Modal Analysis on a Schematic Aerospace Structure: FEM Simulation and Experimental Updating

V. Quaranta(†), I. Dimino(‡), M. d’Ischia(§), G. Davì(‡)

(†) CIRA ScpA (Italian Aerospace Research Centre) – Via Maiorise, 81043 Capua (CE), ITALY
(‡) University of Palermo, Dpt. of Aeronautical Engineering – Viale delle Scienze, 90128 Palermo, ITALY

ABSTRACT

Ground Vibration Tests on modern aircraft structures play a key role for the aeroelastic certification of new airplanes. The dynamic model of an aircraft is essential for flutter predictions and/or for analytical structural model updating and validation. This paper describes a technique aimed to the reliability and accuracy improvement of the numerical modal analysis by means of an experimental dynamic model. The procedure described hereby consists of a finite element analysis of a schematic aerospace structure, a GVT (both Phase Separation and Resonance techniques) and the FE model updating based on the knowledge of the experimental FRFs and natural frequencies. The results show that this procedure allows an effective reduction of the discrepancies between experimental and numerical estimations.

NOMENCLATURE

DOF Degree of Freedom
DPR Driving Point Residue
FE Finite Element
FRF Frequency Response Function
GVT Ground Vibration Test
MAC Modal Assurance Criterion
MIMO Multiple Input-Multiple Output
MIF Mode Indicator Function
MDOF Multiple Degree Of Freedom
SDOF Single Degree Of Freedom
SIMO Single Input-Multiple Output
HTP Horizontal Tail Plane
VTP Vertical Tail Plane
\[V_r\] Mode shape vectors
\[V_r^T\] Trasposed mode shape vectors
\[K_a\] Initial analytical stiffness matrix
\[K_i\] i-th substructure stiffness matrix
\[M_a\] Initial analytical mass matrix
\[M_j\] j-th substructure mass matrix
\[S\] Sensitivity matrix
\[W\] Weighting matrix
\(\alpha\) Stiffness correction parameter
\(\beta\) Mass correction parameter
\(\Delta z\) Residual vector
\(J\) Objective function
\(\Delta p^i\) vector of parameter changes at i-iteration
t test
a analytical
T Transpose of a matrix or vector
x, y, z Translational degree of freedom coordinates
1. INTRODUCTION

Aeroelastic feedback affecting aerospace structures constraints the efficient design for dynamic stability certification of high performance flight vehicles. The partitioned approach is typically performed with the purpose of a simple decomposition of the problem of interaction between unsteady aerodynamic phenomena and elastic and inertial forces. The first class of parameters depends on the modal displacements, the dynamic pressure, the Mach number and the oscillation frequency of the modal forces.

The dynamic behaviour of a structure in a given frequency range can be modelled as a set of individual vibration modes, each of them having a characteristic natural frequency, damping factor and modal shape. Present modal model identification methods include a wide range of contiguous areas such as modal correlation and updating, coupling validated mathematical models and high fidelity dynamic test data. Taking into account the modelling and testing uncertainties, the primary challenge associated with the system identification process is the definition of a refined FE model, that offers good predictions under fixed conditions.

In modern design procedures, model testing, system identification and model updating are conceived as independent steps. The process of both modelling and testing is very time consuming. Theoretical or experimental methodology selection depends on the phase of the project. Preliminary mathematical model and/or experimental modal results give the starting point of a process that can identify and improve the approximations of each individual approach.

The objective of the experimental/numerical integration is the development of better designs by bringing together the best of both worlds. FE model allow both the prediction of the structural behaviour and parametric studies. Experimental techniques (both ground and flight testing) allow the validation of the numerical estimations and the identification of different kinds of aeroelastic instabilities.

The purpose of this research is to investigate a global and integrated approach according to the following steps:

- Pre-test analysis
- GVT (phase separation test and phase resonance test)
- Experimental modal model validation
- FE model updating.

A general procedure for an analytical and experimentally compatible modal investigation is described and applied on a schematic aerospace structure. A prior FE model is developed at the beginning of the process to simulate the system's true vibration behaviour. The pre-test phase includes the enhancement of test conditions by optimising the exciter and sensor locations. The updating process, including model correlation and model updating, uses test measurements of the validated modal database to refine the numerical model. Finally, the updated model is expected to represent the dynamic behaviour of the structure in a more accurate way. This validated numerical model can be used later to match the aerodynamic loads and to evaluate the airplane aerelastic stability parameters.

