STRUCTURAL HEALTH MONITORING USING FREQUENCY RESPONSE FUNCTIONS AND SPARSE MEASUREMENTS

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ABSTRACT. The accuracy of damage detection techniques that are based on using vibration response measurements depends on the number of measurements that are taken over the surface of the structure. This paper examines the accuracy of locating structural damage using the damage vector method and a sparse distribution of sensors over the structure. An analytical beam model is used to compare the accuracy of locating damage using partial and full measurements. It is shown that damage can be located with a spatial resolution equal to the distance between the sensors on the structure. The technique requires measurement of rotational and translation frequency response functions, and that the excitation force be applied at a point on the structure where the response is measured. Experimentation is underway to determine the feasibility of the technique.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>FRF</td>
<td>Frequency Response Function</td>
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<tr>
<td>DOF</td>
<td>Degree of Freedom</td>
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<tr>
<td>FEM</td>
<td>Finite Element Model</td>
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<tr>
<td>x(t)</td>
<td>Displacement vector (nx1)</td>
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<td>f(t)</td>
<td>Random natural excitation vector (nx1)</td>
</tr>
<tr>
<td>H</td>
<td>Receptance FRF matrix (nxn)</td>
</tr>
<tr>
<td>h</td>
<td>Columns of H.</td>
</tr>
<tr>
<td>i</td>
<td>= \sqrt{-1}</td>
</tr>
<tr>
<td>M, C, K</td>
<td>Mass, damping and stiffness matrices (nxn)</td>
</tr>
<tr>
<td>n</td>
<td>number of sensors or measurement points</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
</tr>
<tr>
<td>\omega</td>
<td>= frequency (rad/sec)</td>
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Superscripts

* = complex conjugate transpose

h = healthy structure
d = damaged structure

1. INTRODUCTION

A basic limitation of vibration based damage detection methods is that a large number of DOFs need to be measured to locate damage [1-5], although damage can often be detected using a small number of sensors. To be practical, a damage detection algorithm must use incomplete measurements or else approximate unmeasured coordinates. Approximating unmeasured coordinates puts error in the damage diagnosis and using a small number of measurements on a large structure causes spatial aliasing due to coarse resolution of the structural spatial response.

Since the damage detection method is based on the FRFs of the healthy structure, changes due to the environment must be compensated. These changes are can be modeled as changes in overall stiffness for the whole structure, or a section of the structure. The global changes affect the FRFs in a predictable way, and can be compensated for and are not considered in this paper.

2. DAMAGE DETECTION THEORY

In many structures it is impractical to use a finite-element model to diagnose damage because the model would not be accurate enough, and geometry changes due to cracks and nonlinear effects would be difficult to include. The proposed frequency domain method uses FRF measurements from the healthy structure to identify an input-output frequency domain model of the structure, where the number of model DOFs is equal to the number...
of accelerometers or sensors on the structure. This approach is suitable for detecting and locating damage, but the level and type of damage is not diagnosed.

The linear structure is assumed to be represented by:

\[ M\ddot{x} + C\dot{x} + Kx = f(t) \]  

(1)

Taking the Fourier transform of (1) gives:

\[ A(i\omega)x(i\omega) = f(i\omega) \]  

(2)

where the system matrix is:

\[ A(i\omega) = (K - \omega^2 M + i\omega C) \]

Solving for the displacement vector gives:

\[ x = Hf \]  

(3)

where \( H = A^{-1} \) is the system FRF matrix and the \((i\omega)\) argument has been dropped for brevity. Equation (3) uses all DOFs of the system to detect damage, which is impractical. Thus the damage detection algorithm must use incomplete measurements or else approximate unmeasured coordinates. Typically, a model reduction procedure such as Guyan reduction or dynamic reduction/expansion is used to eliminate or approximate the unmeasured coordinates based on a model of the structure in the healthy condition. However, the reduction/expansion will almost always include DOFs near the damage. This causes an incorrect damage indication because the reduction/expansion based on the healthy model is incorrect for the damaged structure. The approach taken here to the problem of incomplete measurements is to eliminate the unmeasured DOFs from the problem, and measure transverse displacements and estimate rotations using finite-differences only at the sensor locations. This will provide a repeatable approximation that includes the effects of damage and eliminates the error caused by using information from the healthy model to perform a reduction on a damaged structure. In other words, the finite-difference approximation depends only on the geometry of deformation, not the model properties, and this improves the accuracy of damage detection.

