Approach to Dynamic Modeling of Aircraft Landing on Moving Ships

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ABSTRACT
This paper presents an application of the dynamic response and load calculations on a tricycle type landing gear of an aircraft. The formulation based on the Lagrange’s equations of motion treats the aircraft landing on a stationary deck in terms of pitching motion, roll motion, and vertical heaving motion. The landing is simulated as a single point, dual point, and three-point touch down with specified initial conditions. Effect of damping on ensuing motion is discussed. The results for a typical aircraft configuration are presented in the form of state variables and representative dynamic loads are shown in the form of graphs. In order to see the effect of motion of the deck, the equations are modified to include the deck movement into the formulation. Some preliminary results are presented in this case.

Keywords: Landing Gear Dynamic Loads, Ship Motion, Aircraft Carrier Landing

1. INTRODUCTION:
Aircraft landing is a challenging problem in that it involves the complex transfer of loads of the aircraft in flight with all six degrees of freedom motion to the dynamic motion of the aircraft on landing with elastic elements of the landing gear becoming engaged. The loads coming on to the aircraft during various maneuvers on the ground are critical and are to be considered in the design of various structural elements of an aircraft. This study addresses the problem of calculating the dynamic loads associated with aircraft landing based on Lagrange’s equations of motion.

In this paper, we detail an analysis for calculating the dynamic loads arising out of an aircraft landing on a deck. This simplified modeling, useful in the preliminary design of landing gears, treats the landing as single point, two point and three point touch down with the equations of motions developed for a tri-cycle type of landing gear. The aircraft is assumed to have pitching, roll, and vertical heaving degrees of freedom. The Lagrange’s equations of motion are developed for the tri-cycle landing gear, and the resulting equations of motion are solved in a multi-phase solution to accommodate various specific initial conditions during the rollout phase. The equations are uncoupled using assumed modes approach and the resulting uncoupled equations are solved for displacements, velocities, and accelerations of the basic state variables. Some sample calculations showing the dynamic loads are presented to illustrate the use of the present approach.

2. EQUATIONS OF MOTION:
The Lagrange’s equations\(^1-4\) may be written as

\[ \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_j} \right) - \frac{\partial T}{\partial q_j} + \frac{\partial V}{\partial q_j} - Q_j = 0 \]  

(1)

for \( j = 1, 2, \ldots, N \). In the equation (1), \( T \) and \( V \) are the kinetic and potential energies of the
system; \( q_j \) are the generalized coordinates and \( Q_j \) are the corresponding generalized forces for an \( N \) degrees of freedom system.

Consider an aircraft with its mass at the center of gravity having a tri-cycle gear configuration as shown in Figure 1. Let \( k_{LM}, k_{RM}, \) and \( k_N \) denote the stiffnesses of the left main (LM) landing gear, right main (RM) landing gear, and nose (N) landing gear respectively of the aircraft. Let \( z, \theta, \) and \( \phi \) represent the vertical, pitch and roll degrees of freedom. Let \( \dot{z}, \dot{\theta}, \) and \( \dot{\phi} \) represent the vertical, pitch and roll rates, while \( \ddot{z}, \ddot{\theta}, \) and \( \ddot{\phi} \) represent the vertical, pitch and roll accelerations of the aircraft.

The potential energy, the Rayleigh damping function, and the kinetic energy of the system are given by the following expressions:

\[
V = \frac{1}{2} k_{LM} \left( z + b\theta - \frac{c}{2} \phi \right)^2 + \frac{1}{2} k_{RM} \left( z + b\theta + \frac{c}{2} \phi \right)^2 + \frac{1}{2} k_N (z - a\theta)^2
\]

\[
R = \frac{1}{2} c_{LM} \left( \dot{z} + b\dot{\theta} - \frac{c}{2} \dot{\phi} \right)^2 + \frac{1}{2} c_{RM} \left( \dot{z} + b\dot{\theta} + \frac{c}{2} \dot{\phi} \right)^2 + \frac{1}{2} c_N (\dot{z} - a\dot{\theta})^2
\]

\[
T = \frac{1}{2} m(\ddot{z})^2 + \frac{1}{2} I_{yy}(\ddot{\theta})^2 + \frac{1}{2} I_{xx}(\ddot{\phi})^2
\]