2. SMALL-SCALE AIRCRAFT STRUCTURE

In this study, a representative structure of a typical aircraft design, similar to the GARTEUR SM-AG 19 testbed [1], manufactured by CIRA, was used. The model's fuselage, wings and tail are built with standard aluminium beams, jointed together through bolted connections (Fig.1). The overall length of the structure was 1.5 m and the wing span was 2.0 m. In order to obtain realistic damping levels, additional energy dissipation has been introduced through the use of a viscoelastic layer, bonded on the wing's upper surface. The structure has been soft-suspended on the centre of gravity with a soft spring system, aimed to approximate unconstrained boundary conditions.

The procedure for the updating of the computational model is based on the maximisation of the correlation between analytical and experimental FRFs. This operation doesn’t take into account modal parameters, but is based on the use of functions that describe the FRF shape and amplitude correlation over a frequency band. Typically, the success rate of the commonly used approaches to FRF-based updating is largely dependent on the optimal choice of frequency points at which a residual is evaluated and then minimized [2,3]. In this paper, the FRF correlation is evaluated at all frequency points, instead of using a limited number of reference frequencies (preventively chosen). The residual, that needs to be iteratively minimized, includes eigenvalue and frequency response errors. A sensitivity study was carried out in order to determine the influences of mass and stiffness changes on the dynamic behaviour of the structure and on the selection of the updating parameters. The differences between the measured and predicted modal data have been minimized, by using the NASTRAN Sol 200 Solver in combination with LMS/Gateway [5] as pre and post-processor. The result of this analysis was the modification of the selected numerical model parameters (stiffness and/or mass related), in order to improve the correlation between test and FE model dynamic results.
2.1 FINITE ELEMENT ANALYSIS

The FE model of the structure was generated in order to have analytical data for the preparation of the Vibration Test. The requirements for this representation were related to the need for a simplified model with a reduced computational time and a model that allows a global description of its dynamic behaviour. It consists of 502 Euler-Bernouilli beam elements and two lumped masses. Special care was taken when modelling the joints and mass distribution, taking into account the stiffness effect of the junctions. In particular, the joint between fuselage and wing was modelled by estimating the equivalent Young's modulus and mass density of the connecting elements. Some results of the FEM simulation are shown in Fig. 2.

3. PRE TEST ANALYSIS

The objective of an effective integration of FE analysis with structural testing is to combine the advantages of both techniques in a more valuable synergistic approach. Starting from a FE model, the numerical results can be used in order to plan test strategy and to find optimal location for sensors, shakers and suspension, allowing the optimal control and observation of a number of selected target modes. Pre Test analysis is important to maximize the test efficiency, reduce test costs and limit the unavailability of the prototype under test, increasing quality of modal test data. Once the set of target modes has been defined, the measurement locations have to be chosen such that all target modes can be observed. Due to equipment (number of sensors, number of data acquisition channels) or setup (mass-loading) limitations, a large number of
measurements cannot be made for all responses simultaneously, so the challenge is the use of a minimal number of sensors, aimed to a well defined spatial resolution of all target modes. Since the FE model grid is much finer than the test one, a substantial reduction of its size is needed in order to assess the correlation of the numerical predictions with the modal test measurements.

In recent years, several procedures for selecting sensor locations for the purpose of modal testing have been investigated. Some of the most common techniques assume that by placing the sensors at points of maximum kinetic energy, they will have the maximum observability of the structural parameters of interest, because of the associated mass weighting.

The M/K ratio method of sensor placement, for example, is based on the master DOF in Guyan reduction [4,5]. The DOF having the highest diagonal M/K values (high inertia in comparison to stiffness) are retained as masters or candidate sensor locations, working well typically only for the lowest order modes.

### 3.1 OBSERVABILITY CRITERION: OPTIMAL SENSOR SET

One of the most common methodologies for the evaluation of the quality of the Master DOFs selection is the Modal Assurance Criterion (MAC).

The MAC expresses the correlation between a pair of vectors. The MAC value between two mode shape vectors $[V_r]$ and $[V_r']$ is defined as:

$$MAC_{r,r'} = \frac{[V_r][V_r']^2}{[V_r][V_r'][V_r][V_r']}$$

If the off-diagonal terms of MAC matrix are smaller than 0.1 or 0.2, the cross-correlation between the target modes is sufficiently low, enabling all the modes to be easily distinguished from each other. An erroneous or too limited subset of sensor locations will lead to a phenomenon called Spatial Aliasing, due to an incomplete geometric definition of the mode shapes.

The initial group could be iteratively enhanced with extra degrees of freedom such that off-diagonal MAC shows values (MODMAC) below a given threshold.

