Operational PolyMAX for estimating the dynamic properties of a stadium structure during a football game

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ABSTRACT
During a football game, the ambient vibrations at the roof of a football stadium were recorded. A very large data set consisting of 4 hours of data, sampled at 80 Hz, is available. By a data reduction procedure, the complete data set could be analysed at once in a very short time. The data set was also split in shorter segments corresponding to certain events before, during and after the game to investigate the influence of varying operational conditions on the dynamic properties. The new operational PolyMAX parameter estimation method is used and compared with stochastic subspace identification. Stochastic subspace identification requires the correlations between the responses as primary data, whereas PolyMAX operates on spectra or half spectra (i.e. the Fourier transform of the positive time lags of the correlation functions). The main advantage of PolyMAX is that it yields extremely clear stabilisation diagrams, making an automation of the parameter identification process rather straightforward. This enables a continuous monitoring of the dynamic properties of a structure.

1 INTRODUCTION
For more than a year, the University of Sheffield monitored the Midland Road Stand at the Bradford and Bingley Stadium, home of Bradford City Football Club (Figure 1, Left). Twelve accelerometers were used to instrument this stand. They were located at 6 frame locations at the intersection between the column at the back of the seating deck and the cantilever roof beam (Figure 1, Right). Additionally, two video cameras were used to monitor the crowd for correlating its distribution and behaviour with the measured accelerations [1][2]. The key aspects of stadia dynamics that it is hoped to learn about from these data are:

- How does the presence of a crowd affect the modal properties of a stadium (i.e. what is the nature of crowd-structure interaction)?
- Does a crowd introduce significant nonlinearity into the human-structure dynamic system?
- What forces do crowds apply to a stadium and how can these be modelled for future designs?

The University of Sheffield is interested in the innovative techniques that researchers around the world can apply to the data in an effort to understand more fully the nature of crowd excitation and interaction on stadia. Hereto, a part of the continuous monitoring data was made available through the university web site [3]. Participants have the opportunity to disseminate their results during a special session that is being organised at IMAC 23, the International Modal Analysis Conference, Orlando (FL), 2005.

This paper is part of that session and focuses on the first key question raised above, by introducing the innovative PolyMAX method, which accurately and objectively identifies the modal parameters of a structure excited by ambient excitation. Section 2 describes the PolyMAX method. Emphasis is put on the pre-processing of the raw operational time data and the post-processing of the poles and operational reference factors to obtain the mode
shapes. Section 3 discusses the application of PolyMAX to the stadium vibration data and compares it with stochastic subspace identification. In particular the efficiency of the method is highlighted: in a very short time, the modes of a structure are identified, even from very large datasets. The main advantage of PolyMAX is that it yields extremely clear stabilisation diagrams, making an automation of the parameter identification process rather straightforward. This enables a continuous monitoring of the dynamic properties of a structure, which is also demonstrated in Section 3.

2 PolyMAX FOR OPERATIONAL MODAL ANALYSIS

Before the monitoring system was installed, shaker tests were performed to learn more about the road stand [1]. As the purpose of this study is to find out how the stadium during a game interacts with the crowd, shaker tests cannot be performed and only ambient excitation can be used. The ambient excitation came from the people occupying the stand, the traffic on the road behind the stand and the wind. It is practically impossible to measure this ambient excitation and, by consequence, the structural response is the only information available to identify the modal parameters of the structure. In this case one speaks of Operational Modal Analysis. This section discusses how the PolyMAX method, which is very successful in classical modal analysis [4], can be used for Operational Modal Analysis.