Noting that

\[
Q_j = -\frac{\partial R}{\partial \dot{q}_j}
\]

the Lagrange’s equations of motion become

\[
m \dddot{z} + (c_N + c_{LM} + c_{RM}) \dddot{z} + (b c_{LM} + b c_{RM} - a c_N) \dddot{\theta} + \frac{c}{2} (c_{RM} - c_{LM}) \dddot{\phi} + (k_{LM} + k_{RM} + k_N) z + (b k_{LM} + b k_{RM} - a k_N) \theta + \frac{c}{2} (k_{RM} - k_{LM}) \phi = 0
\]

\[
I_{yy} \dddot{\theta} + (b c_{LM} + b c_{RM} - a c_N) \dddot{z} + (b^2 c_{LM} + b^2 c_{RM} + a^2 c_N) \dot{\theta} + \frac{b c}{2} (c_{RM} - c_{LM}) \dot{\phi} + (b k_{LM} + b k_{RM} - a k_N) z + (b^2 k_{LM} + b^2 k_{RM} + a^2 k_N) \theta + \frac{b c}{2} (k_{RM} - k_{LM}) \phi = 0
\]

\[
I_{xx} \dddot{\phi} + \frac{c}{2} (c_{RM} - c_{LM}) \dddot{z} \]

\[
+ \frac{b c}{2} (c_{RM} - c_{LM}) \dot{\theta} + \left( \frac{c}{2} \right)^2 (c_{RM} + c_{LM}) \dot{\phi} + \frac{c}{2} (k_{RM} - k_{LM}) z + \frac{b c}{2} (k_{RM} - k_{LM}) \theta + \left( \frac{c}{2} \right)^2 (k_{RM} + k_{LM}) \phi = 0
\]

The equations of motion may also be written in a matrix form as follows:

\[
\mathbf{M} \ddot{\mathbf{q}} + \mathbf{C} \dot{\mathbf{q}} + \mathbf{K} \mathbf{q} = \mathbf{0}
\]

where \( \mathbf{M}, \mathbf{C}, \) and \( \mathbf{K} \) are the mass, damping, and stiffness matrices respectively.

A MATLAB based program is developed to solve the equations (9). The landing of the aircraft is simulated by performing the following operations:

- The aircraft is assumed to land on one landing gear at touch down. Say, right main landing gear (RMLG) is selected arbitrarily as the first point of contact. Then, the
stiffness of the RMLG, \( k_{RM} \), is non-zero, and other stiffnesses are assumed to be zero. Similarly, only the mass of the aircraft is invoked and the equations are solved for natural frequencies and mode shapes.\(^5,6\) The response is a single degree of freedom system with the appropriate initial conditions.

- The response from the first step is used to solve the landing gear that touches the deck next, using the geometry of the aircraft and landing gear spacing. The state variables corresponding to this state will form the initial conditions for the second phase. The time for the end of the first stage is computed graphically. The state variables at this time are calculated and used as the initial conditions for the second phase.

- In the second phase, the two main landing gears are in contact with the deck, with \( k_{RM} \) and \( k_{LM} \) stiffnesses activated, and \( k_N \) being still zero. The initial conditions are used to solve the two-degree of freedom model. Again the state variables are calculated and pitch response is plotted to observe when the nose gear touches the deck. This time is noted and the corresponding state variables are calculated, which form the initial conditions for the third phase.

- In the third phase, all the three landing gear stiffnesses are activated. The three degree of freedom system with the initial conditions obtained from the termination stage of the second phase is solved for the response.

- Once the three displacement variables are determined, the loads may be computed from the geometry of the layout of the tri-cycle landing gear.

**3. RESULTS:**

The formulation and the method described in the previous sections are illustrated by applying to a typical aircraft.

The basic parameters selected are as follows:

\[
\begin{align*}
\text{Weight} & = 13,700 \text{ lbs.} \\
a & = 12.2867 \text{ ft.} \\
b & = 1.6167 \text{ ft.} \\
c & = 12.7933 \text{ ft.} \\
I_{xx} & = 5200 \text{ slug} \cdot \text{ft}^2 \\
I_{yy} & = 19,000 \text{ slug} \cdot \text{ft}^2 \\
k_{RM} & = 300,000 \text{ lb/ft} \\
k_{LM} & = 300,000 \text{ lb/ft} \\
k_N & = 300,000 \text{ lb/ft} \\
v_z & = 24.6 \text{ ft/s}
\end{align*}
\]

where \( a \) is the distance between the c.g. of the aircraft and the nose landing gear, \( b \) is the distance between the c.g. and the left or right main landing gear, and \( c \) is the distance between the two main landing gears (Figure 1). The mass moments of inertia about x-axis and y-axis are \( I_{xx} \) and \( I_{yy} \) respectively, while the sink velocity or the vertical rate of descent is \( v_z \).