### 3.2 EXCITING PLACEMENT

An important step of a Pretest analysis is the determination of adequate shaker locations for the ground vibration test (GVT). An optimal excitation of all the modes of interest, indeed, would allow to save a valuable amount of time at the test site.

The Driving Point Residue is the chosen technique for selecting the best excitation locations and directions [6]. DPRs are stated to be equivalent to modal participation factors, and they are a measure of how much each mode is excited at the driving point. DPRs are proportional to the magnitudes of the resonance peaks in a driving point FRF measurement. In the parametric model of the FRF, the residue $r_{jk}$ between locations $i$ and $j$ for mode $k$ can be written as the product of a scaling factor $a_k$ and the modal vector components in both locations. If the structure is proportionally damped, the modal vectors of the structure are real and the scaling factor is purely imaginary:

$$r_{jk} = a_k v_{jk}$$

$$a_k = \frac{1}{2j\omega_{dk} m_k}$$

At the driving point, the magnitude of the driving point residue becomes:

$$DPR_k(i) = \frac{v_{ik}^2}{2m_k \omega_{dk}}$$
For shaker placement, the chosen points and excitation directions must allow a maximum value for the averaged DPR for all target modes.

### 3.3 PRE-TEST PHASE

In the Pretest analysis of the small-scale aircraft structure, test was simulated and planned by the use of the FE model. The investigated sensor techniques included three conventional methods: M/K ratio, MODMAC and experience judgement methods. By taking into account the maximum kinetic energy method as observability criterion, master degrees of freedom having the highest mass/stiffness ratio were identified by computing the diagonal M/K values for the model using a MSC/NASTRAN DMAP alter. The number of measurement points were selected in order to enable a good identification of the modes in the frequency range 0-150 Hz. The selected points were chosen between the calculated master DOFs; the adopted criteria were the optimisation of the MAC matrix (MODMAC) and the visual target mode shape inspection. According to this second criterion, a homogeneous distribution of nodes on the structure is assured (according to an engineering judgement approach). The final accelerometer plan is shown in Fig. 8.

![Figure 4: Check of the sensor placement using the MAC comparison: initial (left) and optimised (right) testing meshes](image1)

![Figure 5: DPR values versus FEM DOFs](image2)

![Figure 6: Excitation at right wing tip](image3)

The repeatability and stability of the test were examined by means of the Modal Assurance Criterion. Fig. 4 shows the MAC comparison of the mode set with itself. The new candidate set of sensor locations provides a simpler separation of coupled mode shapes, even if the frequency range of modal analysis was increased. In order to determine the adequate shaker locations for a global excitation of all target modes of the structure, DPR method was applied. The final set of shaker locations was employed in the preliminary operation of frequency scanning and approximate determination of resonance frequencies by using phase separation methods. Fig. 5 shows the DPR values, calculated for different excitation directions. This technique proposed right and left wing tip DOFs as the best points for exciting the global target modes (Fig. 6). This choice is also due to the experimental
difficulties to excite the first wing bending mode and extract its modal parameter from FRFs related to other
directions of excitation. For the subsequent force appropriation step, the sensor and shaker locations validation
technique used is known as the Mode Indicator Function (MIF). MIF indicated how each mode was clearly
identifiable and its rate of purity by using the selected sensor and shaker set to isolate normal modes.

4. EXPERIMENTAL MODAL ANALYSIS

The main goal of the experimental modal analysis consists in identifying the different modes, even when these
modes are very close each other or coupled. Mathematical separation of modes following broadband global excitation (Phase Separation) and physical separation of modes using sine excitation (Phase Resonance or Normal Mode) are two different philosophical
approaches typically used for modal test [7,8]. One method uses single or multiple broadband excitation signals to
global excite the structure and SDOF/MDOF curve-fitting algorithms to extract the modal data from the measured
SIMO/MIMO FRFs. The appropriated excitation method (Normal mode) focuses, instead, on each mode
individually, studying every one in its purest possible form. It uses single/multi-input sinusoidal vibration at a
resonant frequency, fine tuning the distribution of force amplitudes. Once a mode is physically isolated, its shape is
stored from all response locations measured simultaneously, while its generalized parameters are extracted from
the measured mechanical power introduced at the driving points (Complex Power method), and validated from
additional quadrature induced forces.

For a damped MDOF system, with N degree of freedom, the governing equations of motion can be written in matrix
form as:

\[
[M]\ddot{\mathbf{x}} + [C]\dot{\mathbf{x}} + [K]\mathbf{x} = \{F(t)\}
\]

(5)

where \([M]\), \([C]\) and \([K]\) are mass, damping and stiffness matrices respectively.