2.1 Output-only frequency-domain model

Frequency-domain Operational Modal Analysis methods, such as PolyMAX, require output spectra as primary data. In this subsection, it will be shown that, under the assumption of white noise input, output spectra can be modelled in a very similar way as FRFs. It is well known that the modal decomposition of an FRF matrix $[H(\omega)] \in \mathbb{C}^{nm}$ is [5]:

$$[H(\omega)] = \sum_{i=1}^{n} \left\{ \frac{[v_i]}{j\omega - \lambda_i} + \frac{[v_i^*]}{j\omega - \lambda_i^*} \right\}$$

(1)

where $i$ is the number of outputs; $m$ is the number of inputs; $n$ is the number of modes ($2n$ is the system order in (1)); $\bullet^*$ is the complex conjugate of a matrix; $H(\cdot)^*$ is the complex conjugate transpose (Hermitian) of a matrix; $\{v_i\} \in \mathbb{C}^m$ are the mode shapes; $<\cdot^T> \in \mathbb{C}^m$ are the modal participation factors and $\lambda_i$ are the poles, which are occurring in complex-conjugated pairs and are related to the eigenfrequencies $\omega_i$ and damping ratios $\xi_i$ as follows:

$$\lambda_i, \lambda_i^* = -\xi_i \omega_i \pm j \sqrt{1 - \xi_i^2} \omega_i$$

(2)

The input spectra $[S_{uu}] \in \mathbb{C}^{mm}$ and output spectra $[S_{yy}(\omega)] \in \mathbb{C}^{ld}$ of a system represented by the FRF matrix $[H(\omega)]$ are related as:
In case of operational data the output spectra are the only available information. The deterministic knowledge of the input is replaced by the assumption that the input is white noise. A property of white noise is that it has a constant power spectrum. Hence $[S_{s}]$ in (3) is independent of the frequency. The modal decomposition of the output spectrum matrix is obtained by inserting (1) into (3) and converting to the partial fraction form [6][7][8]:

$$[S_{y}] = \sum_{i} \frac{\{v_{i}\} < g_{i}^{*} >}{j\omega - \lambda_{i}} + \frac{\{g_{i}\} < v_{i} >}{j\omega - \lambda_{i}^{*}} + \frac{\{g_{i}^{*}\} < v_{i}^{*} >}{-j\omega - \lambda_{i}} + \frac{\{v_{i}^{*}\} < g_{i} >}{-j\omega - \lambda_{i}^{*}}$$  \hspace{1cm} (4)

where $< g_{i} > \in \mathbb{C}^{l}$ are the so-called operational reference factors, which replace the modal participation factors in case of output-only data. Their physical interpretation is less obvious as they are a function of all modal parameters of the system and the constant input spectrum matrix. Note that the order of the power spectrum model is twice the order of the FRF model. The goal of Operational Modal Analysis is to identify the right hand side terms of (4) based on measured output data pre-processed into output spectra (Section 2.2).

### 2.2 Pre-processing operational data

Power spectra are defined as the Fourier transform of the correlation sequences. The most popular non-parametric spectrum estimate is the so-called weighted averaged periodogram (also known as modified Welch's periodogram). Weighting means that the signal is weighted by one of the classical windows (Hanning, Hamming, ...) to reduce leakage. Welch's method starts with computing the discrete Fourier transform (DFT) of the weighted outputs:

$$Y(\omega) = \sum_{k=0}^{N-1} w_{k} y_{k} \exp(-j\omega k\Delta t) \hspace{1cm} (5)$$

where $w_{k}$ denotes the time window. An unbiased estimate of the spectrum is the weighted periodogram:

$$S_{yy}^{(i)}(\omega) = \frac{1}{\sum_{k=0}^{N-1} |w_{k}|^{2}} Y(\omega) Y^{H}(\omega) \hspace{1cm} (6)$$

The variance of the estimate is reduced by splitting the signal in possibly overlapping segments, computing the weighted periodogram of all segments and taking the average:

$$S_{yy}(\omega) = \frac{1}{P} \sum_{i=1}^{P} S_{yy}^{(i)}(\omega) \hspace{1cm} (7)$$

where $P$ is the number of segments and superindex $i$ denotes the segment index.

