The Instantaneous Frequency Estimation In Rotating Machinery

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ABSTRACT

Stages of run-up or run-down of rotating machinery are unstable random process. The methods of time-frequency analysis, such as short time Fourier transform, wavelet transform and so on, will be employed to analyze the relevant test signals. To obtain the laws of the instantaneous frequency of multi-component signals during the run-up or run-down periods, the method based on peak search is widely used. However, the direct search cannot assure accuracy and precision of the results because of the interference of strong noise and adjacent components of the signal. In this paper, the Hidden Markov Models are used to solve the problem. Simulation indicated that better results can be achieved by the method based on HMM.

Keywords: HMM; time-frequency analysis; instantaneous frequency; peak search

1 INTRODUCTION

In the Order Tracing Method based on Time-Frequency Analysis, how to acquire the Instantaneous Frequency Estimation (IFE) of its relevant Reference Axis Rotational Speed accurately is the key technology[1]. If this method is used to acquire IFE from Time Frequency Representation’s(TFR), when the Signal Noise Ratio(SNR) is low, the false peak value caused by noises may overtake the real value and thus brings significant errors to IFE. Besides, the existing strong components among the multivariable component signals also add difficulties to Accurate Tracing[2,3].

Peak Tracking is used to eliminate the false peak values caused by noises or disturbance, and it can be achieved by the continuity of most Frequency Modulate (FM) signals (i.e., speech signal, rotating machinery signal, etc)[4]. Any high discontinuous value found by the tracking will be removed even though they might have high amplitude.

Hidden Markov Models (abbr. HMM)[5] have been applied to many areas such as speech recognition and estimation of instantaneous frequency, etc[6-15]. The HMM based on Peak Tracking established by this paper has managed to trace Instantaneous Frequency more accurately under Rotary Machinery Speeding up and Down in low SNR.

2 HMM

HMM is a type of important dual random model in mathematics which includes N states that transit from one state to another randomly and the relationship between them is expressed by the state transition probability matrix $A$, see (1)
Where element $a_{ij}$ denotes the probability of transiting from state $i$ to state $j$ such that the following relations (1) is satisfied:

$$
\sum_{j=1}^{N} a_{ij} = 1 \quad (a_{ij} \geq 0)
$$

The results observed are also random and are dictated by the distributed probability matrix $B$:

$$
B = \begin{pmatrix}
    b_{11} & b_{12} & \cdots & b_{1M} \\
    b_{21} & b_{22} & \cdots & b_{2M} \\
    \vdots & \vdots & \ddots & \vdots \\
    b_{N1} & b_{N2} & \cdots & b_{NM}
\end{pmatrix}
$$

Where each element $b_{ij}$ represents the $j$th probability under the observation state $i$, which satisfies the following rule (4)

$$
\sum_{k=1}^{M} b_{ik} = 1 \quad b_{ij} \geq 0, \quad i = 1,2, \ldots, N
$$

$M$ is the probable Observation Result Number in relation to each state; the initial state probability vector $\pi = (\pi_i)_{1 \times N}$ stands for the occurring probability of each state at the initial moment. $A, B, \pi$ compose a HMM, recorded as $\lambda(A, B, \pi)$. HMM represents the rule of a stochastic event.

According to this model, we can put forward the question: Given an observation sequence $O = (o_t)_{T}$, which one of the state sequences $q = (q_1, q_2, \ldots, q_T)$ will most likely correspond to the sequence $O$? In other words, what is the most probable Union Probability $P(O, q | \lambda)$ between the result $O$ and the state consequence $q$.

This paper is to establish a Time-frequency Spectrum HMM based on Rotary Machinery Speeding up/down Testing Signal, and solve the `Peak Value Tracing problem based on TFR.

### 3 HMM OF TIME-FREQUENCY SPECTRUM

The speeding up and down of Rotary Machinery is an unstable random process. In order to obtain the relationship between the rotation speed and time in the speeding up and down process and to ascertain it’s state, Time-Frequency Distribution Analysis methods such as STFT and Wavelet Transformation are required to analyze its test signals. Fig 1 and Fig 2 are the STFT spectrums of a simulated Rotary Machinery signal of several components between the speeding up and down of a rotary machine. STFT spectrums are expressed as $|X_{TFR}(t,f)|$. If the frequency at each time moment is represented by the peak of STFT spectrogram, i.e., the frequency that has the most energy at this time[2,3], it is evident that we can get good results through the method of finding extreme points at this time moment for the STFT spectrogram under low noise and little interference. But the method above is not ideal for signals with high noise and interference because the results have significant errors. Although we can improve the results by finding local peak points in certain frequency segment at each time moment, we must use the basis of observation values at certain time, which will cause the result’s precision to be determined by that of the observation value.