Normal modes \([\Phi]\) are solutions of the undamped eigenvalue problem. Each mode is assumed real-valued,
indicating that all degrees of freedom must move in phase or in phase opposition. It represents a shape, not an
absolute measure of displacement and it's typically orthonormalized with respect to mass.

Written a coordinate transformation \(\{x\} = [\Phi]\{q\}\) and performed a congruence transformation, the equations become:

\[
[\mu]\{\dot{q}\} + [c]\{q\} + [\gamma]\{q\} = [\Phi]^T\{F\}
\]

(6)

where \([\mu]\), \([c]\) and \([\gamma]\) are the generalized matrices and the excitation of these uncoupled equations is the
generalized force vector.

Consider what happens when we chose the physical excitation forces as [9]:

\[
[F] = \text{Re}(f e^{j\omega t})
\]

(7)

where:

\[
[f] = [f^r + jf^i]
\]

(8)

The steady state response is harmonic:

\[
[q] = \text{Re}(-j\bar{q} e^{j\omega t})
\]

\[
\bar{q} = [\bar{q}^r + j\bar{q}^i]
\]

(9)

A mode k is said to be isolated at its resonance frequency \(\Omega_k^r\) if all the velocity responses are exactly in phase or in
opposite phase with the applied forces. The excitation forces are chosen in phase within \(k\pi\) and the generalized
forces exactly balance all the damping forces:

\[
[f^r] = 0
\]

\[
[f^i] = f
\]

(11)

\[
[\Phi]^T\{f\} = [c_k]\Omega_k^r
\]

4.1 GROUND VIBRATION TEST

Experimental modal characterisation of the benchmark structure has been performed with both Phase Separation
and Normal Mode techniques.

Global excitation of modes of the structure with a SIMO random excitation signal was employed in order to identify
the structural resonances and derive an overview of the basic modes within the frequency range of interest.
Measurement and excitation points are showed in Fig. 8. In addition to the approximate estimation of modal frequencies, the individual peaks of Single Mode Indicator function (Fig 9) suggested the suitability of the respective exciter configuration. In order to extract modal parameters from the FRFs, Frequency Domain MDOF modal analysis technique was applied.
Once the structural resonances have been characterized with an approximate measurement, Normal Mode technique was used for the isolation and for the frequency and shape measurement of each mode; this was done by applying sinusoidal signals to the structure at each resonant frequency. To achieve perfect mode isolation, the phase relation between all channels and the master excitation force was checked in real time with various graphical and numerical indicator: MIF, multi Lissajous displays, Phase Scatter diagrams, real and imaginary mode shape plot. In a second step, the generalized parameters (mass, stiffness and damping) of each mode have been evaluated from the total driving power (Complex Power method), measured with a sine sweep test, keeping constant the value and the ratio of the excitation forces, over a narrow frequency range centred on each resonance. The results are shown in Table 1. 14 modes were determined in the frequency range 0-150 Hz. The high MIF value demonstrate the high phase purity of all measured modes. Only two modes were excited with two shakers, confirming the appropriate exciting positions used. Plots of the mode shapes are shown in Fig. 12. Finally, by conducting a series of normal mode test, with different master force level, but with constant force ratio, a linearity test was performed for each mode. The natural frequencies and the maximum response were measured versus the master force value, in order to individuate system nonlinearity. The model showed a good linear behaviour.

![Image](image_url)