Another non-parametric spectrum estimate is the so-called weighted correlogram. It will be shown that this estimate has some specific advantages in a modal analysis context. First the correlations have to be estimated:

$$R_{i} = \frac{1}{N} \sum_{k=0}^{N-1} y_{k+i}^{T} y_{k} \hspace{1cm} (8)$$

High-speed (FFT-based) implementations exist to compute the correlations as in (8); see for instance [9]. The weighted correlogram is the DFT of the weighted estimated correlation sequence:

$$S_{yy}(\omega) = \sum_{k=-L}^{L} w_{k} R_{k} \exp(-j\omega k\Delta t) \hspace{1cm} (9)$$

where $L$ is the maximum number of time lags at which the correlations are estimated. This number is typically much smaller than the number of data samples to avoid the greater statistical variance associated with the higher lags of the correlation estimates. In a modal analysis context, the weighted correlogram has the following advantages:

- It is sufficient to compute the so-called half spectra which are obtained by using only the correlations having a positive time lag in (9):
The relation between the half spectra (10) and the full spectra (9) is the following:

\[ S_{yy}^+(\omega) = S_{yy}^-(\omega) + \left( S_{yy}^+(\omega) \right)^H \]  

(11)

It can be shown (see for instance [6][10]) that the modal decomposition of these half spectra only consists of the first two terms in (4):

\[ S_{yy}^+(j\omega) = \sum_{i=1}^{n} \frac{\{v_i\}^H g_i^*}{j\omega - \lambda_i} + \frac{\{v_i^*\}^H g_i}{j\omega - \lambda_i^*} \]  

(12)

The advantage in modal analysis is that models of lower order can be fitted without affecting the quality.

- Under the white noise input assumption, the output correlations are equivalent to impulse response. So, just like in impact testing, it seems logical to apply an exponential window \( w_j \) to the correlations before computing the DFT (10). The exponential window reduces the effect of leakage and the influence of the higher time lags, which have a larger variance. Moreover, the application of an exponential window to impulses responses or correlations is compatible with the modal model and the pole estimates can be corrected. This is not the case when a Hanning window is used: such a window always leads to biased damping estimates.

Interesting discussions on the estimation of spectra from the measured time histories in a modal analysis context can be found in [6][10][11].

The weighted corregogram approach is illustrated using the Bradford stadium vibration data (More details on the data can be found in Section 3). Figure 2 shows the time history of a typical acceleration channel. Figure 3 (Left) shows how thousands time data samples are reduced to 256 correlation samples using (8). The effect of the exponential window is visible. Figure 3 (Right) shows the half spectrum estimate according to (10).

![Figure 2: Stadium ambient acceleration time data. (Top) 4 hours recording; (Bottom) selection of the last 2400 s for further processing.](image)
2.3 PolyMAX modal parameter estimation

After having pre-processed output data into output spectra (Section 2.2), it is now the task to identify a modal model (Section 2.1). By comparing (12) with (1), it is clear that FRFs and half spectra can be parameterised in exactly the same way. By consequence, the same modal parameter estimation methods can be used in both cases. PolyMAX is such a method. It will be reviewed in the following.

Originally, the least-squares complex frequency-domain (LSCF) estimation method was introduced to find initial values for the iterative maximum likelihood method [12]. The method estimates a so-called common-denominator transfer function model [13]. Quickly it was found that these “initial values” yielded already very accurate modal parameters with a very small computational effort [12][14][15]. The most important advantage of the LSCF estimator over the available and widely applied parameter estimation techniques [5] is the fact that very clear stabilisation diagrams are obtained.

It was found that the identified common-denominator model closely fitted the measured frequency response function (FRF) data. However, when converting this model to a modal model (1) by reducing the residues to a rank-one matrix using the singular value decomposition (SVD), the quality of the fit decreased [14]. Another feature of the common-denominator implementation is that the stabilisation diagram can only be constructed using pole information (eigenfrequencies and damping ratios). Neither participation factors nor mode shapes are available at first instance. The theoretically associated drawback is that closely spaced poles will erroneously show up as a single pole [16]. These two reasons provided the motivation for developing “PolyMAX”, which is a polyreference version of the LSCF method, using a so-called right matrix-fraction model. In this approach, also the participation factors are available when constructing the stabilisation diagram. The main benefits of the polyreference method are the facts that the SVD step to decompose the residues can be avoided and that closely spaced poles can be separated. PolyMAX was introduced and validated using industrial data sets in [4][16][17]. The present paper will indicate how PolyMAX can also be applied to operational data when appropriate pre-processing (Section 2.2) and post-processing (Section 2.4) is applied.