For Speech Signals, HMM Tracing Technology can effectively trace the peak value of Signal Time-frequency
Spectrum and also remove the fake values caused by noise and disturbance. Therefore, we establish a HMM model based on the Observation Signal TF Spectrum of Rotary Machinery Speeding Up/Down Process. The method is illustrated as follows:

In the digitization chart of STFT spectrum, the Time-Frequency plate is composed of discrete time and frequency grids. Assuming that the Frequency Grids are encoded from 0 to $N_f$, we divide them into subsections, i.e. every $s$ grids form a subsection and the total number of subsections is $N = \lceil N_f/s \rceil + 1$ ($\lceil x \rceil$ is an integer no more than $x$). Grid $i$ is in subsection $j = \lceil i/s \rceil + 1$. The purpose is to reduce the state number of HMM, and consequently the order of state transition probability matrix and computing time are reduced. If the maximum frequency of the tested signal is known, the data that exceeds the maximum frequency can be removed from the STFT spectrum, and then the subsection is made, which can also reduce the order of state transition probability matrix and computing time quickly.

During the speeding up and down process of Rotary Machinery, the frequency goes up and down in contrast to time, but every moment the frequency changes with several random variables influenced by various factors. In the whole time period, the probability of Rotary Machinery at a certain frequency subsection is different. Therefore, let’s assume that there are $N$ states, in which state $i$ has higher probability in subsection $i$ and has the same probability in other subsections. In this way, the probabilities in different frequency subsections of all the state form in a state observation probability matrix $B$, where element $b_{ij}$ should satisfy the following relationship (5). The number $M$ of every state observation result equals $N$.

$$\sum_{j=1}^{N} b_{ij} = 1 \quad (b_{ii} > b_{ij} \geq 0, \ j \neq i) \quad (5)$$

The initial state probability vector $\pi = (\pi_i)_{1 \times N}$ signifies the State Occurrence Frequency at the initial moment, which can be determined by the initial state of STFT spectrum ($t=1$). Assume the initial state is $i$, define $\pi_i=1$, and let other vector element be 0. The Observation probability vector $O=(O_t)_{1 \times T}$ is determined by the maximum spectrum value of every moment ($1 \leq t \leq T$). The relevant frequency grid of the maximum spectrum being $k$, let $O_t=\lceil k/s \rceil + 1$.

In order to ensure the State Transition Matrix $A=(a_{ij})_{N \times N}$, we select the STFT spectrum of the signal with low noise.
and little interference to establish it. Let $q_t$ be the state at moment $t$ ($1 \leq t \leq T$), namely $q_t$ is the number of the frequency subsections in which the maximal point of the spectrum at moment $t$ is. $P_{ij}$ is the number of state $i$ at moment $t$ and state $j$ at the moment $t+1$, or the number of the state $i$ transiting to state $j$, i.e.

$$P_{ij} = \sum_{t} (\delta_{q_{t},i} \times \delta_{q_{t+1},j}) \quad (i, j = 1, 2, ..., N)$$

(6)

$\delta$ is Function Dirac delta, i.e.

$$\delta_{m,n} = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases}$$

(7)

Thus the state Transition Matrix is calculated as:

$$a_{ij} = \frac{P_{ij}}{\sum_{j} P_{ij}} \quad (i, j = 1, 2, ..., N)$$

(8)

In order to avoid $\sum_{j} P_{ij} = 0$, we add a positive number to every diagonal and sub-diagonal element of Matrix $P$. It is reasonable because the instantaneous frequency of the testing signal in stages of run-up or run-down of rotating machinery is continuous and therefore every state is easier to transit to its closest state than any other. The integer value can be adjusted according to the signal until a satisfying result is obtained. Fig. 3 represents the State Transition Matrix $A$ of the STFT spectrum corresponding to Fig. 1.

The other method to avoid $\sum_{j} P_{ij} = 0$ for a given $i$ is to do the same as expression (4) with the STFT spectrums of the Rotary Machinery’s different Sampled Signals having low noise and little interference, and to add these matrices acquired for every such STFT spectrum to matrix $P$ till $\sum_{j} P_{ij} \neq 0$ for all $i$. The latter method is closer to reality than the former.

