ORTHOGONAL FUNCTIONS TECHNIQUES FOR THE IDENTIFICATION OF MECHANICAL SYSTEMS

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ABSTRACT

This paper presents orthogonal function techniques for the identification of mechanical systems. For this purpose, mechanical systems are represented by state-space equations and the input and output signals are developed in series of orthogonal functions. The equation of motion can be integrated using numerical techniques together with integration properties specific for orthogonal functions. This procedure permits to obtain a simple algebraic equation, which leads to the determination of the unknown parameters. Different orthogonal functions were tested in numerical and experimental applications, including gyroscopic systems.

NOMENCLATURE

\[ \mathbf{P} \] operational matrix
\[ \mathbf{\Phi}(t) \] vectorial basis of the orthogonal series
\[ \mathbf{M} \] inertia matrix
\[ \mathbf{C} \] viscous damping matrix
\[ \mathbf{K} \] stiffness matrix
\[ \mathbf{\mathbf{x}}(t) \] vector of displacement time responses
\[ \mathbf{f}(t) \] vector of excitation forces
\[ E \] Young's modulus
\[ f \] natural frequency
\[ r \] number of terms in the orthogonal series expansion
\[ \mathit{nas} \] number of assumed stations
\[ \nu \] Poisson's ratio
\[ \xi \] damping factor
\[ \Omega(n) \] rotation speed law

1 INTRODUCTION

In many engineering situations, it is necessary to extract unknown parameters from a mechanical system. The identification of excitation forces is also very important in situations where they can not be measured directly.

The unknown parameters can be modal parameters – natural frequencies, modal damping factors and mode shapes – or structural parameters, which have important applications in Mechanical Engineering, such as: dynamic analysis of complex structures, finite element model updating, evaluation of dynamic loads, prediction of dynamic responses, control, damage detection, etc.

The methods used to solve these problems can be roughly divided into two classes: those which operate in the frequency domain and the ones which operate in the time domain.

Some time domain parameter identification techniques became very popular in the past two decades: Ibrahim Time Domain Method\cite{Ibrahim}, Complex Exponential Method\cite{Complex}, Polyreference Time Domain Method\cite{Polyreference}, Eigensystem Realization Algorithm\cite{Eigensystem} and Autoregressive-Moving Average Model (ARMA)\cite{ARMA} have been successfully applied to different types of mechanical systems.

Another problem that has received attention from researchers and engineers is the identification of excitation forces from the dynamic responses. This procedure is important, mainly, because in many practical situations the direct measurement of forces, using force gauges, proves to be ineffective or even impossible. Such is the case, for example, when the forces are applied at inaccessible locations of the structure or when the introduction of force transducers is likely to significantly change the dynamic characteristics of the mechanical system. In these cases, the indirect identification of input forces from the dynamic responses of the structure - which can generally be easily acquired - appears as a valuable alternative.

Several techniques for force identification, operating either in the time domain or in the frequency domain, have been proposed\cite{Forces}. As for the time domain methods, the most widely known is that named SWAT (Sum of Weighted Accelerations Technique)\cite{SWAT}. This method is based on the modal decomposition of the acceleration time responses and utilizes the modal equilibrium equations for the rigid body modes. Recently, Genaro and Rade\cite{Genaro} studied a method which enables to extend the range of application of SWAT method, by taking into account both rigid body and elastic modes.
Orthogonal functions have been used for analysis, identification and control purposes since the mid seventies, as, for instance, Walsh functions, Laguerre polynomials, Block-Pulse functions, Legendre polynomials, Chebyshev series, Jacobi polynomials, Fourier series and Hermite polynomials.

This paper presents a technique which utilizes important properties of the orthogonal functions, for the parameter identification and force reconstruction, based on time responses of mechanical systems.

In the sequence, the orthogonal functions used are presented, as well as the basic formulation of the identification method. Then, numerical applications to both numerically simulated and experimentally tested mechanical systems are shown, including gyroscopic systems, aiming at illustrating the main features of the method.