In the PolyMAX method, following so-called right matrix-fraction model is assumed to represent the measured half spectrum matrix:
\[
[S^*(\omega)] = \sum_{r=0}^{p} z^{-r} [\beta_r^*] \left( \sum_{r=0}^{p} z^{-r} [\alpha_r] \right)^{-1}
\]

where \([\beta_r] \in \mathbb{C}^{l \times m}\) are the numerator matrix polynomial coefficients; \([\alpha_r] \in \mathbb{C}^{m \times m}\) are the denominator matrix polynomial coefficients; \(p\) is the model order and \(m\) is in this case not the number of inputs as in (1), but the number of outputs selected as references. For estimating the modal parameters, it is indeed sufficient to compute only the cross spectra between all outputs \(l\) and a limited set of references \(m\). From this point forward, \(S^*(\omega)\) is an \(l \times m\) matrix instead of an \(l \times l\) as above. Please note that a so-called \(z\)-domain model (i.e. a frequency-domain model that is derived from a discrete-time model) is used in (13), with \(z = \exp(j\omega \Delta \tau)\) (\(\Delta \tau\) is the sampling time). Equation (13) can be written down for all values \(\omega\) of the frequency axis of the half spectra data. Basically, the unknown model coefficients \([\alpha_r^*], [\beta_r]\) are then found as the least-squares solution of these equations (after linearisation). More details about this procedure can be found in [4][16][17].

Once the denominator coefficients \([\alpha_r]\) are determined, the poles \(\lambda_i\) and operational reference factors \(<g_i>\) are retrieved as the eigenvalues and eigenvectors of their companion matrix, which is rather classical [5][19]. A \(p^{th}\) order right matrix-fraction model yields \(pm\) poles. This procedure allows constructing a stabilisation diagram [18] for increasing model orders \(p\) and using stability criteria for eigenfrequencies, damping ratios and operational reference factors. An efficient construction of the PolyMAX stabilisation diagram is possible by formulating the least squares problem for the maximum model order \(p_{\text{max}}\). Considering submatrices of appropriate dimensions can then solve lower-order problems. Examples of PolyMAX stabilisation diagrams will be shown in Section 3.

2.4 Operational residuals

The interpretation of the stabilisation diagram yields a set of poles and corresponding operational reference factors. The mode shapes can then be found from (12):

\[
S^*_\psi(\omega) = \sum_{i=1}^{n} \left\{ \frac{\psi_i}{j\omega - \lambda_i} \right\} <g_i> + \left\{ \frac{\psi_i^*}{j\omega - \lambda_i^*} \right\} <g_i^*> + \frac{\text{LR}}{j\omega} + j\omega \text{UR}
\]

in which \(\text{LR, UR} \in \mathbb{R}^{l \times m}\), respectively the lower and upper residuals, have been introduced to model the influence of the out-of-band modes in the considered frequency band. The only unknowns in (14) are the mode shapes \(\{\psi_i\}\) and the lower and upper residuals. They are readily obtained by solving (14) in a linear least-squares sense. This second step is commonly called least-squares frequency-domain (LSFD) method [5][19].

The operational residues \(\text{LR, UR}\) in (14) are different from the usual residuals in case of FRFs. They were determined by verifying the asymptotic behaviour of the output spectra of a SDOF mechanical system excited by white noise. The formats of the residuals that have to be applied in the LSFD method are listed in Table 1 for FRFs (1), classical full output spectra (4) and half spectra (12). In general, they depend on the measured output quantity (displacement – velocity – acceleration). Deriving displacements to velocities (and similarly velocities to accelerations) implies multiplication by \(j\omega\) for FRFs and \((j\omega)^2\) for full spectra. Rather surprising is that this is not the case for half spectra: the residuals are independent of the output quantity.

