Comparison of Scanning Laser Measurements and Inverse Boundary Element Method

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NOMENCLATURE

\( p \)  Sound pressure
\( v \)  Normal velocity
\( H \)  Transfer function
\( F_d \) Doppler frequency
\( \lambda \) Wavelength of laser source

ABSTRACT

Two different methods for non-contact structural vibration measurement are compared by experiment. One method is the Scanning Laser Doppler Vibrometry (SLDV) method. This technique provides a point-by-point sampling of the surface of a structure, thereby producing a vibration map. The other method is based on acoustic holography and makes use of acoustic measurements around the structure. The measurement data are fed into an Inverse Boundary Element Method (IBEM) allowing for reconstruction of the structural surface vibration. For this method, however, only an approximation of the vibration pattern is obtained. The quality of the IBEM surface vibration reconstruction is assessed for a small experiment using a loudspeaker and compared to the results from laser measurements. Excellent agreement is found between absolute vibration levels determined by the two different methods.

INTRODUCTION AND THEORY

Non-contact measurement techniques for vibration analysis have the advantage compared to more traditional transducer mounting methods that they totally eliminate mass loading and thereby do not change the structure’s inherent dynamics. Furthermore, they allow for doing measurements under demanding conditions like e.g. measuring on surfaces with high temperatures. As the different techniques have different unique advantages, non-contact measurement techniques are not “just” used for applications where traditional contact transducers are inapplicable, but also as a fast alternative to traditional contact measurement techniques.

A variety of non-contact vibration measurement techniques exist. One of the most well-established and well-proven techniques is based on Laser Doppler Vibrometers (LDVs), either a Single-point LDV, where one point is measured and the laser sensor head is moved to measure another point or a Scanning LDV (SLDV), where an internal system of mirrors position the laser beam at specified scan positions without moving the laser sensor head.

The LDV principle is based on interferometry, where a laser beam is divided into an internal reference beam and a measurement beam. The measurement beam is directed onto a vibrating test surface and the back-reflected light recombined with the internal reference beam. When the test surface moves, the path difference between the routes followed by the reference and measurement beams changes, resulting in light-intensity modulation of the
recombined beam due to interference between the reference and the measurement beams. The frequency of the intensity modulation, known as the Doppler frequency $F_d$, is directly proportional to the surface velocity $v$:

$$F_d = 2v / \lambda$$

The recombined beam is split into two paths. A quarter-wave plate is used in one of the paths so that the two paths are in quadrature allowing the direction of motion to be determined. By mixing and demodulating the signals, a voltage output with mean value proportional to the surface velocity is obtained. This output is then fed into an analyzer for vibration analysis.

In addition to the before mentioned general advantages of using non-contacting measurement techniques, the LDV principle allows for measurement of structures containing inaccessible parts, measurement of large or distant structures and measuring in demanding environments such as high-radiation fields, high-voltage areas, clean rooms or wind tunnels. Furthermore, LDVs support higher frequency ranges than traditional contact transducers. Where a Single-point LDV can be the ideal choice when only a few points have to be measured, a SLDV provides additional benefits such as high spatial resolution and faster and more flexible data acquisition.

Another method for non-contact vibration measurement is the Inverse Boundary Element Method (IBEM). This method uses purely acoustic measurements around the structure to reconstruct the surface normal vibration and is based on a numerical model of the acoustic environment surrounding the structure. The numerical model relates the measured sound field around the structure, as represented by a vector of discrete sound pressure sampling points, to the surface normal velocity of the structure as represented by a vector of discrete surface velocity sampling points. These points correspond to the nodes of a surface mesh discretizing the surface of the structure. The model should thus include the structure under test and any other obstacles close to it and in many cases also the hard ground under the structure. The relation between field sound pressures and surface node velocities may be written in matrix-vector form as

$$
\begin{pmatrix}
  p_1 \\
  p_2 \\
  \vdots \\
  p_m
\end{pmatrix} =
\begin{pmatrix}
  H_{11} & H_{12} & \cdots & H_{1j} & \cdots & H_{1n} \\
  H_{21} & H_{22} & \cdots & H_{2j} & \cdots & H_{2n} \\
  \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
  H_{m1} & H_{m2} & \cdots & H_{mj} & \cdots & H_{mn}
\end{pmatrix}
\begin{pmatrix}
  v_1 \\
  v_2 \\
  \vdots \\
  v_j \\
  \vdots \\
  v_n
\end{pmatrix}
$$

where $p_i$ is the measured sound pressure at the $i$th microphone and $v_j$ is the surface normal velocity at the $j$th node of the boundary element mesh. The relation between $p_i$ and $v_j$ is established through the transfer function $H_{ij}$ determined numerically by the Boundary Element Method [1]. Note that the linear system of equations is only valid at a single frequency and must therefore be recalculated for different frequencies.