2 ORTHOGONAL FUNCTIONS

A set of functions \( \{\phi_i(t)\} \), \( i = 1, 2, 3, ... \) is said to be orthogonal in the interval \([a,b]\) if:

\[
\int_a^b \phi_m(t) \phi_n(t) dt = K_{mn}
\]

where:

\[
K_{mn} = 0 \text{ if } m \neq n
\]

\[
K_{nn} = \delta_{mn}
\]

If \( K_{mn} \) is the Kronecker's delta, the set of functions \( \{\phi_i(t)\} \) is said orthonormal. The following property, related to the successive integration, holds for a set of \( r \) orthogonal functions:

\[
\int_0^t \int_0^t \cdots \int_0^t \phi(t) \span\{\phi_i(t)\} (dt) \text{ } = \span\{P\}^n \span\{\phi(t)\}
\]

where:

\[\span\{P\} \in \mathbb{R}^{r \times r}\] is a square matrix with constant elements, called operational matrix

\[
\span\{\phi(t)\} = \span\{\phi_1(t) \phi_2(t) \cdots \phi_r(t)\}^T\]

is the vectorial basis of the orthogonal series

Pacheco et al.\cite{10} give details about the vectorial basis and operational matrix related to each type of orthogonal function considered in this paper.

3 TIME DOMAIN IDENTIFICATION TECHNIQUE

The proposed identification method can exploit either free or forced time responses, in terms of either displacements, velocities or accelerations. Since the formulations for these three types of responses are quite similar, only the formulation for forced and free systems, in terms of displacements, will be presented in the following.

The equation of motion for a \( N \) d.o.f. system is given by:

\[
[M] \ddot{\mathbf{x}}(t) + [C] \dot{\mathbf{x}}(t) + [K] \mathbf{x}(t) = \mathbf{f}(t)
\]

where \([M], [C] \text{ and } [K] \in \mathbb{R}^{N \times N}\) are, respectively, the inertia, damping and stiffness matrices; \( \mathbf{x}(t) \in \mathbb{R}^{N \times 1}\) is the vector of displacement time responses and \( \mathbf{f}(t) \in \mathbb{R}^{N \times 1}\) is the vector of excitation forces.

Integrating Eq. (2) twice in the interval \([0,t]\), one obtains:

\[
[M] \mathbf{x}(t) - \mathbf{x}(0) t + [C] \int_0^t \mathbf{x}(\tau) d\tau - \mathbf{x}(0) t + [K] \int_0^t \int_0^t \mathbf{x}(\tau_2) d\tau_2 - \mathbf{x}(0) t + \int_0^t \mathbf{f}(\tau) d\tau_2
\]

where \( \mathbf{x}(0) \) and \( \mathbf{x}(0) \) are the vectors of initial displacements and velocities, respectively.

The signals \( \mathbf{x}(t) \) and \( \mathbf{f}(t) \) can be expanded in truncated series of \( r \) orthogonal functions as follows:

\[
\mathbf{x}(t) = \mathbf{X} \mathbf{\phi}(t)
\]

\[
\mathbf{f}(t) = \mathbf{F} \mathbf{\phi}(t)
\]

where:

\[
\mathbf{X} \in \mathbb{R}^{N \times r}\]

is the matrix of the coefficients of the expansion of \( \mathbf{x}(t) \)

\[
\mathbf{F} \in \mathbb{R}^{N \times r}\]

is the matrix of the coefficients of the expansion of \( \mathbf{f}(t) \)

Substituting Eq. (4) in Eq. (3) and applying the property for the integration of orthogonal functions, given by Eq. (1), the following system of algebraic equations is obtained:

\[
[H][J] = [E]
\]

where:

\[
[H] = \begin{bmatrix}
[M] & -[M]\mathbf{x}(0) & -[M]\mathbf{x}(0) & -[M]\mathbf{x}(0) & -[C]\mathbf{x}(0) & -[C] \mathbf{x}(0) & -[K] \mathbf{x}(0)
\end{bmatrix}
\]

\[
[J] = \begin{bmatrix}
\mathbf{X}^T & [\mathbf{e}]^T & [\mathbf{r}]^T [\mathbf{e}] & [\mathbf{p}]^T [\mathbf{x}] & [\mathbf{p}]^T [\mathbf{x}]^T & [\mathbf{p}]^T [\mathbf{x}]^T
\end{bmatrix}
\]

\[
[E] = [\mathbf{F}]^T [\mathbf{e}]
\]

In this equation, \( [\mathbf{e}] \in \mathbb{R}^{r,1} \) is a constant vector whose form depends on the particular choice of the orthogonal series: for the Block-Pulse functions, \( [\mathbf{e}] = [1 \ 1 \ \ldots \ 1]^T \); for Fourier, Chebyshev, Legendre, Jacobi and Walsh series, \( [\mathbf{e}] = [1 \ 0 \ \ldots \ 0]^T \).
Solving system (5) for matrix \( H \) one obtains the structural
model of the system, represented by matrices \( M \), \( C \) and
\( K \) and the set of initial conditions. A computationally stable
solution to (5) can be achieved by using the least square
method combined with the singular value decomposition
technique.