### Table 1: Lower and upper residuals.

<table>
<thead>
<tr>
<th>Output quantity</th>
<th>FRFs (1)</th>
<th>Full spectra (4)</th>
<th>Half spectra (12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>(\frac{\text{LR}}{j\omega^2})</td>
<td>UR</td>
<td>(\frac{\text{LR}}{j\omega^4})</td>
</tr>
<tr>
<td>Velocity</td>
<td>(\frac{\text{LR}}{j\omega})</td>
<td>((j\omega) UR)</td>
<td>(\frac{\text{LR}}{j\omega^2})</td>
</tr>
<tr>
<td>Acceleration</td>
<td>(\frac{\text{LR}}{j\omega^2})</td>
<td>(\frac{\text{LR}}{j\omega})</td>
<td>(\frac{\text{LR}}{j\omega^4})</td>
</tr>
</tbody>
</table>
3 STADIUM DATA ANALYSIS

The stadium structure was introduced in Section 1. The University of Sheffield made a 4-hours recording of stadium accelerations available [3]. The data acquisition started about 1 hour before the football game between Bradford City and Sheffield United on 23 November 2002. The data includes following events (Figure 4):

- Filling up of the stand
- People seated during the game
- Half time
- Emptying of the stand
- Empty stand
- 5 goals (by the way, all scored by Sheffield United)

These events represent different crowd-structure interaction scenarios and allow investigating how the presence of a crowd affects the modal properties of the stadium. The original data was sampled at 80 Hz. As we are mainly interested in the modal parameters below 10 Hz, the data was resampled to 20 Hz. By consequence, 288000 samples (20 Hz x 14400 s) are available for each of the 12 accelerometers. The "sliding" RMS values represented in Figure 4 are based on a frame of 1000 samples and an overlap between the frames of 50%.

Figure 4: Typical acceleration time history and “sliding” RMS value with an indication of the events during the game.

3.1 Analysis of the complete data set

Although it is expected that the different crowd configurations identified in Figure 4 will influence the modal parameters of the stadium, a first analysis will be performed on the complete data set. This allows illustrating the efficiency of the PolyMAX operational modal parameter estimation process and getting an idea of the modes that can be expected from a more detailed analysis. Evidently, the risk of inconsistencies exits due to changing modal parameters during the data acquisition.

Two sensors are selected as reference sensors and, as explained in Section 2.2 and illustrated in Figure 2 and Figure 3, the 12 raw time histories are reduced to a 12 x 2 cross correlation (8) and cross spectrum matrix (10). An exponential window of 1% has been applied to the cross-correlations before computing the half spectra by a DFT. The number of time lags at which the correlations have been computed was 256. Afterwards, the PolyMAX method was applied to estimate the modal parameters. The whole process required only 4 s of computation time on a year-2003 PC (Figure 5). Figure 6 compares the PolyMAX stabilisation diagram with the one obtained using
Stochastic Subspace Identification. This method is, for instance, described in [7][8] and uses the correlations as primary data. Although both methods are known to yield accurate modal parameter estimates, it is obvious that PolyMAX considerably facilitates the identification process; in the sense that it is much easier to pick the stable poles from the diagram. Section 3.2 will compare PolyMAX and Stochastic Subspace Identification results in more detail. As an overall quality indicator of the parameter estimation process, the sum\(^1\) of measured cross spectra is compared with the sum of spectra that are synthesised from the identified modal parameters using (14). The good correspondence indicates that all major dynamic properties have been extracted from the data.

<table>
<thead>
<tr>
<th>Raw time data</th>
<th>Cross correlations and cross spectra</th>
<th>Set of PolyMAX modal parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 x 288000 samples</td>
<td>12 x 2 x 256 time lags</td>
<td>2 s</td>
</tr>
</tbody>
</table>

Figure 5: Reduction of 4 hours of time data to a set of modal parameters using the PolyMAX method requires 4 seconds of computation time.

Figure 6: Stabilisation diagram obtained by applying PolyMAX (Left) and Stochastic Subspace Identification (Right) to the complete data set.

Figure 7: Complete data set: sum of measured spectra (red/black) and sum of synthesised PolyMAX spectra (green/grey).