**Table 1: Experimental modal analysis results**

<table>
<thead>
<tr>
<th>Nr.</th>
<th>Mode</th>
<th>MIF</th>
<th>ξ [%]</th>
<th>$M_{mom}$ [Kg m²]</th>
<th>Frequency [Hz]</th>
<th>Ph Separation</th>
<th>Frequency [Hz]</th>
<th>Ph Resonance</th>
<th>Excitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2N wing bending</td>
<td>0.9749</td>
<td>1.476</td>
<td>3.27</td>
<td>5.30</td>
<td>5.2656</td>
<td></td>
<td></td>
<td>Gart 22 z</td>
</tr>
<tr>
<td>2</td>
<td>Fuselage rotation</td>
<td>0.9955</td>
<td>1.4</td>
<td>4.46</td>
<td>14.25</td>
<td>13.750</td>
<td></td>
<td></td>
<td>Gart 13 z</td>
</tr>
<tr>
<td>3</td>
<td>Antisymmetric wing torsion</td>
<td>0.9559</td>
<td>0.6</td>
<td>0.61</td>
<td>33.25</td>
<td>28.17</td>
<td></td>
<td></td>
<td>Gart 22 z</td>
</tr>
<tr>
<td>4</td>
<td>Symmetric wing torsion</td>
<td>0.95</td>
<td>0.5</td>
<td>1.71</td>
<td>32.86</td>
<td>28.637</td>
<td></td>
<td></td>
<td>Gart 22 z</td>
</tr>
<tr>
<td>5</td>
<td>3N wing bending</td>
<td>0.9990</td>
<td>1.084</td>
<td>2.06</td>
<td>31.83</td>
<td>31.5</td>
<td></td>
<td></td>
<td>Gart 13 z</td>
</tr>
<tr>
<td>6</td>
<td>4N wing bending</td>
<td>0.9998</td>
<td>0.510</td>
<td>2.8</td>
<td>42.81</td>
<td>41.4843</td>
<td></td>
<td></td>
<td>Gart 13 z</td>
</tr>
<tr>
<td>7</td>
<td>Antisym. Fore/Aft wing bending</td>
<td>0.9525</td>
<td>2.186</td>
<td>7.14</td>
<td>41.15</td>
<td>40.20</td>
<td></td>
<td></td>
<td>Gart 3 y</td>
</tr>
<tr>
<td>8</td>
<td>Symm. Fore/Aft wing bending</td>
<td>0.9989</td>
<td>0.8</td>
<td>3.84</td>
<td>51.09</td>
<td>50.6718</td>
<td></td>
<td></td>
<td>Gart 7 x</td>
</tr>
<tr>
<td>9</td>
<td>5N wing bending</td>
<td>0.9905</td>
<td>2.1</td>
<td>3.76</td>
<td>55.62</td>
<td>54.00</td>
<td></td>
<td></td>
<td>Gart 13 z</td>
</tr>
<tr>
<td>10</td>
<td>Tail torsion</td>
<td>0.9980</td>
<td>0.2</td>
<td>0.48</td>
<td>61.90</td>
<td>56.5312</td>
<td></td>
<td></td>
<td>Gart 25 z</td>
</tr>
<tr>
<td>11</td>
<td>Lateral fuselage bending</td>
<td>0.9990</td>
<td>0.840</td>
<td>1.02</td>
<td>100.48</td>
<td>97.3437</td>
<td></td>
<td></td>
<td>Gart 3 y</td>
</tr>
<tr>
<td>12</td>
<td>2nd tail bending</td>
<td>0.9788</td>
<td>1.280</td>
<td>3.38</td>
<td>120.00</td>
<td>117.3906</td>
<td></td>
<td></td>
<td>Gart 13 z</td>
</tr>
<tr>
<td>13</td>
<td>6N wing bending</td>
<td>0.9960</td>
<td>0.963</td>
<td>2.17</td>
<td>126.66</td>
<td>123.3750</td>
<td></td>
<td></td>
<td>Gart 13 z</td>
</tr>
<tr>
<td>14</td>
<td>7N wing bending</td>
<td>0.9568</td>
<td>1.358</td>
<td>1.73</td>
<td>134.34</td>
<td>132.1718</td>
<td></td>
<td></td>
<td>Gart 13 z</td>
</tr>
</tbody>
</table>

**Figure 7: Test setup - Vibration & Acoustics Test facility of CIRA**
Figure 8: Measurement mesh and excitation points (black arrows) used for the broadband test.

Figure 9: Driving point FRF and Single mode Indicator function (Excitation at dof 13-z).

Figure 10: Phase Scatter Diagram and Lissajous display during the appropriation of the first wing bending mode (master force 7N).

Figure 11: Real and Imaginary values of Complex Power for the first wing bending mode (master force 7N).

Figure 12: Experimental mode shapes (Normal Modes).
5. CORRELATION ANALYSIS

Correlation analysis allows the identification of corresponding modes, as well as of relations between the analytical and experimental modal data. The spatial incompatibility due to the measurement of mode shapes through a limited set of physical sensors can be resolved by using expansion/reduction algorithms in order to obtain the same number of DOFs for both models.

Numerical/experimental correlation is typically expressed in terms of resonance frequencies and mode shapes. Proportionality check on mode shapes of both models is a simple criterion to analyse the correspondence between test and analytical results. Cross-Orthogonality Test, deriving from the eigenvectors’ orthogonality properties, Modal Assurance Criterion (MAC) and natural frequency comparison are the most common approaches.

