ABSTRACT

The vibrations of the Hämensilta Bridge in the centre of Tampere, Finland have exceeded the recommendations on the sidewalk. An experimental modal analysis was performed in order to study the effects of the renovation on the vibration performance of the bridge. The stochastic subspace identification was applied using the response data from traffic excitation. The measurement data was, however, corrupted with spurious harmonic components. Fortunately, two additional vibration transducers were used together with the modal accelerometers and these were used to estimate the natural frequencies and damping ratios. The natural modes were estimated from the corrupted data using spectral analysis. The system identification was also performed on the corrupted data resulting in models comprising both structural and spurious modes. The ability to extract the modal parameters from signals comprising harmonic components is an important result for rotating machine applications and is a subject for further study.

NOMENCLATURE

\( \Delta t \) sampling interval
\( \omega_i \) natural frequency of mode \( i \) [rad/s]
\( \zeta_i \) damping ratio of mode \( i \)
\( \lambda_i \) \( i^{th} \) eigenvalue of matrix \( A \)
\( \mu_i \) \( i^{th} \) eigenvalue of matrix \( A^t \)
\( \overline{\mu}_i \) conjugate pair of \( \mu_i \)
\( U(t), \dot{U}(t) \) displacement, velocity, and acceleration
\( M \) mass matrix
\( D \) damping matrix
\( K \) stiffness matrix
\( P(t) \) excitation force
\( x(t) \) state vector

response (output) vector
measurement noise vector
process noise vector
state matrix, continuous time model
state matrix, discrete time model
output matrix
eigenvectors of matrix \( A \)
mode shapes

1 INTRODUCTION

The vibrations of the Hämensilta Bridge in the centre of Tampere have been seen to exceed the recommendations on the sidewalk, causing annoyance to the pedestrians. Vibration measurements were performed in order to study the vibration levels of the bridge deck at different temperatures and during repair. The bridge was renovated in 1997, and the effects of the renovation on the vibration performance were studied with ambient vibration tests. Two types of loading were used: a single heavy vehicle crossing the bridge, and normal traffic. One part of the study was to determine experimentally the modal parameters of the bridge: natural frequencies, damping ratios, and mode shapes.

The dynamic properties of structures are usually measured using forced vibration testing, where both the excitation and the response are measured. However, for large civil engineering structures, this technique may be very difficult and expensive for the following reasons: The natural frequencies may be very low, and to excite them would require a huge impact exciter or shaker. There is no easily applicable practical method available to excite large structures with natural frequencies below 1 Hz [1]. Forced vibration testing is expensive as it involves sizeable equipment and considerable time and work. Moreover,
structures must be closed down for testing, which may not be feasible for many structures such as highway bridges. In addition, ambient excitation is always present, which would cause disturbance to the measurements.

For large civil engineering structures like bridges, high-rise buildings, masts, oil platforms, etc., the excitation comes from wind, traffic, waves, or micro-seismic tremors and even at low amplitude it can buffet the structure and cause measurable vibrations. These ambient excitations are usually random in nature, comprising the frequency range of interest. Therefore, no artificial excitation is needed, although the excitation is not specifically known. Fortunately, identification techniques for output-only systems have been developed and are quite reliable if the excitation can be considered filtered white noise. Such identification techniques include parametric or non-parametric methods in the frequency or time domain.

This study is restricted to the identification of the modal parameters from the response data of the renovated bridge due to normal traffic excitation. The excitation was not known, but it was assumed to include all frequencies of interest exciting the lowest natural modes.

2 DESCRIPTION OF STRUCTURE AND MEASUREMENT DATA

The Hameensilta Bridge in the centre of Tampere is a reinforced concrete backfilled arch-bridge with a span of 40 m and an effective width of 28.5 m (Figure 1). There is a single hinge at the top of the arch. The bridge was built in 1929 and renovated in 1997.

The data used in this study was ambient response measurements from traffic excitation. The vertical responses of the bridge deck were measured in three set-ups at 13 points (Figure 2). Each set-up consisted of five HBM B12/500 accelerometers including one reference sensor, which was fixed in all set-ups at point 10. The record length of the data was 9 600 samples with a sampling period of 0.0125 s. It was later re-sampled with a sampling period of 0.025 s.

