Characterizing Damage in Plates through Beamforming with Sensor Arrays

Shankar Sundararaman\(^a\), Douglas E. Adams\(^a\) and Elias J. Rigas\(^b\)

\(^a\)School of Mechanical Engineering, Purdue University, Ray W. Herrick Laboratories, West Lafayette, IN 47907-2031, USA

\(^b\)U.S. Army Research Laboratory, Aberdeen Proving Grounds, MD 21005-5066, USA

NOMENCLATURE

- \(E\) Elastic Modulus
- \(\nu\) Poisson’s Ratio
- \(G\) Shear Modulus
- \(\mu\) Bulk Modulus (or) Convergence parameter
- \(\mathbf{F}\) Body Force
- \(\rho\) Density
- \(v_{ph}\) Phase velocity
- \(d\) Half the thickness of the plate
- \(k_s\) S-wave wave-number
- \(\zeta\) Ratio of Shear and Lamb wave velocities
- \(\xi\) Ratio of Shear and Longitudinal wave velocities
- \(\omega\) Angular Frequency of interest
- \(n\Delta t\) \(n^{th}\) time sample
- \(w_e\) Complex weights
- \(\lambda\) Wavelength
- \(\theta_{\text{max}}\) Maximum azimuthal scan angle
- \(\theta_e\) Desired/Incident azimuthal angle
- \(d_e\) Desired signal
- \(\bar{d}\) Product of the S-wave wave-number and half the thickness of the plate
- \(x_e\) Element data
- \(X_e\) Discrete Fourier Transform of the Element data
- \(e_{r=1/2}\) Constraint error
- \(C\) Frost constraint
- \(u_e\) Distance between sensor \(e\) and reference sensor
- \(\|u_e\|\) Distance between reference sensor and source
- \(y\) Array Factor (time-domain)
- \(Y\) Array Factor (frequency-domain)
- \(S\) Frequency averaged array factor
- \(E[\cdot]\) Expectation of random variable
- \(u\) Displacement vector
- \(\nabla[\cdot]\) Gradient of a vector
- \(\nabla_\cdot[\cdot]\) Divergence of a vector
- \(N_s\) Number of sensors

ABSTRACT

This paper discusses work in the use of sensor arrays for the online nondestructive evaluation of heterogeneous and homogeneous plates. The theory of narrowband beamforming and broadband beamforming of propagating plate waves in damaged elastic media is discussed. Design issues including sensor spacing, and frequency of operation are considered apart from signal processing issues like aliasing, leakage and sampling and wave propagation issues such as scatter. Beamforming is achieved by using a tapped delay line to steer the beamformer or complex weights to manipulate both gain and phase. High-frequency acoustic waves are used to excite the plate and the data is collected from a sparse array of sensors. Damage is detected and located by comparing the spatially filtered results of ‘baseline’ and ‘damage’ data. Experiments on composite and steel plates are used to demonstrate the technique.
BACKGROUND

Damage can be defined as a perturbation in material or structural properties resulting in degradation and a reduction in the operational capability of the structure as designed. Common forms of material and structural degradation include cracks, corrosion and loosened fixtures. Each material form of degradation can be further classified as brittle damage, creep damage, fatigue damage or a combination of these types depending upon the materials and operational and environmental conditions [1]. It is important to determine the integrity of structures to provide adequate performance/safety margins and to reduce life-cycle costs as damage accumulates.

Heterogeneous structures, which are lightweight yet strong with extended fatigue lives, contain a mixture of composite materials (e.g., laminated composite, filament wound), ceramics, plastics, metal-alloys and fabrics and are being used increasingly by the defense industry to design faster, more maneuverable and lethal weapon systems and vehicles (e.g., multi-layer armor, rocket motor casings, rotary and fixed-wing aircraft, etc.) [2,3]. Such structural systems of materials are particularly difficult to maintain because they often look fine from the outside when damaged, meaning routine visual inspections are ineffective. Manufacturing and operational variabilities make it nearly impossible to detect damage without implementing some form(s) of online/offline damage characterization method(s).