\(^1\) The real (imaginary) part of the "sum of spectra" is in fact the sum of the absolute values of the real (imaginary) parts of the spectra. This is the reason why its phase is always between 0° and 90°.
3.2 Empty stadium

In this section, we will discuss in detail the Operational Modal Analysis of the empty stadium. The data used here involves the last 2400 s of the records, as indicated in Figure 2 and pre-processed in Figure 3. Figure 8 shows the PolyMAX stabilisation diagram and the quality of the parameter estimation process by comparing the sum of measured and synthesised cross spectra. Again, the correspondence is excellent. It is noteworthy that not only the absolute levels of vibration are much smaller when comparing the spectra of the empty stadium (Figure 8, Right) with the spectra computed from the complete data set (Figure 7), but that also the relative contribution of the modes is changing. So, it can be expected that, depending on the selected event (Figure 4), the modes will be better or less well excited and the mode extraction process will be easier or, on the contrary, more difficult.

Some of the identified mode shapes are represented in Figure 15 at the end of the paper. The correspondence between measured and synthesised half spectra (Figure 8, right and Figure 9) already confirms the accuracy of the PolyMAX estimates, but it still interesting to compare them with estimates from the Stochastic Subspace Identification method. Table 2 contains this comparison. The eigenfrequency agreement is excellent, whereas the damping ratio differences fall within the typical uncertainty on these estimates. Both PolyMAX and Subspace estimate modes with low complexity as indicated by the MPC values close to 100%. The correspondence between the mode shapes is very good (very high MAC values in last column). The complete matrix of MAC values is represented in Figure 10.

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**Figure 8:** Empty stadium: (Left) stabilisation diagram obtained by applying PolyMAX; (Right) sum of measured (red/black) versus synthesised PolyMAX spectra (green/grey).

**Figure 9:** Empty stadium: measured (red/black) versus synthesised PolyMAX spectra (green/grey).
Table 2: Comparison of modal parameters from PolyMAX and Stochastic Subspace Identification (SSI). Eigenfrequencies ($f$) and damping ratios ($\xi$) for both sets are given as well as their differences ($\Delta$). MPC stands for "mode phase colinearity" and expresses the complexity of a mode shape. MAC stands for "modal assurance criterion" and equals the squared correlation between two mode shapes.

<table>
<thead>
<tr>
<th></th>
<th>$f_{SSI}$ [Hz]</th>
<th>$f_{PolyMAX}$ [Hz]</th>
<th>$\Delta f$ [Hz]</th>
<th>$\xi_{SSI}$ [%]</th>
<th>$\xi_{PolyMAX}$ [%]</th>
<th>$\Delta \xi$ [%]</th>
<th>MPC$_{SSI}$ [%]</th>
<th>MPC$_{PolyMAX}$ [%]</th>
<th>MAC [%]</th>
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<tbody>
<tr>
<td>1</td>
<td>3.34</td>
<td>3.34</td>
<td>0.00</td>
<td>1.22</td>
<td>1.18</td>
<td>0.04</td>
<td>99</td>
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<td>100</td>
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<tr>
<td>2</td>
<td>3.59</td>
<td>3.59</td>
<td>0.00</td>
<td>1.17</td>
<td>1.23</td>
<td>0.05</td>
<td>100</td>
<td>100</td>
<td>100</td>
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<tr>
<td>3</td>
<td>4.21</td>
<td>4.20</td>
<td>0.02</td>
<td>1.64</td>
<td>1.73</td>
<td>0.09</td>
<td>99</td>
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<td>100</td>
</tr>
<tr>
<td>4</td>
<td>5.01</td>
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<td>1.00</td>
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<td>0.83</td>
<td>0.17</td>
<td>37</td>
<td>84</td>
<td>73</td>
</tr>
</tbody>
</table>

Figure 10: MAC values assessing the mode shape correlation between Subspace and PolyMAX mode shapes.