The purpose of the IBEM algorithm is to reconstruct the normal velocity at every node from the measured field sound pressures. This is essentially an inverse problem involving inversion of the transfer function matrix of (2). The matrix inversion is done by imposing some sort of regularisation since the solution would otherwise become meaningless. This is a consequence of the matrix being almost singular for nearly every case. The physical explanation for this near-singularity is that evanescent source radiation information (radiation components corresponding to fast varying surface velocity components) dies out quickly when radiating from the structure, i.e. it doesn’t give rise to any pressure field far away from the source. Consequently this information can hardly be measured by the microphones and is therefore difficult to reconstruct especially in the presence of noise. In such a case it is better to avoid some of the evanescent information at the price of a reduced surface resolution.

In practice the linear system of equations is most often solved using an approach based on Singular Value Decomposition (SVD) of the matrix of transfer functions followed by a regularisation step essentially grouping the
SVD components into useful and unwanted components. Computing the desired solution, i.e. the structural surface velocity vector is then done via the set of useful components. Deciding on the number of useful components is, however, not trivial but some methods for automatic detection of this subset can be found [2]. Throughout this work, we will be using the so-called L-curve criterion [3], which seeks to determine a solution that fits well to the measured data and at the same time has limited energy. Further details about the IBEM method may be found in [4] and [5].

The numerical sound field model used in IBEM assumes a harmonic, totally coherent sound field as input. In general, this is achieved by deriving a cross spectral sound field description from the measured field sound pressures, based on a number of reference signals representing the uncorrelated or partially correlated radiation mechanisms of the test object under investigation. To deal with partially correlated source mechanisms a principal component decomposition [6] of the sound field is carried out and each component then processed in turn by IBEM. Finally the individual results are added on a power basis to form the total result. However, when using a single reference it is in principle possible to include phase information in the resulting surface vibration output.

At this point it should be noted that while SLDV provides measurement of the actual surface vibration, IBEM provides an estimate of the vibration based on acoustic near-field measurements. Only the part of the vibration causing an acoustic signal at the receiver microphones may be reconstructed by the method. However, when considering structure-borne noise problems such an estimate is indeed relevant. Moreover, the method provides overall source location on arbitrary geometries without need for line-of-sight, and allows for estimation of acoustic quantities like surface sound pressure and intensity. In addition, since conventional BEM can be used to simulate the radiated sound field if the structural vibration pattern is known, the result of an IBEM analysis provides an acoustic source model that may be used in simulations of acoustic radiation in different environments or of the effect on sound radiation of damping parts of the vibrating surface.

In this paper, the two different non-contact techniques are compared: Scanning Laser Doppler Vibrometer (SLDV) and Inverse Boundary Element Method (IBEM). The result of both methods is a map of the structural vibration, which will be compared for the case of a small loudspeaker source. The SLDV is by now considered to be a well-established and accurate measurement vibration measurement technique. Therefore the SLDV method serves here as a reference method for determining the validity of vibration distributions and absolute levels obtained by the IBEM method.

LOUDSPEAKER SOURCE

A 2-way loudspeaker system consisting of a 6" woofer and a 1" tweeter with a crossover frequency around 1.6 kHz and mounted in a closed enclosure (0.23m X 0.33m X 0.18m) was used as sound source for the experiments made. The loudspeaker was fed with a white noise signal having a 3.2 kHz frequency range. In this paper only measurements on the woofer are compared. Consequently, only frequencies below 1.6 kHz are considered. The sound pressure level of the loudspeaker was chosen as a compromise between avoiding non-linear behaviour and excessive background noise. For all frequencies of interest it was between 58 and 64 dB at 50 mm.

LASER MEASUREMENT & ANALYSIS

The setup for the SLDV measurement is shown in Figure 1. The instrumentation consist of the Ometron VPI+ Sensor Head, the Portable PULSE™ 3560-C front-end and a laptop PC running dedicated PULSE™ data acquisition and analysis software. The measurement distance between the sensor head and the woofer is approximately 1.033 m corresponding to 5 coherence lengths. This gives a sufficient focus depth of 41 mm and a maximum scanning angle of less than ± 5 degrees.

A circular scanning mesh consisting of 415 points with a resolution of 5 mm was created. This mesh is shown to the right in Figure 1. Operating Deflection Shapes (ODS) measurements were made using the output from the generator as reference signal and performing FFT analysis in 3.2 kHz, 800 frequency lines and with 10 averages per mesh point. Dithering – a technique by where the laser automatically moves the beam to a nearby position in a controlled manner until sufficient light is reflected - was used to optimize the signal-to-noise ratio and modest smoothing of the deflection shapes using a 5th order polynomial fitter was applied to account for the higher spatial
resolution in the SLDV measurement compared to the spatial resolution in the actual IBEM measurement. No cosine correction was applied and no treatment of the woofer’s surface (paint, powder, reflective tape etc.) was necessary to apply to enhance reflectivity.

**Figure 1.** The setup for the SLDV measurement (left) and the resulting scanning mesh on the woofer (right).

**IBEM MEASUREMENT & ANALYSIS**

The setup for the IBEM measurement is shown in Figure 2. Three planar arrays, each configured with 9 x 6 B&K Type 4935 microphones, were placed in the vicinity of the box, parallel to the box surfaces in order to spatially sample the sound field radiated by the test source. In principle, for a general sound source, the sound field on all sides of the source should be sampled. However, since the present source was known to radiate mainly from the front it was decided to place microphones on three sides only. To avoid spatial aliasing effects the microphone spacing should, as a guideline, be less than half the acoustic wavelength at the maximum analysis frequency. In the present setup the microphone spacing was chosen to be 50 mm, thus corresponding to an upper analysis frequency limit of 3.4 kHz.

**Figure 2.** The setup for the IBEM measurement (left) and the numerical mesh model and field point positions (right).
The average distance to the source was also kept at 50 mm. This distance was chosen as a compromise between ensuring the highest possible surface vibration resolution with the given microphone spacing and at the same time ensuring sufficient decay of very fast spatially varying radiation components so that the spatial sampling criterion would indeed be fulfilled at the microphone positions [4]. The positions of all field sampling points relative to the test object geometry were furthermore measured, since this information is required for setting up the numerical BEM model relating the surface vibration to the field pressures.

Each of the 162 microphones were connected to individual input channels of a B&K Type 3561 Intelligent Data Acquisition (IDA) front-end, which in turn was connected by LAN to a laptop PC running dedicated B&K data acquisition software. All microphones were individually calibrated before measurement. With the speaker source running, simultaneous acquisition from all microphones was done using a sampling rate of 8192 Hz, corresponding to an upper frequency limit of 3.2 kHz. The measurement time was 30 seconds. Following the measurement, a cross-spectral sound field description suitable as input for IBEM was calculated by FFT averaging. One of the array microphones closest to the speaker woofer was used as reference, since the output signal from this microphone was assumed to be highly correlated with the speaker driver signal.

A boundary element surface mesh of the loudspeaker box was constructed from a rough sketch but still including the diaphragm of the large loudspeaker. The triangular mesh was designed to a maximum edge length of 45 mm. As a rule of thumb, the edge length should not exceed one quarter of the acoustic wavelength at the maximum analysis frequency [4]. This means that the upper frequency limit for BEM with this mesh was 2 kHz. The resulting mesh consisted of 330 elements and 167 nodes as shown in Figure 2. Note that this figure also shows the microphone positions relative to the test object geometry.

RESULTS & DISCUSSION

The left-hand side of Figure 3 shows the ODS at 500 Hz as obtained with the SLDV method. The plot clearly shows how the woofer at this frequency moves as a piston (close to uniform velocity distribution over the complete diaphragm). The small pyramid shown in the centre of the mesh marks a point where the velocity level is read out to compare with IBEM. The right-hand side of Figure 3 shows the ODS at 1240 Hz. At this frequency the membrane no longer moves as a piston, rather parts of the diaphragm are vibrating out of phase.

Figure 3. Surface velocity results of the SLDV measurements at 500 Hz (left) and 1240 Hz (right).
Figure 4 shows examples of surface velocity contour plots obtained via the IBEM method. The left-hand side of the figure shows the result at 500 Hz. As this frequency is well below the crossover frequency of the 2-way speaker, the woofer is the dominating sound source and the only part of the speaker vibrating significantly. While the overall source location is excellent, note that in contrast to the corresponding SLDV result at the same frequency, the IBEM result does not show a uniform velocity distribution over the diaphragm. Instead, the velocity distribution has a peak at the woofer’s centre and decays towards its edge. This behaviour is a result of the fact that only an estimate of the surface vibration is obtained by the IBEM method. As previously mentioned, only the slowly spatially varying (smooth) surface vibration components may be reconstructed by the method, since the fast varying components generate exponentially decaying acoustic waves that are difficult to pick up and reverse. The obtainable resolution depends on the signal-to-noise ratio of the decaying waves at the microphone positions. Thus the estimated surface velocity distribution seeks to emulate the real physical step change in normal velocity at the woofer edge by producing a slowly varying distribution. The right-hand side of the figure shows the result at 1.24 kHz. At this frequency the surface normal velocity of the tweeter is significantly higher than that of the woofer. Again the general source location is very good.