Wave propagation methods are non-modal, localized acoustic-based techniques for identifying damage. Elastic wave techniques are attractive because waves propagate long distances and can be used to inspect large regions. Even a small number of sensors can be sensitive to distant flaws, which act as impedance changes along the paths of the waves. The ability to steer certain types of waves (e.g., Lamb waves) along structural components to reach hidden or buried (subsurface) parts also makes these methods attractive. Different wave propagation based techniques are available that exploit certain characteristics of waves to detect damage. Before using wave propagation based methods, the nature of waves in different materials involving reflection, refraction, diffraction, and mode splitting must be understood. Wooh and Shi [4], who reviewed the literature on phased arrays for ultrasonic imaging, note that the successful implementation of phased arrays for nondestructive testing is hindered by the high speeds of waves in elastic solids. Material property gradients in anisotropic and heterogeneous solids make it even more difficult to interpret elastic waves. Alleyne and Cawley [5] were among the first to discuss the automation of non-destructive testing based on Lamb waves. Saravanos et al. [6] demonstrated the possibility of detecting delaminations in composites using Lamb waves. Wu [7] discussed some of the work involving the use of waves to measure elastic properties as well as flaws in solid specimens. Pines [8] showed that global defects can be detected by a change in the wavenumber-frequency relation while local defects can be detected by a change in the scattering matrix elements of a beam.

Beamforming was originally developed in the communications industry to enable continuous spatial filters for attenuating interference from transmitted propagating electromagnetic waves. Other applications of beamforming include radar, sonar, imaging, geophysical exploration, astrophysical exploration and medical diagnostics [9]. Some of the pioneering work in this field including the development of the Least Mean Squares (LMS) algorithm was carried out by the group at Stanford under Widrow [10,11]. There is a large body of reference works in this area including those by VanVeen [9], Godara [12,13], Naidu [14] and more recently by Van Trees [15].

A phased array is a multi-element device, which can be used to provide directional control of the transducer characteristic. The use of phased arrays requires that high sample rates be used in conjunction with expensive instrumentation and complex electronic circuitry and digital boards. McNab and Campbell [16] studied the feasibility of using a phased array approach for nondestructive testing (NDT) applications based on the above considerations. Researchers have investigated the use of phased arrays in NDT for detecting defects in composites [17] and characterizing defects in welds [18]. Wooh and Wang [19] discussed an ultrasonic array scheme for pitch-catch defect identification in a metal railhead. Giurgiuju and Bao [20] have developed the Embedded-Ultrasonics Structural Radar (EUSR) concept for in-situ non-destructive evaluation. In their work, piezoelectric-wafer active sensors were used to generate guided plate (Lamb) waves for structural damage identification. More recently, Purekar et al [21] have used a phased array filter based on piezoelectric sensors along with a wave propagation approach to scan different directions in a plate.

The research presented here utilizes a newly developed beamforming [22-25] method for transmitting, receiving and processing propagating bulk and surface stress/strain waves to detect and locate material discontinuities along and within members (i.e., debonding, delamination, fiber breakage, matrix cracking, cracks, voids).
Beamformers consisting of sensor arrays are used to extract directional damage signatures. In this diagnostic technique, propagating waves are sent out into a finite elastic homogeneous or heterogeneous media and then various scattering patterns appear as the waves are partially reflected, refracted or transmitted by the damage. In this context, damage is equivalent to an acoustic impedance discontinuity in the solid medium. Scatter patterns are then analyzed to detect, locate and quantify damage. It is also interesting to note that the common orb web spider uses its legs as a beamforming array to transmit and receive elastic waves along orb webs to locate trapped prey [26].