In some applications, the use of FRFs for correlation can be more attractive than eigenvectors. FRFs are not subject to curve-fitting errors, particularly when dealing with structures that have high modal density and high damping, and provide information on damping characteristics over the entire spectrum. Both approaches were considered in this application.

For the test/analysis correlation in the modal domain, MAC criterion was used [10]:

\[
MAC = \frac{\left| \Phi_t^T \Phi_a \right|^2}{\left( \Phi_t^T \Phi_t \right)^{1/2} \left( \Phi_a^T \Phi_a \right)^{1/2}}
\]  

(12)

Moreover, it was relevant to compare the mode shapes qualitatively using side-by-side or animated viewing, auto-MAC plot and frequency comparison.

For the frequency domain correlation the Frequency Response Assurance Criterion (FRAC) was applied. At each frequency point \( \omega_i \), the level of correlation between all corresponding experimental and predicted FRFs was evaluated as:

\[
FRAC(j) = \frac{\left| H(\omega_j)_{FEM}^T H(\omega_j)_{ext} \right|^2}{\left( \left| H(\omega_j)_{FEM} \right|^2 \right) \left( \left| H(\omega_j)_{ext} \right|^2 \right)}
\]

(13)

where \( \left| H(\omega_j) \right| \) is a vector describing the frequency response function at DOF \( j \) for a frequency \( \omega_j \). This function is a measure of shape correlation with values ranging between 0 and 1. It is very sensitive to changes of mass and stiffness modelling. If a stiffness factor of \( \alpha \) is applied to the structure, then this will result in a frequency shift of \( \sqrt{\alpha} \) and an amplitude shift of \( \frac{1}{\alpha} \). By defining a frequency factor \( \beta = \sqrt{\alpha} \) and computing the FRAC values for a given range of values of \( \beta \), the value of \( \beta \) at the maximum value of FRF indicates the frequency shift and the global stiffness factor required to improve the correlation between the analytical and experimental FRFs [5]. This method was very helpful to achieve reconciliation between the models, giving qualitative indications about their discrepancies and updating strategy.
6. MODEL UPDATING: MATHEMATICAL BACKGROUND

Model updating is a step in the validation process that modifies some of the parameters in the FE model in order to minimise the discrepancies between FE model predictions and experimental data. Two interconnected philosophies are presently accepted as state of the art in aeronautical applications [11]. The first one states that model updating consists of two distinct stages, namely error localization and error correction. It advocates a parameter updating technique that tries to obtain an improved numerical model by changing parameters, preserving the physical meaning of the model. The second philosophy consists on a global correction in a curve fitting sense in order to minimize an objective function subject to some constraints. The corrections don’t correspond to specific modelling errors, but they directly regard the global mass and stiffness matrices sequentially. Methods belonging to the first group are generally iterative and they have emerged as the most widely used in general practical applications [12]. Their basis is the following parameterisation of the model matrices:

\[
[K] = [K_0] + \sum_i \alpha_i [K_i]
\]

\[
[M] = [M_0] + \sum_j \beta_j [M_j]
\]

with:

- \([K_0], [M_0]\) initial analytical stiffness and mass matrices respectively
- \(\{\alpha_i, \beta_j\}\) vector of unknown design parameters
- \([K_i], [M_j]\) given substructure matrices defining location and error of model uncertainties

Substructure matrices can be considered as the first derivative of the updated matrices with respect to a physical or geometrical model parameter:

\[
[K_i] = \alpha_i \frac{\partial[K]}{\partial\alpha_i}
\]

\[
[M_j] = \beta_j \frac{\partial[M]}{\partial\beta_j}
\]

(15)

where \(\alpha_i, \beta_j\) denote the initial parameters used to make the parameter changes dimensionless.

The computational model updating procedures are based on a penalty function minimization. This parameterisation allows the updating of a wide range of simultaneous parameters in uncertain model areas. By using appropriate residuals, the optimisation of the following objective function \(J\) can iteratively update the design parameters \(p\):

\[
J = \{\Delta z\}^T W \{\Delta z\} \rightarrow \min
\]

(16)

with:

- \(\Delta z\) residual vector
- \(W\) a positive definite weighting matrix.

The residual vector \(\Delta z = z_t - z(p)\), containing the difference between test data vector and the analytical results, usually depends in a nonlinear way on the design parameters. Thus the minimization problem is also nonlinear and must be solved iteratively.