If the free responses are used, a system of equations similar
to (5) is obtained, with:

\[
H = \begin{bmatrix}
\{x(0)\} & \{\dot{x}(0)\} & [M]^{-1}[C]\{x(0)\} & -[M]^{-1}[C] & -[M]^{-1}[K]
\end{bmatrix}
\]

\[
J = \begin{bmatrix}
\end{bmatrix}
\]

As can be seen in the equations above, it is not possible to
identify, separately, matrices \( M \), \( C \) and \( K \) when the
free responses are used. However, regardless of the nature
of the responses, it is always possible to form the following
state matrix, whose eigensolutions provide the natural
frequencies, modal damping factors and complex vibration
modes of the system:

\[
[A] = \begin{bmatrix}
0 & [I] & [0] & [-[M]^{-1}[K]] & -[M]^{-1}[C]
\end{bmatrix} \in \mathbb{R}^{2N,2N}
\]

Due to practical constraints it is generally impossible to use
the same number of sensors as the vibration modes
contributing in the response. Thus, in order to create
oversized mathematical models with a reduced amount of
instrumentation, a technique named Transformed Stations
Technique has been used, together with the Modal
Confidence Factor (MCF). The MCF is used to separate
the structural modes from computational ones. Details are given

4 FORCE IDENTIFICATION USING ORTHOGONAL
FUNCTIONS

The approach for force identification is similar to that
presented in the previous section for the identification of
modal parameters. Based on the assumption that matrices
\( M \), \( C \) and \( K \) are known, it is only needed to rearrange
Eq. (5) for estimating matrix \( F \) which contains the
coefficients of the excitation forces. In this case, the matrices
in (5) are given by:

\[
H = \begin{bmatrix}
{F} & {M}[x(0)] & {M}[\dot{x}(0)] & {F}[x(0)] + [C][x(0)]
\end{bmatrix}
\]

\[
J = \begin{bmatrix}
{P}^T & \{e\} & [P]^T[2\{x\}]^T
\end{bmatrix}
\]

\[
{e} = [M][x] + [C][x][P] + [K][x][P]^2
\]

5 APPLICATIONS

5.1 Free-free aluminum Beam

This application contains a numerical part and an
experimental one. The system analyzed is a free-free
aluminum beam (Figure 1) with rectangular cross section
(dimensions along x axis: 38.3 mm; dimension along z axis:
4.7 mm). This beam was modeled using 54 finite elements
(55 nodes) for the transversal vibration analysis (translation
along z axis and rotation around x axis).

![Figure 1: Physical model of the aluminum beam](image)

a) Modal Parameter Identification: Numerical Simulation

From the global model of the structure, the time response
was calculated using partial fractions expansions, in which
the 15 first modes were retained. Initial conditions
corresponding at \( z(0) = 0 \) and \( \dot{z}(0) = 1 \text{ m/s} \) were applied
at the second node. The acceleration response was
determined in the interval from 0 to 0.06 s.

In the identification process, the responses at the nodes 2, 4,
6 and 8 were used together with 16 transformed stations to
create an oversized mathematical model. The values of
natural frequencies and damping factors obtained in the
FEM simulations (\( \omega_{mod} \) and \( \xi_{mod} \)) and the identified ones (\( \omega_{id} \)
and \( \xi_{id} \)) - using Fourier series \( r = 81 \) - are presented in Table
1.

<table>
<thead>
<tr>
<th>MODE</th>
<th>( \omega_{mod} ) [Hz]</th>
<th>( \omega_{id} ) [Hz]</th>
<th>( \xi_{mod} ) [%]</th>
<th>( \xi_{id} ) [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
<td>0.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0.0</td>
<td>0.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>13.0</td>
<td>13.0</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>4</td>
<td>35.9</td>
<td>35.9</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>5</td>
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<td>70.3</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>6</td>
<td>116.3</td>
<td>116.3</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>7</td>
<td>173.7</td>
<td>173.7</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>8</td>
<td>242.5</td>
<td>242.5</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>9</td>
<td>322.9</td>
<td>322.9</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>10</td>
<td>414.7</td>
<td>414.7</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>11</td>
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<td>518.0</td>
<td>0.00</td>
<td>0.00</td>
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<tr>
<td>12</td>
<td>632.7</td>
<td>632.7</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>13</td>
<td>758.9</td>
<td>758.9</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>14</td>
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<td>896.6</td>
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<tr>
<td>15</td>
<td>1045.7</td>
<td>1045.7</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 1: Modal parameters of the aluminum beam
b) Force Reconstruction: Numerical Simulation