**WAVE PROPAGATION**

In a homogeneous unbounded elastic medium, bulk waves exist (one longitudinal and the two shear waves); however, in finite media (e.g., steel or composite plate), Rayleigh waves, Love waves, plate modes and Lamb waves are also known to exist. Some of these waves are more difficult to deal with because they are dispersive (i.e., velocity is a function of frequency). Steady-state extensional (symmetrical) and flexural (anti-symmetrical) Lamb waves (also called plate or guided waves) are propagating elastic disturbances in a plate with free boundaries and exist as a combination of longitudinal and shear waves. Extensional waves can be produced by “bending” a plate locally at certain frequencies whereas flexural waves are produced by “stretching” the plate. The acoustic field equations are used to describe waves in any given elastic medium. Bulk waves of dilatation and distortion can be produced using in- or out-of-plane tractions and/or initial conditions. If the displacement and velocity gradients are small, the equations of motion can be expressed explicitly in terms of the displacements. In a homogeneous, isotropic body, the Navier’s equations [27] in Eq. (1) govern propagating elastic waves:

\[
GV^2 u + (\mu + G) \nabla (\nabla \cdot u) + F = \rho \frac{\partial^2 u}{\partial t^2}
\]

The phase speeds can be obtained by directly substituting material properties into the Christoffel matrix [28] and solving the eigenvalue problem given the elastic constants from the stiffness matrix. The Christoffel matrix can then be used to characterize slowness surfaces that represent the relation between the inverse of the phase velocity and direction of propagation. These slowness surfaces also help in understanding wave scatter and mode conversion phenomena. In real-life/laboratory situations, group velocity is more relevant than phase velocity because group velocities characterize the velocity of the most commonly excited groups of waves traveling between different points. The Rayleigh-Lamb equation, which is used to describe the dispersion relationship between the phase velocity, wavelength and frequency-thickness, is given by:

\[
\tan \sqrt{1 - \xi^2 \frac{d}{d}} + \left( \frac{4\xi^2 \sqrt{1 - \xi^2} \sqrt{\xi^2 - \xi^2}}{(2\xi^2 - 1)^2} \right)^{1/2} = 0.
\]

where the exponents +1 and -1 correspond to symmetrical and anti-symmetrical modes respectively. Lamb waves have been successfully used to detect damage in materials. In fact, researchers have selectively used certain Lamb modes to detect particular damage-mechanisms. The factors that help determine which Lamb mode to use and at what frequency can be classified in two ways based on the properties suitable for inspection and transduction. Properties that are determined by the inspection scheme are dispersion, attenuation and sensitivity while properties determined by the transduction scheme are excitability, detectability and selectivity [29]. The wave speed of the Ao mode Lamb wave, which is used in this work, can be approximated at low frequencies by its asymptote, the flexural wave, as:

\[
V_{Ao} = \left( \frac{Ed^2}{12\rho(1 - \nu^2)} \right)^{1/4} \sqrt{\omega}
\]

**BEAMFORMING OF PHASED ARRAYS**

A beamformer is a spatial-temporal filter that can be used to ‘look’ in the direction of transmitted signal while eliminating interference, or jamming signals, that cannot be removed through temporal filtering or carrier demodulation alone [22]. In conventional beamforming, the phase of the signals from different elements is combined to point a beam in a desired direction while in adaptive beamforming both the gain and phase are changed by an adaptive algorithm [12-13]. Some design rules for beamforming of plane waves were developed in [22] based on the nomenclature and scheme of [12-13].
By delaying, weighting and summing (or, by equivalent filter operations), the transmitted and received waves at various actuator and sensor locations, a beamformer similar to that used by the orb web spider provides the capability of pin-pointing the direction and distance to a given defect. Damage is assumed to be equivalent to a local impedance change. The technique for characterizing damage signatures from sensor data that is collected from elastic waves is similar to the method adopted by the communications industry to extract electromagnetic signals lost in a noisy environment using large radio telescope arrays. The array output for arbitrary pairs of azimuth and elevation angles is given by:

\[ y(n\Delta t, \theta, \phi) = w^T x e^{j\omega t} \]

where, the time delay, \( \Delta t \), based on a plane wave assumption at a frequency, \( \omega \), is given by:

\[ \Delta t(\omega) = u \sin \theta / c(\omega) \]

It is often difficult to launch plane waves, especially in large spaces like sheets or plates, using just a single actuator because the transmitted wave front is circularly diverging. The time delay, \( \Delta t \), based on a circularly diverging wave assumption is given by:

\[
\Delta t(\omega) = \sqrt{\frac{1}{2} \left( \frac{2}{l^2} \sin \theta - 2 \frac{u}{\|u\|} \sin \theta \right)}^{1/2} - 1
\]