As mentioned earlier, the regularization of the inverse problem was done using the so-called L-curve criterion in which solution accuracy is weighted against solution norm. As it turned out, the regularization determined by this method was found to be a little bit too “aggressive” for the problem at hand, causing the determined surface solution distributions to be non-smooth. The reason for this behaviour can be related e.g. to phase error in the measurement chain, to position measurement errors or to the mesh density relative to the microphone spacing. By relaxing the determined regularization by a fixed small amount, a smooth solution was obtained.

Comparing the two methods, it is already clear that they do not yield identical surface vibration maps since the IBEM method provides an estimate based on sound radiation. However, it is interesting to use SLDV as a reference method in order to investigate the accuracy of the absolute surface normal velocity estimate obtained by the IBEM method. Figure 5 shows the measured surface velocity at the centre of the woofer as determined by the SLDV and the IBEM methods. For the SLDV method the level was determined by simply reading out the value in the laser sampling point closest to the woofer’s centre (as marked by the cursor pyramid). For the IBEM method the level was determined by reading out the value in the node placed at the woofer’s centre. The agreement between the absolute levels as determined by the two methods is excellent in the frequency range between 40 Hz and 1.2 kHz. Above 1.2 kHz the velocity estimated by the SLDV method is larger than the one determined by the IBEM method. Part of the explanation for this can be found by looking again at the IBEM result in the right-hand side of Figure 4. Due to the limited dynamic range of the IBEM method (as determined by the signal-to-noise ratio) the estimate of the woofer vibration level may be assumed to be inaccurate at this frequency because the tweeter is the dominating source and is furthermore placed quite close to the woofer. Also, above this frequency the woofer no longer behaves as a simple piston. Whereas the SLDV method is able to measure this behaviour, the resolution of the IBEM method prevents representation of such complicated diaphragm vibrations. This will naturally result in deviations between the SLDV and IBEM results in this frequency range.
Figure 5. Comparison of surface velocity results from IBEM and SLDV methods at the woofer centre.

Figure 6. Comparison of surface velocity results from IBEM and SLDV methods halfway between centre and edge.

Figure 7. Comparison of surface velocity results from IBEM and SLDV methods at the woofer edge.
Figure 6 shows the measured surface velocity halfway between the centre and the edge of the woofer. Again the agreement between the results of the two methods is quite good. Note, however, that the velocity determined by the SLDV is in general slightly higher than the one determined with IBEM. This corresponds well with the SLDV showing uniform velocity distribution over the diaphragm and the IBEM distribution decaying towards the edge of the woofer.

Finally figure 7 shows the measured surface velocity near the edge of the woofer. Again the two results show the same trend as a function of frequency. Note that the difference between the absolute levels is here even larger, corresponding to the IBEM solution decaying further at the edge position. It should be noted that the IBEM results above 1.3 kHz are associated with considerable uncertainty since the difference between tweeter and woofer levels is here comparable with the dynamic range of the method.

CONCLUSIONS & FUTURE WORK

Two different methods for non-contact surface vibration measurement have been compared by experiments on a small speaker source: Scanning Laser Doppler Vibrometry and the Inverse Boundary Element Method. Although totally different in principle and implementation, the two methods have shown very good agreement in terms of absolute measured vibration levels. Since the SLDV method is well established it is considered as reference and thus the measurements have served to verify the accuracy of the IBEM method in terms of absolute vibration levels.

The IBEM method provides a limited resolution estimate of the surface vibration from acoustic measurements. This means that for detailed surface vibration analysis the SLDV method is superior – especially at high frequencies where the demands on mesh density and microphone spacing in IBEM become prohibitive. However, for overall source location and structure-borne sound analysis, the IBEM method is a viable alternative to SLDV. Moreover, the surface BEM solution obtained with IBEM may be used directly in further simulations.

Future work may include measurements on more complicated structures like vibrating panels and other industrial applications. Moreover, it is the plan to compare the results obtained with SLDV and IBEM to results of traditional planar Near-field Acoustical Holography for further verification of this method.

REFERENCES