For the problem of force reconstruction, a periodic excitation force containing two harmonics was applied at node 2:

\[ f_3(t) = 5 \sin(2000 \pi t) + 2 \cos(1600 \pi t) \text{ [N]} \]

Using the fourth order Runge-Kutta method, the system time response was determined in the interval [0 : 0.004 s]. In this case the velocity response was used in the identification process.

Figure 2 shows the plots of exact and identified forces using Fourier series (r=151). The relative error between the RMS value of the difference between these two forces and the RMS value of the exact force was 3.7%.

![Figure 2: Exact and identified force – Fourier series](image)

The result obtained using the Block-Pulse functions expansion (r=512) is shown in Figure 3. The relative error in the RMS value was 3.1%.

![Figure 3: Exact and identified force – Block-Pulse functions](image)

c) Modal Parameter Identification – Experimental Part

The beam is the same of the previous sections and it was now suspended through two strings, as shown in Figure 4.

![Figure 4: Experimental set-up](image)

The acceleration response along the z axis was measured at the eight nodes shown in Fig. 4, due to an impulsive excitation applied at the node number 2. The sampling frequency was 26600 Hz and 2048 points were used. The acceleration of node 2 is presented in Figure 5.

![Figure 5: Acceleration of node 2 along the z axis](image)

The natural frequencies obtained from finite element calculation \( f_{\text{mod}} \) and the experimental ones \( f_{\text{exp}} \) are shown in Table 2.

<table>
<thead>
<tr>
<th>Mode</th>
<th>( f_{\text{exp}} ) [Hz]</th>
<th>( f_{\text{mod}} ) [Hz]</th>
<th>Error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.6</td>
<td>13.0</td>
<td>3.50</td>
</tr>
<tr>
<td>2</td>
<td>36.0</td>
<td>35.9</td>
<td>0.36</td>
</tr>
<tr>
<td>3</td>
<td>70.2</td>
<td>70.3</td>
<td>0.21</td>
</tr>
<tr>
<td>4</td>
<td>115.8</td>
<td>116.3</td>
<td>0.44</td>
</tr>
<tr>
<td>5</td>
<td>173.0</td>
<td>173.7</td>
<td>0.43</td>
</tr>
<tr>
<td>6</td>
<td>242.1</td>
<td>242.5</td>
<td>0.17</td>
</tr>
<tr>
<td>7</td>
<td>323.4</td>
<td>322.9</td>
<td>0.15</td>
</tr>
<tr>
<td>8</td>
<td>416.3</td>
<td>414.7</td>
<td>0.37</td>
</tr>
</tbody>
</table>

### Table 2: Numerical and experimental natural frequencies

The natural frequencies identified using Fourier series are in Table 3. The identification process was carried out in two steps: in the first one 121 expansion terms \( r=121 \) were considered together with 88 assumed stations \( \text{nas}=88 \); in
the second step \( r = 101 \) and \( \text{nas} = 8 \). The error was calculated with respect to experimental values.

<table>
<thead>
<tr>
<th>Mode</th>
<th>( f_d ) [Hz]</th>
<th>Error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13.07</td>
<td>4.08</td>
</tr>
<tr>
<td>2</td>
<td>36.00</td>
<td>0.09</td>
</tr>
<tr>
<td>3</td>
<td>70.32</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>116.08</td>
<td>0.25</td>
</tr>
<tr>
<td>5</td>
<td>173.50</td>
<td>0.32</td>
</tr>
<tr>
<td>6</td>
<td>242.23</td>
<td>0.06</td>
</tr>
<tr>
<td>7</td>
<td>322.84</td>
<td>0.17</td>
</tr>
<tr>
<td>8</td>
<td>416.58</td>
<td>0.08</td>
</tr>
</tbody>
</table>

**Table 3: Identified natural frequency – Fourier series**

The MAC number determined using identified eigenvectors and those obtained through FE calculation are presented in Table 4.

