) is to distinguish that this matrix is the a-set partition of K_{ff} rather than the reduced a-set stiffness matrix.

Assuming that the loads on the o-set DOF are negligible ($P_o = 0$), the upper partition of (2) can be solved:

$$U_o = -K_{oo}^{-1} K_{oa} U_a \tag{3}$$

The transformation matrix from the FEM DOF to the TAM DOF is:

$$\begin{Bmatrix} U_o \\ U_a \end{Bmatrix} = \begin{bmatrix} -K_{oo}^{-1} K_{oa} \\ I_{aa} \end{bmatrix} U_a \tag{4}$$

or

$$T_{Guyan} = \begin{bmatrix} -K_{oo}^{-1} K_{oa} \\ I_{aa} \end{bmatrix} \tag{5}$$

The reduced stiffness and mass matrices can be formed using the original FEM matrices and the transformation matrix T:

$$K_{aa} = T^T K_{ff} T \qquad M_{aa} = T^T M_{ff} T \tag{6}$$

IRS REDUCTION

The Improved Reduced System (IRS) method developed by O'Callahan [2] improves upon the Guyan reduction by including a first order estimate for mass effects in the development of the transformation matrix. The first step in the IRS method is identical to the Guyan reduction. The next step uses the static transformation matrix to estimate the mass effects of the omitted DOF.

The basis for the IRS reduction method is the standard eigenvalue equation:

$$K_{ff} \phi_f = M_{ff} \phi_f \lambda \tag{7}$$

Using Guyan reduction (5, 6), (7) can be reduced to the a-set DOF and solved:

$$K_{aa} \tilde{\phi}_a = M_{aa} \tilde{\phi}_a \tilde{\lambda} \tag{8}$$

Note that, due to the mass matrix approximation of the Guyan reduction, the mode shapes and frequencies from (7) and (8) will not be identical. The degree of difference between the two solutions will depend on the a-set DOF.

The f-set approximate mode shapes can be recovered using (8), (4), and (5):

$$\tilde{\phi}_f = T_{Guyan} \tilde{\phi}_a \tag{9}$$

The key feature of the IRS method is the development of a correction term for modal analysis similar to the correction term used by the Guyan reduction for static analysis. An "inertia" force term can be formed by combining (2), (8), and (9):

$$K_{ff}\,\tilde{\phi}_f = P_f = M_{ff}\,\tilde{\phi}_f\,\tilde{\lambda} \qquad (10)$$

The dynamic correction term similar to the static correction term can be written as:

$$\phi_f^{*} = K_{ff}^{-1} M_{ff}\, T_{Guyan}\,\tilde{\phi}_a\,\tilde{\lambda} \qquad (11)$$

Improved mode shapes can be written using (9) and (11):

$$\hat{\phi}_f = T_{Guyan}\,\tilde{\phi}_a + K_{ff}^{-1} M_{ff}\, T_{Guyan}\,\tilde{\phi}_a\,\tilde{\lambda} \qquad (12)$$

(8) can be rewritten as:

$$M_{aa}^{-1} K_{aa}\,\tilde{\phi}_a = \tilde{\phi}_a\,\tilde{\lambda} \qquad (13)$$

Substituting (13) into (12),

$$\hat{\phi}_f = T_{Guyan}\,\tilde{\phi}_a + K_{ff}^{-1} M_{ff}\, T_{Guyan}\, M_{aa}^{-1} K_{aa}\,\tilde{\phi}_a \qquad (14)$$

Rearranging terms in (14), the final transformation matrix used by the IRS reduction method can be written as:

$$T_{IRS} = \begin{bmatrix} G_{Guyan} + G_{IRS} \\ I \end{bmatrix} \qquad (15)$$

where

$$G_{Guyan} = -K_{oo}^{-1} K_{oa} \qquad (16)$$

$$G_{IRS} = K_{oo}^{-1}\left[M_{oa} + M_{oo}\, G_{Static}\right] M_{aa}^{-1} K_{aa} \qquad (17)$$

The stiffness and mass matrices are reduced to the TAM DOF using the IRS transformation matrix (15) and the transformation equation (6).

DYNAMIC REDUCTION

Another approach to improve on the Guyan reduction method is called Dynamic reduction. This method begins with the eigenvalue equation:

$$\begin{bmatrix} K_{oo} & K_{oa} \\ K_{ao} & K_{aa} \end{bmatrix} \begin{Bmatrix} \phi_o \\ \phi_a \end{Bmatrix}^i - \lambda^i \begin{bmatrix} M_{oo} & M_{oa} \\ M_{ao} & M_{aa} \end{bmatrix} \begin{Bmatrix} \phi_o \\ \phi_a \end{Bmatrix}^i = 0 \qquad (18)$$

Equation (18) could be solved using techniques similar to those used in the Guyan reduction method to calculate a transformation matrix relating the o-set DOF to the a-set DOF. However, the transformation matrix would be unique for each mode.

The alternative proposed by Paz [3] is to use a user-specified constant value Λ in place of the eigenvalue in (18). The transformation matrix calculated using Dynamic Reduction can be calculated by solving the upper partition of (18):

$$T_{DynRed} = \begin{bmatrix} -\left[K_{oo} - \Lambda M_{oo}\right]^{-1}\left[K_{oa} - \Lambda M_{oa}\right] \\ I_{aa} \end{bmatrix} \qquad (19)$$

COMPARISON OF REDUCTION METHODS

The Guyan, IRS, and Dynamic reduction methods have strengths and weaknesses. Some aspects of the methods are discussed in the following paragraphs. Additional discussion of TAM reduction and expansion is presented in [4] and [5].

Guyan Reduction. The major advantages of Guyan reduction are that it is computationally efficient and easy to implement. Guyan reduction is a standard option in many commercial FEA programs including NASTRAN and SDRC I-DEAS™. The method has been used for many years and is widely accepted in the testing community. Finally, Guyan reduction is sufficiently accurate for many systems as long as the accelerometer set is well selected.

The major weakness of Guyan reduction is that it does not explicitly account for mass effects at omitted DOF. This can lead to reduced accuracy if the o-set mass is significant or if the accelerometer set is poorly selected or inadequate. The Guyan reduction method is unacceptable for systems with high mass/stiffness ratios.

IRS Reduction. IRS reduction is not difficult to implement. For example, the IRS reduction method can be easily performed in NASTRAN using a rigid format alter [6, 7]. The method is

generally more accurate than Guyan reduction since a first order correction is included for mass effects at the omitted DOF.

IRS reduction has weaknesses. As shown by Gordis [8], IRS reduction can produce inaccurate results if the modes of the omitted DOF (restrained at the a-set DOF) approach the frequency range of the a-set DOF. The method requires additional computation resources. Finally, the IRS method has not yet been widely used, and the method should still be used with caution. Some success has been reported for selected systems such as the NASA Langley Research Center ten bay truss for control-structure interaction studies [9].

Dynamic Reduction. The primary advantage of Dynamic reduction is that it explicitly includes an approximation for mass effects. While the user must select a value for the "effective eigenvalue," any reasonable value is probably better than the Guyan reduction approach that assumes an eigenvalue of zero and neglects mass effects in determining the transformation matrix.

The weaknesses of the Dynamic reduction method are similar to those of the IRS method. Dynamic reduction is more computationally intensive and requires modification to commercial FEA codes. The user must supply a value for the "effective eigenvalue," and it is not clear what is the best value for this constant. The method may have accuracy limitations similar to the IRS method if the a-set DOF are poorly selected or inadequate. Finally, the IRS method has not yet been widely used, and the method should still be used with caution.

MODE SHAPE EXPANSION

The transformation matrix developed for each reduction method can also be used to expand the test modes from the accelerometer locations to the FEM DOF:

$$\phi_f = T\phi_a \qquad (20)$$

A question that often arises is whether it is better to reduce the mass and stiffness matrices to the TAM a-set DOF or to expand the test mode shapes to the FEM DOF. If the transformation matrix is used consistently for reduction or expansion, the numerical results will be identical. For practicality, it is usually more convenient to reduce the mass and stiffness matrix prior to the test. This provides a check as to the adequacy of the accelerometer DOF and reduces the volume of data that must be stored on the test data reduction computer.

A variation of the dynamic reduction method can sometimes provide unique capabilities in mode shape expansion. The upper partition of (18) can be written as:

$$\phi_o^{\,i} = -\left[K_{oo} - \lambda^i M_{oo}\right]\left[K_{oa} - \lambda^i M_{oa}\right]\phi_a^{\,i} \qquad (21)$$

For mode shape expansion, this approach has the advantage of explicitly using the available test data (frequencies and mode shapes) and FEM mass and stiffness matrices to provide a best estimate of the test mode shapes expanded to the FEM DOF. The method can be numerically intensive since (21) must be solved uniquely for each mode. However, this is usually not a major factor using modern computers. The Dynamic expansion method has been shown to dramatically improve orthogonality and cross-orthogonality results for selected problems [10, 11].

CONCLUSIONS

There are many methods for reducing FEM mass and stiffness matrices to the accelerometer DOF in a Test-Analysis model. Guyan reduction is the most widely used but least accurate method. Greater accuracy can often be obtained using the IRS and Dynamic reduction methods. However, these more advanced methods can encounter numerical problems if the accelerometer set is inadequate. In general, Guyan reduction should be used due to simplicity and greater experience base. The more advanced methods can be used when the Guyan reduction approach is inadequate. Dynamic Expansion can be employed in special cases to provide further accuracy in FEM/test orthogonality and cross-orthogonality calculations.

REFERENCES

1. Guyan, R.J., "Reduction of Stiffness and Mass Matrices," AIAA Journal, Vol. 3, February, 1965.

2. O'Callahan, J., "A Procedure for an Improved Reduced System (IRS) Model," 7th International Modal Analysis Conference, January, 1989.

3. Paz, M., "Dynamic Condensation," AIAA Journal, pp. 724-727, 1984.

4. Friswell, M.I. and Mottershead, J.E., Finite Element Model Updating in Structural Dynamics, Kluwer Academic Publishers, ISBN 0-7923-3431-0, 1995.

5. Freed, A.M., Flanigan, C.C., "A Comparison of Test-Analysis Model Reduction Methods," 8th International Modal Analysis Conference, January, 1990.

6. Flanigan, C., "Implementation of the IRS Dynamic Reduction Method in MSC/NASTRAN," MSC/NASTRAN World Users Conference, Los Angeles, CA, Mar., 1990.

7. Flanigan, C., "Development of the IRS Component Dynamic Reduction Method for Substructure Analysis," AIAA-91-1056, AIAA 32nd Structures, Structural Dynamics, and Materials Conference, April, 1991.

8. Gordis, J.H., "An Analysis of the Improved Reduced System (IRS) Model Reduction Procedure," 10th International Modal Analysis Conference, February, 1992.

9. Tran, M.K., "Evaluation of Reduced Model Concepts for the Ten Bay CSI Truss Modal Test," 11th International Modal Analysis Conference, 1993.

10. Flanigan, C., and J. Freymiller, "Using Dynamic Expansion to Improve Test-Analysis Correlation," 12th International Modal Analysis Conference, Honolulu, Hawaii, Jan. 31 - Feb. 3, 1994.

11. Botos, C.D., and Flanigan, C., "Improvement of the Centaur Modal Survey Test Correlation using Dynamic Expansion," 15th International Modal Analysis Conference, Orlando, Florida, February 3-6, 1997.