A Study of the Order Tracking Methodology Applied to Gear Whine Noise Analysis

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NOMENCLATURE

\( \sigma_{\text{rad}} \) = average radiation ratio for the panel at the frequency \( \omega \)  
\( \{v^2(\omega)\} \) = space-time average mean-square velocity of the panel at the frequency \( \omega \) in the band \( \Delta \omega \), \( \text{m}^2/\text{sec}^2 \)  
\( S \) = panel area, \( \text{m}^2 \)  
\( \rho \) = casing material density, \( \text{kg/m}^3 \)  
\( c \) = velocity of sound, \( \text{m/sec} \)  
\( f_c \) = critical frequency, Hz  
\( t \) = thickness of the plate, \( \text{m} \)  
\( E \) = Young's modulus, \( \text{N/m}^2 \)  
\( r \) = distance from the source, \( \text{m} \)  
\( Q \) = directivity factor of the source=2, (the sound is radiating over a hemi-spherical surface)  
\( R \) = room constant

ABSTRACT

This paper discusses the application of order tracking analysis to study the dynamic response of geared systems. The procedure used to analyze sound pressure level and vibration signals obtained experimentally is presented with the help of a test signal generated through a simple simulation model. The accuracy of the results obtained is investigated. The inferences gained from the simulation are then applied to experimental data to better understand the resulting noise analysis.

INTRODUCTION

Order tracking allows the correlation of RPM with specific harmonics of the dynamic signals [1] and is thus, an effective method for presenting and analyzing sound and vibration signals created by rotating machinery. It is a proven tool to identify system resonances and critical speeds that may be encountered during the operation cycle of the machinery. Fig. 1 outlines the gear whine noise generation process. Excitations generated at the gear mesh such as transmission error, friction and axial shuttling forces [2], induce torsional and bending vibrations in the shafting. The bearing forces resulting from the shaft vibrations excite the housing which then radiates noise. The first part of this study involves simulating this dynamic system using a transfer function approach and then analyzing the influence of the signal processing on the calculated results. Order tracking analysis is then applied to experimental data and the results are incorporated into a simple noise generation model that further describes the influence of the excitation sources, and the resulting forces and vibration, on the sound.
SYSTEM SIMULATION

The system described by the blocks in Fig. 1 is simulated by passing a known dynamic signal through an assumed transfer function representing the shaft-bearing-housing structure. The output signal is then processed using order-tracking analysis based on Vold-Kalman filtering [3]. Fig. 2 shows a block diagram of a system created to generate the test signal. The input signal, created using time domain simulation [4], is a sawtooth wave with constant amplitude but frequency increasing linearly with time. The system simulates a speed run-up from 12 to 960 RPM over a period of 79 seconds (12 RPM/sec). The first ten harmonics of the input signal were considered for the analysis and the relative amplitudes of the input signal harmonics are shown in Fig. 3. The signal is fed through three 2nd order transfer functions with resonant frequencies at 300, 1300 and 2800 Hz, respectively and Fig 4 shows the system transfer function. Band limited white noise is added to simulate the presence of background noise. Noise is added to the input signal and the sum is then fed through the transfer function.

Figs. 5 and 6 show the waterfall plots of the respective output signals for cases without and with noise added at the input. The response of the three resonant frequencies of the system transfer function are clearly evident. The waterfall plot with noise added displays behavior very similar to that observed in physical systems. A waterfall plot provides a good overview of the system response as the speed is varied, but it is typically only useful when the ordinate scale is linear.

A more useful plot that has much greater dynamic range, tracks the amplitude of each harmonic as a function of speed (time varying frequency) using a logarithmic or decibel ordinate scale [5-9]. This type of signal processing is known as order tracking and to achieve this type of plot, the transient data of this paper is processed using Vold-Kalman filtering techniques [3]. Examples of an order-tracked display for the data are given in Fig. 7 which shows the order-tracked harmonics of the output signal obtained by adding the noise to the test signal. Note that the abscissa in these figures is frequency and any one of the functions plotted may be visualized as projecting the corresponding cursor line from Fig. 6 towards the frequency axis. The lower harmonics, with high amplitudes relative to the noise, are similar to those calculated without noise. However, the higher harmonics with low amplitudes as compared to the noise, are overwhelmed by the noise. In fact, the behavior of the higher harmonics is similar to that observed when only noise is passed through the transfer function (Fig. 8).
Fig. 3: FFT of sawtooth input signal.

Fig. 4: Transfer function of the system.

Fig. 5: Waterfall plot of the output signal with no noise added.

Fig. 6: Waterfall plot of the output signal with noise added.

Fig. 7: Output signal obtained during speed run-up with noise added to the test signal.

Fig. 8: Output signal obtained during speed run-up with only noise fed to the transfer function.
This highlights the importance of a high signal-to-noise ratio. The ability of the order tracking algorithm to extract the raw signal from the noise floor is limited and may not be effective in physical situations as shown in Fig. 9 below. The experimental data displayed here is compromised by a sizable noise floor and the corresponding order plots may not be an effective representation of the system.

Fig. 10 shows the effect of doubling the slew rate of the speed run-up (24 RPM/sec). The low frequency range of the higher harmonics is disrupted. Even when noise is not added in the simulation, the signal processing itself adds noise to the calculated results. There is also the appearance of step-like behavior in the graphs. These notches are investigated further by comparing the actual behavior of the order-tracked harmonics of the sawtooth signal with those calculated using signal processing (Fig 11). These order plots are similar to those observed earlier but in this case, time has been substituted for frequency on the abscissa. The actual behavior shows that harmonic amplitudes do not vary with time, however the processed results show a time varying amplitude and the appearance of steps, which do not occur in the actual signal.