<table>
<thead>
<tr>
<th>( f_0 ) [Hz]</th>
<th>( a_0 ) [Hz]</th>
<th>( a_1 ) [Hz]</th>
<th>( a_2 ) [Hz]</th>
<th>( a_3 ) [Hz]</th>
<th>( a_4 ) [Hz]</th>
<th>( a_5 ) [Hz]</th>
<th>( a_6 ) [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>98.02</td>
<td>2.01</td>
<td>19.60</td>
<td>0.02</td>
<td>7.29</td>
<td>0.44</td>
<td>18.64</td>
<td>0.09</td>
</tr>
<tr>
<td>0.50</td>
<td>98.48</td>
<td>0.12</td>
<td>5.99</td>
<td>1.35</td>
<td>7.97</td>
<td>0.34</td>
<td>9.90</td>
</tr>
<tr>
<td>20.31</td>
<td>0.26</td>
<td>98.79</td>
<td>0.99</td>
<td>26.95</td>
<td>0.57</td>
<td>4.76</td>
<td>9.49</td>
</tr>
<tr>
<td>0.18</td>
<td>8.00</td>
<td>0.56</td>
<td>97.43</td>
<td>1.18</td>
<td>0.83</td>
<td>4.40</td>
<td>9.52</td>
</tr>
<tr>
<td>3.06</td>
<td>1.08</td>
<td>25.54</td>
<td>1.14</td>
<td>97.71</td>
<td>5.05</td>
<td>22.01</td>
<td>0.86</td>
</tr>
<tr>
<td>0.88</td>
<td>4.65</td>
<td>0.67</td>
<td>4.51</td>
<td>96.97</td>
<td>0.86</td>
<td>34.33</td>
<td></td>
</tr>
<tr>
<td>7.56</td>
<td>0.77</td>
<td>1.41</td>
<td>4.04</td>
<td>12.86</td>
<td>1.56</td>
<td>95.25</td>
<td>35.50</td>
</tr>
<tr>
<td>0.55</td>
<td>0.06</td>
<td>5.02</td>
<td>2.50</td>
<td>2.14</td>
<td>31.33</td>
<td>41.45</td>
<td>91.31</td>
</tr>
</tbody>
</table>

**Table 4: MAC – Fourier series**

The identification results using Block-Pulse functions (1\(^{st}\) step: \( r = 184 \) and \( \text{nas} = 88 \); 2\(^{nd}\) step: \( r = 1286 \) and \( \text{nas} = 8 \)) and the corresponding MAC number are shown in Table 5 and 6, respectively.

<table>
<thead>
<tr>
<th>Mode</th>
<th>( f_0 ) [Hz]</th>
<th>Error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.56</td>
<td>4.88</td>
</tr>
<tr>
<td>2</td>
<td>36.03</td>
<td>0.01</td>
</tr>
<tr>
<td>3</td>
<td>70.15</td>
<td>0.33</td>
</tr>
<tr>
<td>4</td>
<td>115.79</td>
<td>0.30</td>
</tr>
<tr>
<td>5</td>
<td>172.95</td>
<td>0.41</td>
</tr>
<tr>
<td>6</td>
<td>242.08</td>
<td>0.19</td>
</tr>
<tr>
<td>7</td>
<td>323.39</td>
<td>0.02</td>
</tr>
<tr>
<td>8</td>
<td>416.25</td>
<td>0.25</td>
</tr>
</tbody>
</table>

**Table 5: Identified natural frequency – Block-Pulse functions**

The Campbell diagram of the rotor is given in Figure 7. The operation speed (\( \Omega_{op} \)) is 21000 rpm.

The displacement time response of the rotor in the transient state, i.e., the response during the start up of the rotor using an exponential function for the rotation speed, was obtained using Newmark method combined with the pseudo-modal method\(^{[3]}\), in which the first six modes were retained.
The response was obtained in the interval from 0 to 3 s using 8192 points and the following rotation speed law:

\[ n(t) = 22000 - 22000 e^{-0.66 t} \text{ [rpm]} \]

The amplitude displacement of disk 2 (node 6) is shown in Figure 8.

In the identification process, only the response after the rotor had reached the operation speed was used, since, during the transient motion, the modal parameters change at each time.

The orthogonal functions used in the identification procedure were the following: Fourier series, Chebyshev series, Legendre polynomials, Jacobi polynomials, Block-Pulse functions and Walsh functions. The results obtained are listed in Tables 11 to 16. For Fourier series, Block-Pulse and Walsh functions two responses were used (displacements \( x(t) \) and \( z(t) \) of disk 2), for the others four responses (displacements \( x(t) \) and \( z(t) \) of disks 2 and 3) were taken into account.