3.3 Other events

Based on Figure 4, certain events are isolated and an Operational Modal Analysis using the PolyMAX method was performed. The pre-processing and processing parameters were the same in all cases: correlations up to lag 256 were computed, an exponential window of 1% was applied, and the PolyMAX method was running up to model order 60 between 2 and 8 Hz. Figure 11 shows the PolyMAX stabilisation diagrams for the events: filling up – 1st goal – sitting – half time – leaving. Figure 12 compares the sum of measured and synthesised cross spectra for all these events. Again it can be concluded that PolyMAX is able to extract the main dynamic features from the data of all events.
Figure 11: PolyMAX stabilisation diagrams for different events: Filling – 1st goal – sitting – half time – leaving.

Figure 12: Sum of measured spectra (red/black) and sum of synthesised PolyMAX spectra (green/grey) for different events: Filling – 1st goal – sitting – half time – leaving.
3.4 Automatic modal analysis

As discussed earlier in this paper, the main advantage of PolyMAX is that it yields extremely clear stabilisation diagrams. This makes an automation of the parameter identification process rather straightforward and enables a continuous monitoring of the dynamic properties of a structure. An overview and discussion on automatic modal analysis techniques can be found in [18].

In this section, a method has been used which is able to autonomously interpret stabilisation diagrams based on a set of heuristic rules [8][21]. Unlike in previous sections, no attempts were made to isolate certain events, but the data was simply cut in segments of equal length. Two different segmentations were applied:

- Segments of 4096 samples (204.8 s @ 20 Hz) without overlap
- Segments of 16384 samples (819.2 s @ 20 Hz) with 75% overlap, to obtain the same time resolution as in previous case.

Figure 13 shows the automatic PolyMAX results. From these diagrams following observations can be made:

- The 3 lowest modes are easiest to track.
- The higher modes can be tracked when the stadium is filling up or when it is emptying or empty.
- During the football game, the higher modes are not well excited and/or not consistently estimated and/or not well tracked.
- Enlarging the segment length yields more stable results (Bottom diagram of Figure 13 vs. Top diagram) and an improved tracking. Of course, larger segments will make it more difficult to identify changes in modal parameters that only occur during a short time.

Figure 14 shows the tracking of mode 3 based on Figure 13. During the game, when people are sometimes heavily interacting with the structure, the frequencies tend to decrease and the damping is increasing. The estimated mode shapes may also be change during the game, as evidenced by the drops in MAC values. The most stable conditions occur when the stadium is empty, filling up with people before the game or when it is emptying after the game.

Despite the fact that the analysis of individual events is quite successful (Section 3.3), tracking is difficult for some of the modes. Possible causes are that modes change too much at certain crowd-structure interaction events or that they may not be well excited in certain data segments (For instance, the crowd is mainly exciting the lower modes.)
Figure 13: Unusual stabilisation diagrams representing automatic modal analysis results: the y-axis is not the model order, but represents the time. Automatic PolyMAX analyses were performed on data segments of 204.8 s and no overlap (Top) and 819.2 s with 75% overlap (Bottom). The background image is the normalised 1st singular value (CMIF) of the spectrum matrix computed for each segment. The curves on the Left represent the sliding RMS values (Figure 4).
4 CONCLUSIONS

The paper discussed the use of the operational PolyMAX method for monitoring a stadium structure during a football game. The main advantage of PolyMAX is that it yields very clear stabilisation diagrams, easing dramatically the problem of selecting the model order and the structural system poles. The analysis results compare favourably with current best-of-class commercially available methods, without increasing the computational effort.

Although having a completely different mathematical background, both PolyMAX and Stochastic Subspace Identification approximately yield the same modal parameter estimates. The main difference is that PolyMAX is easier to automate. Such an automated processing was successfully applied to the whole data set split in different segments. Although the theoretical white noise excitation assumption is very questionable when people are interacting with the structure, the application of Operational Modal Analysis confirmed that the frequencies of the structure are decreasing when the number of people is increasing (added mass) and that the damping ratios are generally increasing. Both PolyMAX and the Stochastic Subspace Identification (“Op. Time MDOF”) are implemented in the LMS Test.Lab Structures software [22].

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Figure 15: PolyMAX mode shapes extracted from ambient data while the stadium was empty (Section 3.2).