The procedure for damage detection using sparse measurements is developed by partitioning the system into measured and unmeasured DOFs as:

\[
\begin{bmatrix}
  x_1 \\
  x_2
\end{bmatrix}
\begin{bmatrix}
  H_{11} & H_{12} \\
  H_{21} & H_{22}
\end{bmatrix}
\begin{bmatrix}
  f_1 \\
  f_2
\end{bmatrix}
\]  

(4)

where \( x_1 \) represents the DOFs where the response is measured and \( x_2 \) represents the DOFs where the response is not measured. The procedure assumes that \( f_1 \) is known and \( f_2 = 0 \) in the bandwidth where we are looking for damage. This could be accomplished by using actuators at the DOFs where measurements are taken. Then the first equation in (4) becomes:

\[ x_1 = H_{11}f_1 \]  

(5)

The damage location procedure to be developed requires that the \( H \) matrix be identified for the healthy structure in the bandwidth used to detect damage to the structure. The dimension of \( x \) is \( nx1 \) for \( n \) measurements on the structure, and thus the \( nxn \) \( H \) matrix must be determined experimentally for the healthy structure. The individual \( M, C, \) and \( K \) matrices do not need to be known.

To identify the \( H \) matrix for the healthy structure, a force, either random, sine, or impulse, is applied at each DOF one at a time. For example, for a random input the elements of the force vector are \( f_i = 0, (i \neq k) \), and \( f_k \) is a random force. Then by multiplying both sides of (5) on the right by the complex conjugate force and taking expectations, the equation becomes:

\[ h_k = g_{sf} / |f_k|^2 \]

(6)

where \( h_k \) is the \( k \)th column of \( H \), \( g_{sf} \) is the cross-spectral density vector between the displacement vector and the input force, and \( |f_k|^2 \) is the mean-square value of the random force. Thus from (6) each column and hence the full \( H \) matrix can be identified for the healthy structure. Note that the \( H \) matrix is symmetric unless gyroscopic forces are present. A practical problem with computing the \( H \) matrix is that it is difficult to apply moments to obtain the FRFs for rotational DOFs. Offset fixtures to apply moments are currently being tested with a finite-difference approximation of angles to obtain rotational FRFs. Once \( H \) is determined, the \( A_{11} \) matrix is calculated as \( A_{11} = H_{11}^{-1} \) at each frequency point in the bandwidth that is used to detect damage, and the \( A_{11} \) matrices are stored as historical data. Now a damage vector is defined as:

\[ d_i = A^h x^d_i - f^d_i \]

(7)
where \( h \) and \( d \) represent the healthy and damaged structure. If damage occurs the \( d_i \) vector will have non-zero forces only at the DOFs that are connected to the damaged elements. For example, if damage causes changes to the \( i \)th row of the stiffness matrix, only the \( i \)th entry of \( d \) will be nonzero. To locate damage a damage matrix is formed as:

\[
D = \lim_{T_o \to \infty} \frac{2}{T_o} \mathbb{E}(d_i d_i^*)
\]

where \( \mathbb{E} \) is the expectation. The diagonal entries of \( D \) are the auto-spectral densities of the damage force. The integral of the diagonal entries of \( D \) is the rms damage at each of the measured DOFs, and is used to locate damage. The RMS damage forces are computed as:

\[
d_{\text{rms}} = \int_0^\infty \text{diag}(D)df.
\]

Global changes in the structural properties due to environmental effects such as changes in temperature and pressure will be interpreted as damage in (9) and this problem is being worked on considering normalization and frequency shifting. The advantage of this approach for damage detection is that, after surveying the healthy structure, only the random structural accelerations and excitation forces need to be measured to locate damage. The technique is also simple and suitable for continuous damage monitoring during operation of the structure. Based on (9), damage can be located using only one input point and all response measurements at the sensor locations (i.e. by identifying one column of the reduced FRF matrix).

3. DAMAGE DETECTION SIMULATION

The damage detection technique uses the input-output FRFs measured on the healthy structure, and no model of the structure is used. A FEM model is used only to test the health monitoring system and for guiding the design of the system. The cantilever beam model shown in Figure 1 is used for the simulation. The beam is 44 inches long, 0.25 inches thick, and 3 inches high. The planar model has 15 elements and 30 DOFs, with translation and rotational DOFs at each node. An excitation force is applied perpendicular to the beam at DOF 19 in the model. The response is computed for the frequency range of 0.1 Hz to 100 Hz. Damage is modeled as a 5% reduction in the elemental stiffness matrix individually at each element in the model. Modeling damping and the effect of the damages on the structural natural frequencies are discussed first, and then the damage location simulation is performed.

3.1 Damping Modeling

Damping is modeled using a frequency dependent damping ratio method. This overcomes the problem of using modal damping which couples all the DOFs in the model and causes the damage to be smeared over the whole structure, and the problem of using proportional damping that causes the damping to be too large at high frequencies. The approach starts with the \( i \)th damping ratio based on proportional damping which is [6]:

\[
\eta_i = \frac{\alpha}{2\omega_i} + \frac{\beta}{2}\omega_i
\]

where \( \alpha \) and \( \beta \) are constants, \( \omega_i \) is the \( i \)th natural frequency, and \( C = \alpha M + \beta K \). It is seen here that proportional damping preserves the FEM connectivity, but only two damping ratios can be chosen independently and all others will be dependent. At high frequencies the damping ratios will become too large, and the damage simulation will be somewhat unrealistic. The approach taken here is to make \( \alpha \) and \( \beta \) functions of frequency so that a unique damping ratio can be chosen at every frequency point. In this study the constants chosen are: \( \alpha = 0 \) and \( \beta = 2\zeta(\omega)/\omega \) where \( \zeta(\omega) = 0.03 \) is constant, and \( \omega \) is the forcing frequency.

3.2 Natural Frequency Changes due to Damage

The first six natural frequencies of the beam for the undamaged case, and the cases of 5% damage in each element of the beam individually are shown in Table 1. It is noted that this small damage causes at most a 0.8 Hz or 0.4% change in any of the lowest six natural frequencies. Thus the first six natural frequencies of the beam are not very sensitive to small damage.

3.3 Locating Damage using Sparse Measurements

The RMS damage vector (9) is used to locate damage using sparse measurements. The two DOFs at every fifth node, that is DOFs 9, 10, 19, 20, and 29,30 are used as the reduced model. The FRFs in the healthy condition are computed using the full model and the FRFs at the reduced DOFs are used as historical data. Damage is put in the model and the response is computed using the full model and the response at the reduced DOFs is used in the damage calculation. The auto-spectral densities of the damage force for the six DOFs in the reduced model are shown in Figure 2, for the case of 5% damage to element 4. Note that only DOFs 9,10 show a nonzero force, indicating damage somewhere between the fixed end and the fifth node of the beam. With sparse measurements, the
damage location is bounded between two nodes at which measurements are taken, but the exact location of damage cannot be determined. The damage force is greatest at the resonant frequencies of the beam.

Damage is now put into all elements of the beam individually, starting with element 1 at the support. The sparse measurements at DOFs 9,10,19,20,29,30 are used to detect damage. The RMS damage vector is calculated for each case and is shown in Figure 3. Note that for all 15 cases, damage is correctly bounded between the two closest measurement nodes, and the DOFs at nodes not nearest the damage have a zero damage force. Thus the RMS damage vector (9) can correctly detect damage, and also locate damage between the two nearest measurement points. When the simulation was performed using only one DOF per node, either translation or rotation, the damage location was not always correct. Thus both translation and rotational measurements are needed at each node to locate damage. Note that the rotational DOFs have a larger damage force than translation DOFs.

3.4 Locating Damage using Full Measurements

Finally, damage is now put into all elements of the beam individually, starting with element 1 at the support, and all DOFs of the beam are used as measurements in the damage algorithm. The result is shown in Figure 4, where the damage is correctly located for all cases. Note that the damage force is maximum for the element near the support, and decreases as the damage is moved toward the free end. Note that the damage force is greater when full measurements are used. Thus the damage force for partial measurements is an indicator of damage severity, but is not the exact damage force.

4. DISCUSSION

The method for damage detection proposed depends on historical data to represent the structural characteristics for the no damage condition. Thus temperature and other environmental effects that change the FRFs must be corrected. An approach to partly correct the effects of global and localized temperature changes is being investigated, but not included in this paper. The damage vector method also uses both translation and rotational FRFs. This requires a technique to apply moments to the structure and a technique to accurately measure rotations. Approaches are being developed to overcome the difficulties of making these measurements.

A subtle point in using this technique is that the excitation force can only be applied at the DOFs being measured. Otherwise the damage location will be incorrect. For the case where there is unmeasured natural excitation over the entire structure, artificial forces would have to be applied at one or more of the measured DOFs at a higher frequency range that the natural excitation. Alternatively, the artificial excitation must be uncorrelated with the natural loading and the technique extended to use the correlation between the artificial excitation and the response.

5. CONCLUSION

A method for locating damage using sparse measurements has been presented and demonstrated using a FEM of a cantilever beam. The technique is being tested in the laboratory and if a method of making accurate rotational measurements can be developed, the technique may be one solution to the long standing problem of not having sufficient spatial measurements to locate damage.

ACKNOWLEDGEMENT

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REFERENCES


Figure 1. Fixed-free beam element numbers and nodal DOFs for simulation

Table 1. Changes in natural frequency due to 5% damage for all elements individually

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<tr>
<th>FRQ</th>
<th>EL1</th>
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Figure 2. Damage force PSD for sparse measurements and 5% damage to element 4 (response measured at DOFs 9, 10, 19, 20, 29, 30, force at DOF 19, 0.1-100Hz freq. range)
Figure 3. Peak damage indicator using sparse measurements for 5% damage to each element individually (response measured at DOFs 9, 10, 19, 20, 29, 30, force at DOF 19, and 0.1-100Hz freq. range)
Figure 4. Peak damage indicator using full measurements for 5% damage to each element individually (response measured at DOFs 1-30, force at DOF 19, and 0.1-100Hz freq. range)