<table>
<thead>
<tr>
<th>Mode</th>
<th>( f_{id} ) [Hz]</th>
<th>Error [%]</th>
<th>( \delta_{id} ) [%]</th>
<th>Error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>54.48</td>
<td>0.71</td>
<td>0.026</td>
<td>0.22</td>
</tr>
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<td>2</td>
<td>68.48</td>
<td>0.49</td>
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<td>1.18</td>
</tr>
<tr>
<td>3</td>
<td>157.37</td>
<td>1.90</td>
<td>0.297</td>
<td>4.22</td>
</tr>
<tr>
<td>4</td>
<td>198.96</td>
<td>1.47</td>
<td>0.284</td>
<td>4.25</td>
</tr>
</tbody>
</table>

Table 11: Modal parameters identified – Fourier series (\( r=101 \) and \( nae=16 \))

<table>
<thead>
<tr>
<th>Mode</th>
<th>( f_{id} ) [Hz]</th>
<th>Error [%]</th>
<th>( \delta_{id} ) [%]</th>
<th>Error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>54.49</td>
<td>0.73</td>
<td>0.023</td>
<td>12.49</td>
</tr>
<tr>
<td>2</td>
<td>68.16</td>
<td>0.03</td>
<td>0.070</td>
<td>6.57</td>
</tr>
<tr>
<td>3</td>
<td>155.99</td>
<td>1.01</td>
<td>0.340</td>
<td>9.84</td>
</tr>
<tr>
<td>4</td>
<td>197.00</td>
<td>0.47</td>
<td>0.259</td>
<td>4.96</td>
</tr>
</tbody>
</table>

Table 12: Modal parameters identified – Chebyshev series (\( r=91 \) and \( nae=8 \))

<table>
<thead>
<tr>
<th>Mode</th>
<th>( f_{id} ) [Hz]</th>
<th>Error [%]</th>
<th>( \delta_{id} ) [%]</th>
<th>Error [%]</th>
</tr>
</thead>
<tbody>
<tr>
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<td>54.50</td>
<td>0.75</td>
<td>0.026</td>
<td>0.75</td>
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<td>2</td>
<td>68.48</td>
<td>0.49</td>
<td>0.067</td>
<td>2.47</td>
</tr>
<tr>
<td>3</td>
<td>157.28</td>
<td>1.84</td>
<td>0.301</td>
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<tr>
<td>4</td>
<td>198.87</td>
<td>1.42</td>
<td>0.286</td>
<td>4.96</td>
</tr>
</tbody>
</table>

Table 13: Modal parameters identified – Legendre polynomials (\( r=81 \) and \( nae=8 \))

<table>
<thead>
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<th>Mode</th>
<th>( f_{id} ) [Hz]</th>
<th>Error [%]</th>
<th>( \delta_{id} ) [%]</th>
<th>Error [%]</th>
</tr>
</thead>
<tbody>
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<td>198.85</td>
<td>1.4128</td>
<td>0.287</td>
<td>5.47</td>
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</tbody>
</table>

Table 14: Modal parameters identified – Jacobi polynomials (\( r=71 \) and \( nae=8 \))

<table>
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<th>Mode</th>
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<th>Error [%]</th>
<th>( \delta_{id} ) [%]</th>
<th>Error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0.027</td>
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<tr>
<td>4</td>
<td>203.21</td>
<td>3.64</td>
<td>0.299</td>
<td>9.90</td>
</tr>
</tbody>
</table>

Table 15: Modal parameters identified – Block-Pulse functions (\( r=256 \) and \( nae=12 \))
6 CONCLUSIONS

It has been shown how different types of orthogonal functions can be conveniently employed for the identification of modal parameters and excitation forces in mechanical systems, using straightforward computational procedures.

Combination of the transformed station technique and the orthogonal function identification method provides a more suitable identification scheme for practical implementation, since a reduced amount of instrumentation is required.

All sets of orthogonal functions used have demonstrated the capability of providing fairly accurate results, though Legendre, Jacobi and Chebyshev polynomials provided less accurate force identification results, as compared to other orthogonal functions.

The successful identification of a gyroscopic system was included as a simulation case study.

Simulation and experimental results show the efficiency of the methodology developed.

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REFERENCES