Fig. 9: Waterfall plot of sound data in the presence of significant background noise

Fig. 10: Output signal obtained during speed run-up at twice the slew rate (with noise).
A second scenario involves passing only the sawtooth signal through the transfer function and adding noise to the output signal i.e. after the transfer function. Fig. 12 shows the order-tracked harmonics of the final signal obtained. Due to the large amplitudes of the resonant peaks the noise seems to have the greatest effect in the region between the resonances and the noise floor is raised.

A test is performed to check how accurately the system transfer function can be re-created using the order-tracked harmonics of the output signal. The composite plot displayed in Fig. 13 shows the behavior of the 1<sup>st</sup> harmonic of the output signal had the experiment been run to a sufficiently high RPM. By adding up the effects of each harmonic, it is possible to obtain a composite frequency response plot over the entire frequency range. The actual system transfer function may be obtained by dividing the composite signal by the amplitude of the 1<sup>st</sup> harmonic of the input signal. Fig. 14 compares the actual system transfer function with those obtained using the order-tracked data, with and without noise. Even the data with noise provides a good approximation of the actual system.
EXPERIMENTAL STUDY

For the purpose of this paper we shall consider the data acquired from a back-to-back gear test rig during a speed run-up experiment. Sound pressure level is obtained using a binaural head that is placed a few meters away from the gearbox and vibration data is recorded from an accelerometer mounted in the axial direction. The results shown in Fig. 15 are for a speed sweep from 90 to 1200 RPM with data taken over a time interval of about 1 minute at an output load that corresponds to about 220% of the rated load of the gears. The waterfall plot of the noise signal shows resonances in the 500 to 3000 Hz range. Fig. 16 shows the corresponding order plot for this data. In this case, only selected harmonics are presented so that the information would not be too congested on the plot. The resonance at about 650 Hz appears to slightly decrease in frequency with the higher order harmonics. The frequency shift appears to be due to the signal processing methodology as a similar shift can be observed in the order plot created for the test signal (Fig 7).
Fig. 17 shows the creation of a composite plot that predicts the behavior of the 1st harmonic of the sound data up to 4kHz by shifting and adding the higher harmonic extensions to the first harmonic plot. Analysis up to these high frequencies could not be obtained experimentally due to the limitations of the test equipment. This procedure assumes that the system behaves linearly over the frequency range of interest.

In order to determine the contribution of the axial vibration to the radiated noise, a predicted sound pressure level curve is generated by integrating the experimentally obtained axial housing acceleration data to obtain the housing velocity which can be related to the radiated sound power via a radiation efficiency equation [10].

\[ W_{rad} = \rho c S \sigma_{rad} \langle v^2(\omega) \rangle \]  

(1)

The propagation speed of the bending wave in the plate equals the speed of sound in air at the critical frequency. The critical frequency depends upon the plate thickness and material properties. The radiation efficiency in the region below the critical frequency may be approximated as a line with a slope of about 6 dB/decade and the radiation efficiency above the critical frequency is 1.

\[ f_c = \frac{c^2}{1.8t} \sqrt{\frac{\rho}{E}} \]  

(2)

The resulting radiation efficiency curve is shown in Fig. 18. The gearbox casing thickness is quite large (~5 cm), and this results in a low critical frequency of approximately 200 Hz.

The resulting sound pressure level can be calculated as follows.

\[ L_p = L_w + 10 \log_{10} \left( \frac{Q}{4\pi r^2} + \frac{4}{R} \right) \]  

(3)

Fig. 17: Composite plot of the 1st harmonic order-tracked sound data
Kartik, et. al. [11] proposed a fairly simple model to predict behavior similar to that observed in the experimental sound data. Transmission error and friction force values generated by LDP [12] are used as an input to the SHAFT program [13], which is a finite element code developed at the Gear Dynamics and Gear Noise Research Laboratory used to simulate the shaft and bearing system dynamics. The program predicts the dynamic forces at bearings. The casing is modeled as a linear second order system and the overall casing velocity is obtained by calculating the response due to the bearing forces. Theoretical sound pressure level is then calculated using the radiated sound power equations described earlier.

Fig. 19 compares the overall sound pressure level from the experiment (fabricated composite first harmonic data), with the SPL predicted by the methods described above. The model using the experimental vibration data goes through similar resonances as those observed in the experimental sound data but predicts amplitudes that are about 10 dB lower. Most of the resonant frequencies match for the two curves. However at 500, 720 & 1100 Hz the experimental sound data shows resonances while the predicted SPL from the vibration data shows anti-resonances. This is an indication that the accelerometer location may be behaving as a node at those frequencies. The average slope of the rise in amplitudes is about the same for the three curves at about 45 dB/decade. The analytical model using the simple housing mobility transfer function shows amplitudes that are quite similar to those found experimentally. It is obvious that the experimental data has more resonant activity that is not captured by this simple model.

![Fig. 18: Radiation efficiency of the housing panel.](image1)

![Fig. 19: Comparison between the experimentally obtained and predicted sound pressure levels.](image2)
SUMMARY AND FUTURE NEEDS

This paper has presented the application of order tracking analysis to study the noise and vibration response of geared systems. Analysis of the test signal showed the ability of the signal processing methodology to create a fairly accurate representation of the system although the reliability of the results may be reduced in the presence of significant background noise. Also, for the example studied, there is a reasonably good correlation between predicted sound pressure levels and experimentally obtained values despite the simple models used.

The following have been identified as future needs,

1. Further development of the signal processing techniques, first to identify when the harmonic data integrity is bad and then to come up with enhancement procedures that allow one to bring the harmonic data out of the noise floor.

2. Development of better models of the gearbox casing transfer function as well as development of improved models of the bearing and shaft dynamic compliances.

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REFERENCES