In this work, we are interested in the estimation of damage along the plane of the plate; hence, an estimate of the array output that varies with the azimuthal angle, \( \theta \), is sufficient. Damage is estimated based on a comparison between the discrete Fourier transform of the baseline and damage array outputs as a function of the wavenumber. Alternatively, damage can be estimated based on the difference between the discrete Fourier transforms of the baseline and damage data and then computing the array factor as,

\[ Y(n\Delta \omega, \theta, \phi) = w^T X e^{j\omega t} \]

The above form of the array factor, referred to as frequency-domain beamforming, is suitable for broadband beamforming as well. The extension from the narrowband case to the broadband case can be written as,

\[ S(\theta, \phi) = E[Y(n\Delta \omega, \theta, \phi)] \]

The performance of the beamformer depends on a number of factors. They include spacing between sensors, number of sensors, array geometry (linear, planar etc.) and even array transducer dimensions. Spatial aliasing in the form of grating lobes is illustrated in Figure 1(a) for three values of \( d = \lambda / 2 \), \( \lambda \), and \( 2\lambda \). Note that an increase in the sensor element spacing creates grating lobes at \(-30^\circ\) for \( d = \lambda \) and then at \( 0^\circ \) and \( 90^\circ \) for \( d = 2\lambda \). It is straightforward to show as in [22] that the sensor spacing must satisfy \( d < \lambda / (1 + \sin \theta_{max}) \) to avoid grating lobes. This condition can be relaxed further if circular waves are used as illustrated in the absence of grating lobes in that case for \( d = \lambda \) (Figure 1(b)) because of the nonlinearity of the circular wave.

Changes in the beamsteering array factor for three values of \( N_s \) (\( N_s = 6, 4, \) and \( 2 = \text{number of sensors} \)) are shown in Figure 2(a). Note that as the number of elements is decreased, the array becomes less directionally sensitive as the width of the main lobe of the spatial filter and the amplitudes of the side lobes increases. This leakage of sensitivity to adjacent azimuth angles is analogous to leakage in temporal digital signal processing. The shape of the array also plays an important role. In a linear array, there is always a main lobe at the desired steer or 'look' direction, \( \theta_{\text{des}} \) as well as at \(-\theta_{\text{des}} \). Furthermore, the sensitivity of the array for waves arriving parallel to the axis of the array is poor. To overcome these two effects, two arrays, which are placed at right angles to each other (e.g., L-shaped array) as discussed in [22], can be used. These arrays have different planes of symmetry by which one common look angle can be established. Figure 2(b) illustrates this effect through which the array is readily steered to eliminate the companion lobe. The effects of the sampling frequency are shown in Figure 3. The use of low sampling frequencies yields rather poor resolution (Figure 3(a)). While high sampling rates (Figure 3(c)) help improve the results, a simpler solution would be to artificially interpolate the data and it seems to give as good a result as sampling the data at a higher frequency (Figure 3(b)).
Experimental Setup and Results

Experiments were performed on steel, Aluminium and S-2 glass/epoxy woven composite plates. An AVC Instruments 790A01 PZT actuator was used to generate the flexural wave used in the experiments. A small point mass was placed on the surface of the plates to simulate a local reversible impedance change in the form of a spot corrosion or delamination. A 4 channel Tektronix TDS3014B oscilloscope and a 4 to 1x4 switch box was used to acquire data at several sensor locations, which measure the out-of-plane acceleration response. An
Agilent 33220A arbitrary waveform generator was used to generate a windowed sinusoidal pulse centered at 21 kHz with a bandwidth of 5 kHz. See Figure 4 for details of the experimental setup and processing.

The data acquired was processed using a Matlab® code. Circular waves were sent out by each of the actuators radially from their center points. Beamforming of the data was carried out through the process outlined in Figure 4 for the “damaged” (with introduction of local impedance discontinuity) and “undamaged” (base specimen with no discontinuities) conditions and then the results of the two cases were compared to detect and locate damage. A peak in the gain difference (i.e., difference in gain between the damaged and undamaged conditions) as a function of steer angle represents the direction in which the damage is located. These experiments were performed while assuming one of the sensors to be the reference sensor (i.e., the sensor about which data from all sensors are processed). The exact position of the damage is located based on a triangulation process.