The sensitivity approach is based on the use of a Taylor series of the modal data, expanded as a function of the updating parameters and truncated after the first term in order to linearize the equation:

\[
\{z(p)\} = \{z_0\} + [S_0]\{\Delta p\}
\]

(17)
with:
\[
\{z_0\} \quad \text{model vector at linearization point 0}
\]
\[
[S_0] = \frac{\partial z}{\partial p} \quad \text{sensitivity matrix at linearization point 0}
\]
\[
\Delta p = p - p_0 \quad \text{design parameter change.}
\]
By proceeding in this way the result is:
\[
\{\Delta z\} = \{\Delta z_0\} - [S_0] \{\Delta p\} \quad (18)
\]
where:
\[
\Delta z_0 = z_f - z_0.
\]
The minimization problem of Eq. (16) yields the following linear system (19) which has to be solved in each iteration step for the actual linearization point:
\[
[S_0]^T \left[ W \right][S_0] \{\Delta p\} = [S_0]^T \left[ W \right] \{\Delta z_0\} \quad (19)
\]
After each iteration step, an updated estimate of the unknown design parameter vector can be obtained by:
\[
\{p^{i+1}\} = \{p^i\} + \{\Delta p^i\} \quad (20)
\]
where:
\[
\{\Delta p^i\} = \left[ S^T \right]^T \left[ W \right] \left[ S \right]^{-1} \left[ S^T \right] \left[ W \right] \{\Delta z^i\} 
\]
\[
(21)
\]
7. SENSITIVITY ANALYSIS AND MODEL UPDATING

The most important step in computational model updating is the selection of the updating parameters. Once the modal discrepancies of two models are quantified, a sensitivity study is performed in order to localize sensitive parameters (physical and/or geometrical parameters) that have a significant influence on the dynamic behaviour of the structure. However, although modal sensitivities provide information on the gradient of the modal parameters with regard to correction parameters, it doesn’t mean that the sensitive parameters are also the erroneous parameters. Sensitivity study allows to improve the procedure efficiency and sensitivity matrix provide information on the potentially sensitive and insensitive zones of the structure.

The fundamental FEM uncertainties, due to the structure modelling, were assumed to be in the joints, the local mass distribution and the wing, VTP and HTP stiffness. Sensitivity analysis regarded the amplitude of FRFs, synthesized in the 4-160 Hz spectrum, and modal frequencies of the FE model. It was performed by the MSC/NASTRAN optimization module (SOL 200) with an output after the gradient calculation of the first iteration.

Table 2 shows the selected parameters used for the computational model updating. Figg 16, 17 show the frequency sensitivities and a sensitivity vector at each frequency line for a selected FRF. Two global parameter changes, proportional to the model matrices (e.g. mass densities), were selected for updating process in addition to local stiffness parameters. The 2n and 3n wing bending modes were sensitive to parameter \(I_{x_2}\) (wing’s area moment of bending inertia), while the symmetric and the antisymmetric wing torsion parameter showed a sensitivity to \(I_{x_2}\) (wing’s area moment of torsional inertia). The VTP bending stiffness had an influence on the fifth mode and parameter \(I_{x_5}\) had the most important influence on the tail torsion mode.

<table>
<thead>
<tr>
<th>Table 2: Updating Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Wing</td>
</tr>
<tr>
<td>Plate of junction</td>
</tr>
<tr>
<td>Vertical tail plane</td>
</tr>
<tr>
<td>Horizontal tail plane</td>
</tr>
<tr>
<td>Model density</td>
</tr>
<tr>
<td>Plate density</td>
</tr>
</tbody>
</table>
The updating process was focused on design parameter changes and it was based on the minimization of the dynamic response residue vector. It was decided to use only design parameters for updating, because the physical interpretation of changes is obvious and easier to assess. Various updating runs were performed and the results with the smallest parameter variations are presented. Table 3 shows parameter changes after updating and Table 4 depicts the definitive frequency deviation after updating, compared with the initial correlation.

The optimization gave a good matching for the eigenfrequencies, and the correct dynamic behaviour, demonstrated by MAC correlation, was maintained, Fig. 19. FRFs, compared before and after model updating in correspondent DOFs of both models, covered test data in most areas, confirming the efficiency of the presented approach, Fig. 20. The fine-tuned model was able to reproduce the measured data and these results were sufficient to consider as validated the FE model of the structure.