After working out matrix $P$, the state Transition Matrix $A$ is obtained as (8). Accordingly we’ve made the HMM Model: $\lambda = (A, B, \pi)$

### 4 PEAK TRACING

The purpose of Peak Tracing is to get IF (Instantaneous Frequency) at any given moment. IF is defined as the derivative of signal phases[16]. A signal is given as follows:

$$x(t) = \exp\left(\int_{\infty}^{t} f_i(\tau)d\tau + N(t)\right)$$

(9)

Its sampling function (with high sampling frequency) is $x(n)$. In (7), $N(t)$ is complex Gauss white noise with mean 0 and variance $\sigma^2$, and $f_i(\tau)$ is the actual IF (the modulation signal in the process). To estimate $f_i(n)$ of $x(n)$, for example finding $\hat{f}_i(n)$, we let $\hat{f}_i(n) \approx f_i(n)$.

One method to get $\hat{f}_i(n)$ is to use TFR peak values based on TFR. Mathematically, this estimate is

$$\hat{f}_i(n) = \arg \max_{f} \left\| X_{TFR}(t,f) \right\|$$

(10)

This is because the frequency that has the highest energy density at every moment is the most probably IF. In (10), $X_{TFR}(t,f)$ is the TFR of $x(t)$, namely TFS. $\arg \max$ denotes the maximum values parameter. The vibration signal related to rotational speed in run-up or run-down of rotating machinery is also Frequency Modulation Signals
and thus can make IFE by TFR.

For complicated Rotary Machinery, the signals include multi-rotators’ frequency components and noise-resulted frequency components. In order to get the necessary components, calculating the maximum of the related frequency components or local maximum directly from its signal STFT spectrum will introduce significant error. Utilizing HMM established above to make peak search has translated the problem of seeking the frequency components at every moment to the problem of seeking the best path.

First we give an observed result \( O = (o_t)_{1\leq T} \) and an HMM \( \lambda \), then seek a path \( (q_1^*, q_2^*, \ldots, q_T^*) \), namely the subsection that IF at every moment located in, to make it process maximal probability \( P \) under HMM \( \lambda \) and observed result \( O \), as follows:

\[
P(q_1^*, q_2^*, \ldots, q_T^* \mid O, \lambda) = \text{Max}_{q_1, q_2, \ldots, q_T} P(q_1, q_2, q_3, \ldots, q_T \mid O, \lambda)
\]

(11)

Where every element of \( O \) is determined by the maximum of the spectrum at every moment \( t \) \((1 \leq t \leq T)\). If the maximum of spectrum corresponds to frequency grid \( k \), we can get \( o_t \) as follows:

\[
o_t = \left[ k / s \right] + 1
\]

(12)

\( q_1, q_2, \ldots, q_T \) correspond to possible states at the moment \( t \) \((1 \leq t \leq T)\).

We employ Viterbi Algorithm to trace frequency peaks at each moment to get the required rotators’ frequency components at each moment. Assume \( q_1, q_2, \ldots, q_T \) are probable states at moment \( t \) \((1 \leq t \leq T)\), \( o_1, o_2, \ldots, o_T \) are observed values, and \( \delta(t) \) is the maximum one in all conditional probabilities \( P(q_1, q_2, \ldots, q_T \mid q_{t-1}, o_1, \ldots, o_T, \lambda) \) with sequence \( q_1, q_2, \ldots, q_T \), i.e.

\[
\delta_t(i) = \text{max}_{q_1, q_2, \ldots, q_{t-1}} P(q_1, \ldots, q_t = i, o_1, \ldots, o_T \mid \lambda)
\]

(13)

The Process of Viterbi Algorithm is as follows:

1. Initialization

\[
\begin{cases}
\delta_t(i) = \pi_i b_{o_t,i} \\
\psi_t(i) = 0
\end{cases} \quad (1 \leq i \leq N)
\]

(14)

2. Recursion

\[
\begin{cases}
\delta_t(j) = \text{max}_{1 \leq i \leq N} \left[ \delta_{t-1}(i) a_{ij} b_{o_t,j} \right] \\
\psi_t(j) = \text{arg max}_{1 \leq i \leq N} \left[ \delta_{t-1}(i) a_{ij} \right]
\end{cases} \quad (2 \leq t \leq T, 1 \leq j \leq N)
\]