A 6 sensor array consisting of PCB U352C22 high frequency miniature accelerometers was used to detect and locate damage on the steel plate (Figure 5). A 14-sensor array consisting of 7 PCB U352C22 high frequency accelerometers and 7 APC850 material PZT sensors as seen in Figure 6 (a) was used to detect and locate damage on the composite plate shown in Figure 6(a). The results of the 14 sensor array are outlined for one damage case in Figure 6 (c-g). Note that in these cases, two sets of peak angles were detected for processing with respect to each reference sensor (first and last sensors of arrays are used as reference sensor). As a result, two potential damage sites are located. For the purposes of illustration, one of the damage sites was excluded from the analysis. To eliminate the second damage site from the analysis, an L-shaped array as outlined in Figure 7(b) can be used in the analysis. Typical results from the 7 sensor L-shaped array (effectively, 8 sensors) are outlined in Figure 7.

Figure 4. Schematic of the process of acquisition of data and digital beamforming
Figure 5: (a) Snapshot of Steel plate instrumented with accelerometers and PZT actuator. (b) Schematic of Damage Location; Normalized Array Gain Factor Plots ((---) Undamaged Spectra, (-----) Damaged Spectra, and (-----) Difference Spectra) by Beamforming with (c,d) reference sensor: 1 and (e,f) reference sensor: 6. (g) Triangulation to Identify Damage.

Figure 6: (a) Snapshot of Composite plate instrumented with accelerometers, PZT sensors and PZT actuator, (b) Schematic of Damage Location; Normalized Array Gain Factor Plots ((---) Undamaged Spectra, (-----) Damaged Spectra, and (-----) Difference Spectra) by Beamforming with (c,d) reference sensor: 1 and (e,f) reference sensor: 14; (g) Triangulation to Identify Damage.
Figure 7: (i) Snapshot of Al plate instrumented with accelerometers, PZT sensors and PZT actuator, (j) Schematic of Damage Location; Normalized Array Gain Factor Plots ((___) Undamaged Spectra, (….) Damaged Spectra, and (-.-.-) Difference Spectra) by Beamforming with (I) Vertical Array: (a-b) reference sensor: 8, (c-d) reference sensor: 11; (II) Horizontal Array: (e-f) reference sensor: 14, (g-h) reference sensor: 11; (III) L-shape array: (k,l) reference sensor: 8 and (m,n) reference sensor: 11; (o) Triangulation to Identify Damage.

SUMMARY

A method for damage characterization in homogeneous and heterogeneous structures using beamformers consisting of sparse phased sensor/actuator arrays and spatio-temporal transversal adaptive filters has been discussed. Diagnostic plate waves that propagate in any direction can be generated using beamforming in the actuator array to produce more uniform scatter at the damage site. The array gain as a function of steer angle is
dependent on the number of elements, inter-element spacing, array geometry and element size; also, more elements produce higher fidelity spatio-temporal filters while larger spacing can introduce grating lobes, and orthogonal arrays eliminate the duplicate side lobes formed about an axis of symmetry. Damage features are extracted from the data using digital signal processing in the frequency domain. Experimental results for a few cases in a composite and steel plate have been presented. More results on the experimental investigation include selectively sending out anti-symmetric and symmetric Lamb waves using piezoelectric actuators and higher frequency accelerometers and strain gauges will be forthcoming.

ACKNOWLEDGMENTS
The authors gratefully acknowledge the Department of the Army, Army Research Office for their support of this work under grant DAAD19-02-1-0185 with Dr. Gary Anderson as technical monitor as well as the Presidential Early Career Award for Scientists and Engineers program within the Department of Defense. The authors also gratefully acknowledge the Air Force Research Laboratory Materials and Manufacturing Directorate for their support of the experiments on the Aluminum plate with Dr. Kumar Jata as technical monitor.

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