Later, however, it was noticed that the measurement data was corrupted with additional disturbance at certain frequencies. The reason for these spurious harmonics was not clear. Fortunately, two HBM B2 vibration transducers, located at points 3 and 11 (Figure 2), were used in addition to the five used for the modal measurements. The signals from these two sensors were not corrupted and could be used to analyse the natural frequencies and damping ratios. Typical simultaneous time signals from two different sensors at point 11 are shown in Figure 3, and their spectra in Figure 4. The differences in the spectra can be clearly seen. It seems that the spurious frequencies appear at equal intervals.
Figure 4. Spectrum from accelerometer (top) and HBM B2 vibration transducer (bottom). Set-up 3, point 11.

3 SYSTEM IDENTIFICATION

The system identification, in this case also called experimental modal analysis, results in a modal model, or the dynamic properties of the structure, by using experimental data. The data from ambient vibration tests consists of output-only data without knowledge of the excitation. Several output-only identification techniques are available, for example, spectral methods [1, 2], vector autoregressive moving average (ARMAV) models [3], polyreference least-square complex exponential method [4], random decrement technique (RD) [5], and stochastic subspace method (SSI) [6]. The stochastic subspace technique has proved to be very reliable and relatively fast. The free parameter of the technique is the model order, which can be chosen by using the stabilisation diagram.

The stochastic subspace identification technique is based on the state-space formulation. Dynamically excited systems can be described by the following matrix equation:

\[ M \ddot{U}(t) + D \dot{U}(t) + KU(t) = P(t) \]  

(1)

where \( M, D, \) and \( K \) are the mass, damping, and stiffness matrices, respectively; \( P \) is the excitation force vector and \( U \) is the displacement vector. It can be shown (e.g. [7]) that the dynamic system in Equation 1 can be expressed in a state space formulation if the excitation is not measured, and stochastic components (noise) are included, Equation 1 can be written in a discrete-time state space formulation

\[ \begin{align*}
  x_{k+1} &= Ax_k + w_k \\
  y_k &= Cx_k + v_k
\end{align*} \]

(2)

where \( x_k \) is the discrete time state vector, \( y_k \) is the response data vector, \( A \) is the system matrix, and \( C \) is the output matrix. Vectors \( w_k \) and \( v_k \) are the process noise and measurement noise, respectively. In the case of ambient vibration testing, \( w_k \) is the only excitation. The system identification consists of estimating the discrete matrices \( A \) and \( C \). The stochastic subspace method identifies the state space matrices from measurements by using robust numerical techniques such as QR factorisation, singular value decomposition, and least squares [4]. Once the matrices are found, the extraction of the modal parameters is quite straightforward. The eigenvalues \( \lambda_i \) of matrix \( A \) are related to the eigenvalues \( \mu_i \) of the continuous time state matrix \( \hat{A} \) by

\[ \mu_i = \frac{1}{\Delta t} \ln \lambda_i \]

(3)

where \( \Delta t \) is the sampling interval. The natural frequencies \( \omega_i \) and damping ratios \( \zeta_i \) of the structure can be extracted from the eigenvalues \( \mu_i \) by the following relation [6]:

\[ \mu_i, \bar{\mu}_i = -\xi \omega_i \pm j \omega_i \sqrt{1-\xi^2} \]

(4)

where \( j \) is the imaginary unit \((j^2 = -1)\). The mode shapes can be obtained from the eigenvectors \( \Phi \) of \( \hat{A} \) [6]:

\[ \Phi = C \Psi \]

(5)

4 RESULTS

The two vibration transducers only were used to define the natural frequencies and damping ratios, whereas the natural modes were defined using the transfer functions between the measurement point and the reference point at the identified natural frequencies [1]. This method is assumed to work well if there are no closely spaced modes and if the disturbance does not coincide with the natural frequencies. However, some of the modes may have been missed, because the two vibration transducers used for the system identification were located at the mid-span, which is a possible nodal line of asymmetric modes.

In the present study, the stabilisation diagram was used to select the optimal model order. In the stabilisation diagram, the identified poles are plotted against the model order. Five different stabilisation degrees were defined: new pole; stabilised frequency; stabilised frequency and mode; stabilised frequency and damping; and stabilised frequency, damping, and mode. A typical stabilisation diagram is shown.
in Figure 5. In this study, the stabilisation conditions were: 0.5% difference for the frequency, 5% for damping ratio, and 2% for natural mode. It can be seen that the model was well stabilised at model orders between 10 and 15. The poles satisfying the following conditions only were plotted:

- Eigenvalues of matrix A must appear in conjugate pairs;
- Natural frequency must be greater than zero;
- Damping ratio must be between 0 and 10%.

Another identification was performed using the five accelerometers producing disturbances, and a typical stabilisation diagram is shown in Figure 6. It can be seen that the analysis resulted in more poles, and a higher model order had to be used to extract the structural modes. The harmonic spurious modes appeared at equal intervals and could be distinguished as having a very low damping ratio. The mode shapes were also extracted from the stochastic subspace analysis.

It was seen that the identified natural frequencies differed between each set-up, decreasing from set-up 1 to set-up 3. It can be assumed that all the variations in the frequencies were due to temperature variation during the day. Many studies have shown that temperature variation may cause high variation in the natural frequencies of civil engineering structures \[E-11\]. The identified damping ratios varied considerably between each set-up.

The comparison between the mode shapes from the two analyses is shown in Figures 7–11. The identified natural frequencies and damping ratios from each set-up are also shown. One additional structural mode was found when using all the accelerometers because the nodal points of the mode were located close to the mid-span. It should be noted that the modes from the spectra led to operational modes rather than natural modes, as can be seen on top of Figure 10, where mode 3 also affected the fourth mode shape.

![Figure 5](image1.png)

**Figure 5.** Stabilisation diagram of set-up 2, record 16 using the HBM 82 vibration transducers only. '+' is a new pole; 'o' stabilised frequency; 'x' stabilised frequency and damping ratio; and '0' stabilised frequency, damping ratio and natural mode.

![Figure 6](image2.png)

**Figure 6.** Stabilisation diagram of set-up 2, record 16 using five accelerometers. '+' is a new pole; 'o' stabilised frequency; 'x' stabilised frequency and damping ratio; and '0' stabilised frequency, damping ratio and natural mode.

![Figure 7](image3.png)

**Figure 7.** Mode 1 from spectral (top) and ssi (bottom) analyses. First longitudinal bending mode, 4.9 – 5.3 Hz.
Figure 8. Mode 2 from spectral (top) and ssi (bottom) analyses. First torsion mode, 6.4 – 6.8 Hz.

Figure 9. Mode 3 from ssi analysis. Asymmetric mode, 8.4 – 8.5 Hz.

Figure 10. Mode 4 from spectral (top) and ssi (bottom) analyses. First lateral bending mode, 8.7 – 8.9 Hz.

Figure 11. Mode 5 from spectral (top) and ssi (bottom) analyses. Second lateral bending mode, 11.1 – 11.5 Hz.
5 CONCLUSION

An experimental modal analysis of the Hämensilta Bridge was performed using responses due to unknown traffic excitation. The stochastic subspace technique was used for the identification of the modal parameters. Since the signals from the accelerometers consisted of spurious frequencies, two analyses were performed — the first using two vibration transducers only, the signals of which were not corrupted, and the second using all five accelerometers. The latter analysis was made at a higher cost, because a higher model order had to be used, but it resulted in an additional structural mode. It also led to natural modes rather than operational modes. The estimated natural frequencies decreased from setup 1 to setup 3, which is probably due to temperature variation. The reason for the spurious frequencies was not clear. The corrupted signals should be detected and avoided already on site, but very often, spurious frequencies can not be avoided, e.g. rotating machines cause harmonic vibrations not corresponding to their natural frequencies. The ability to distinguish these vibrations in a rotating machine is an important application [12, 13] and was studied recently by the author in an industrial assignment.

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REFERENCES