Figure 1 shows the lay out of the aircraft and landing system as a three degrees of freedom system. The system is first solved for natural frequencies and the modes shapes.\(^5,7\) Then mode superposition method is used to compute the response. The equations are uncoupled by transforming them using the modal matrix.

The first phase when RMLG is active, the stiffnesses \( k_{LM} = 0 \) and \( k_N = 0 \). The eigen value problem yields one non-zero eigen value and the natural frequency is given by \( \omega_1 = 3106 \text{ rad/sec} \).

The state variables, displacement, pitch, roll, and the corresponding rates are shown in Figure 2 and 3 respectively.

Using the geometry of the main landing gear spacing, as shown in Figure 1, the end of phase I is characterized by the solving the geometric constraint problem when the angle of tilt (roll) becomes zero. This signifies that the left main landing gear touches the deck. The time \( \tau_1 \) to achieve this is read from the Figure 4, where the y-ordinate crosses the x-axis.

The initial conditions at time \( \tau_1 = 0.008 \) is
found to be
\[
\begin{align*}
z &= 1.9215 \text{ in} \\
\theta &= 0.1211 \text{ rad} \\
\phi &= 0.0517 \text{ rad} \\
\dot{z} &= 219.84 \text{ in/s} \\
\dot{\theta} &= -0.1984 \text{ rad/sec} \\
\dot{\phi} &= -3.2858 \text{ rad/sec}
\end{align*}
\]  \hfill (10)

The equations of motion for the phase II is now solved with $k_{LM}$ and $k_{RM}$ active and the initial conditions given by (10). Figures 5 and 6 give the response displacement and rate quantities as computed using the developed program. The end of phase II is characterized by the pitch angle variation such that the nose gear touches the deck. This is calculated again by the geometry of the aircraft and landing gear layout given in Figure 1. The time $\tau_2$ at which the phase II ends, signifying the nose gear touch down is obtained from the graph shown in Figure 7. The state variables corresponding to this value of $\tau_2$ becomes the initial conditions for the third and final phase, and is given by
\[
\begin{align*}
z &= 1.0069 \text{ in} \\
\theta &= -0.0419 \text{ rad} \\
\phi &= 0.000 \text{ rad} \\
\dot{z} &= 12.368 \text{ in/sec} \\
\dot{\theta} &= -0.8243 \text{ rad/sec} \\
\dot{\phi} &= -0.00609 \text{ rad/sec}
\end{align*}
\]  \hfill (11)

The initial conditions (11) are used in computing the phase III response of the system. The three displacements and there rates are shown in Figures 8 and 9. Once these displacements are known, the loads on the landing gear may be computed in a straight forward manner. The loads calculated in each of the three phases are shown in Figures 10-12, from which the maximum loads are used in the design of the landing gears.

The method developed here may easily be changed to accommodate the motion of the ship. There are several ways of incorporating the ship motion described by the pitch, roll, and heaving degrees of freedom. Each of these quantities prescribed affect the initial conditions of various phases. Results from such studies are underway and will be reported elsewhere at a later date.

\section{4. CONCLUSIONS}

An approach to compute dynamic loads on the tricycle type landing gear of an aircraft is presented. The equations of motion are derived from Lagrange’s equations. The landing is accomplished in three phases and the final stages of each phase corresponds to the initial conditions of the following phase. Calculation of the landing gear loads are described, which may prove to be of use in the preliminary design and sizing of the landing gears.

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\section{REFERENCES}

Figure 1 Aircraft Landing Gear Layout

Figure 2 Displacement, Pitch, and Roll in Phase I

Figure 3 Displacement, Pitch, and Roll Rate – Phase I
Figure 4 Roll Response in Phase I (t1=0.008 secs.)

Figure 5 Displacement, Pitch, and Roll – Phase II

Figure 6 Displacement, Pitch, and Roll Rate – Phase II
Figure 7 Pitch Response – Phase II; zero cross-over at t = 0.186 sec

Figure 8 Displacement, Pitch, and Roll – Final phase

Figure 9 Displacement, Pitch and Roll Rates – Final Phase
Figure 10 Loads History in Phase I

Figure 11 Loads History – Phase II

Figure 12 Loads History – Phase III