### Table 3: parameter changes after updating

<table>
<thead>
<tr>
<th>Parameter</th>
<th>A</th>
<th>Iy</th>
<th>Iz</th>
<th>Ix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wing</td>
<td>-7%</td>
<td>-19.81%</td>
<td>-30.36%</td>
<td>-15.49%</td>
</tr>
<tr>
<td>Plate of junction</td>
<td>+2.66%</td>
<td>+4.86%</td>
<td>-10.23%</td>
<td></td>
</tr>
<tr>
<td>Vertical tail plane</td>
<td>+24.54%</td>
<td></td>
<td>-21.20%</td>
<td>-9.86%</td>
</tr>
<tr>
<td>Horizontal tail plane</td>
<td>+28.06%</td>
<td>+23.71%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model density</td>
<td>+10%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plate density</td>
<td>+10%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 4: experimental and finite element model comparison

<table>
<thead>
<tr>
<th>EMA [Hz]</th>
<th>FEA [Hz]</th>
<th>deviation [%]</th>
<th>FEA [Hz]</th>
<th>deviation [%]</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.27</td>
<td>5.82</td>
<td>10.62</td>
<td>5.06</td>
<td>-3.95</td>
<td>1.00</td>
</tr>
<tr>
<td>13.38</td>
<td>16.81</td>
<td>25.63</td>
<td>13.53</td>
<td>1.13</td>
<td>0.92</td>
</tr>
<tr>
<td>28.17</td>
<td>31.35</td>
<td>11.28</td>
<td>27.92</td>
<td>-0.87</td>
<td>0.60</td>
</tr>
<tr>
<td>28.64</td>
<td>31.51</td>
<td>10.02</td>
<td>28.10</td>
<td>-1.89</td>
<td>0.77</td>
</tr>
<tr>
<td>31.50</td>
<td>38.61</td>
<td>22.57</td>
<td>32.27</td>
<td>2.46</td>
<td>0.56</td>
</tr>
<tr>
<td>41.48</td>
<td>46.74</td>
<td>12.68</td>
<td>41.32</td>
<td>-0.39</td>
<td>0.99</td>
</tr>
<tr>
<td>40.20</td>
<td>53.92</td>
<td>34.12</td>
<td>44.19</td>
<td>9.92</td>
<td>0.89</td>
</tr>
<tr>
<td>50.67</td>
<td>57.11</td>
<td>12.71</td>
<td>46.29</td>
<td>-8.65</td>
<td>0.97</td>
</tr>
<tr>
<td>54</td>
<td>65.44</td>
<td>21.18</td>
<td>55.56</td>
<td>-4.55</td>
<td>0.90</td>
</tr>
<tr>
<td>56.53</td>
<td>67.94</td>
<td>20.18</td>
<td>53.96</td>
<td>2.89</td>
<td>0.89</td>
</tr>
<tr>
<td>97.34</td>
<td>105.07</td>
<td>7.94</td>
<td>90.22</td>
<td>-7.32</td>
<td>0.88</td>
</tr>
</tbody>
</table>
Table 4: Correlation between initial and updated analytical model and test data sets

<table>
<thead>
<tr>
<th>Deviation % FE model versus GVT</th>
<th>Updated FEM</th>
<th>Initial FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>natural frequency</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.27</td>
<td>13.4</td>
<td>28.2</td>
</tr>
<tr>
<td>28.6</td>
<td>315</td>
<td>50.7</td>
</tr>
<tr>
<td>54</td>
<td>56.5</td>
<td>97.3</td>
</tr>
<tr>
<td>117</td>
<td>123</td>
<td>132</td>
</tr>
<tr>
<td>132.17</td>
<td>136.74</td>
<td>14.69</td>
</tr>
<tr>
<td>121.30</td>
<td>114.72</td>
<td>-2.28</td>
</tr>
<tr>
<td>-1.68</td>
<td>0.73</td>
<td>0.96</td>
</tr>
<tr>
<td>0.15</td>
<td>0.62</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 18:** Frequency evolution of the updating process

**Figure 19:** MAC matrix between the experimental (Y axes) and updated model (X axes)

**Figure 20:** Comparisons between FRFs of experimental and initial model (top) and FRFs of experimental and updated model (down)
CONCLUSION
This paper illustrates the procedure employed for the dynamic characterization of a small-scale aircraft structure. Modal analysis, performed in the frequency range of 4-150 Hz, was carried out by means of the phase separation and phase resonance techniques. In this way it was possible to verify the testing mesh that was optimised in the PreTest analysis and to appropriate each normal mode. Correlation of initial model predictions and test results regarded both modal models (MAC) and Frequency Response Functions (FRAC). Computational model updating used sensitivity analysis to select the updating parameters; it reduced drastically the frequency deviations and matched the measured FRFs of the test structure. The generalized use of this combined test/analysis technique is suitable for modal analysis requiring high number of measurement channels, and it is limited by the different hardware and software platforms only. It can assure more accurate results, lower cost in a shorter design cycle.

REFERENCES