(15)

3. Results

\[
q_T^* = \text{arg max}_{1 \leq i \leq N} [\delta_T(i)]
\]

(16)

\[
q_t^* = \psi_{t+1}(q_{t+1}^*) \quad (t = T - 1, T - 2, \ldots, 2, 1)
\]

(17)

\( q_1^*, q_2^*, \ldots, q_T^* \) are the subsections of the required frequency grids. \( q_1^* \)s or maximum of every subsection is the frequency at each time.
5 SIMULATION AND RESULTS

Let the instantaneous frequency for tracing be determined by the following function:

\[ \alpha_i(t) = \exp(\alpha t) \]  

(18)

Make the following Rotary Machinery speeding up state Multi-component signal Model

Signal 1 (Low noise, low disturbance)

\[ x(t) = \sin\left( \int_{-\infty}^{t} \exp(\alpha \tau) d\tau \right) + 0.2 \sin\left( 1.5 \int_{-\infty}^{t} \exp(\alpha \tau) d\tau \right) + 0.2 \sin\left( 5 \int_{-\infty}^{t} \exp(\alpha \tau) d\tau \right) + 0.2 \eta(t) \]  

(19)

Signal 2 (High noise, high disturbance)

\[ x(t) = \sin\left[ \int_{-\infty}^{t} \exp(\alpha \tau) d\tau \right] + 0.8 \sin\left( 0.5 \int_{-\infty}^{t} \exp(\alpha \tau) d\tau \right) + 0.8 \sin\left( 5 \int_{-\infty}^{t} \exp(\alpha \tau) d\tau \right) + 0.8 \sin\left( 0.5 \int_{-\infty}^{t} \exp(\alpha \tau) d\tau \right) + \eta(t) \]  

(20)

As stated earlier, Fig.1 is the STFT spectrum of Rotary Machinery simulated signal 1 under low noise and disturbance, whose time and frequency component relationship is shown in graph 4. Based on this STFT spectrum and using the above methods, we get the probability Transition Matrix A.

In the real order test, we can get such signal in Reference Axis Vibration Sensing Places. For example, if the Reference Axis is the main shaft, we make the testing point at the radial position of the main shaft in the spindle box. Fig.2 represents the STFT spectrum of the simulated signal 2 with high noise and much interference. Fig.4 shows the results obtained by directly seeking the extreme point of the STFT spectrum shown by Fig 1. Fig 5a denotes the results obtained by directly seeking extreme point of the STFT spectrum shown by Fig 2.

Apply Viterbi Algorithm to the STFT spectrum shown by Fig 2 and we get its time and frequency relationship graph in Fig 5b. The results, compared with those from direct extreme point calculation, have lower errors and are much closer to the reality.

![Fig 4 The relation of time and frequency under signals with low noise](image1)

![Fig 5. Results of a) direct peak search. b) Viterbi arithmetic under signals with high noise and much interference](image2)
Experiments have been done using real-world signals. The sampling signal is the simple beam's vibrating signal generated by an eccentric electric machine during its run-up and run-down phases. The tested signal is the accelerating signal. The B&K 4370 accelerating sensor and B&K 2635 charge amplifier are used. The original sampling frequency is 50,000Hz. The signal is re-sampled by 1/32 sampling rate and filtered by the corresponding invariable cut-off frequency to avoid the blending-frequency. Fig 6 and Fig 7 are STFT spectrum of the original signal and the signal with noise added which is down-sampled and filtered by the same method above. To obtain the TF relationship of the low order component in the signal with high noise, we applied the traditional method of directly seeking extrema and our own as discussed in this paper. The results are shown in Fig 8a) and 8b). Compared with low order component TF relationship corresponding to low noise signal STFT spectrum, the method established by this paper can effectively remove interfering peaks introduced by noise and superior results have been achieved.
6 CONCLUSION

HMM has been widely applied to many fields such as signal processing and recognition, mechanical failure diagnosis, encephalogram(EEG) detection and so on. In this paper, HMM is applied to signal analysis in rotating machinery and the work has been explored in IFE. As shown by the experiments, this paper uses STFT spectrum as the input information and establishes the hidden Markov Model. It has made a significant breakthrough in acquiring the time and frequency relationship for analyzing run-up and run-down speed signal of rotary machines.

REFERENCES:


