Wavelet-Based Test-Simulation Correlation Analysis for the Validation of Biodynamical Modeling

Zhiqing Cheng
Advanced Information Engineering Services, A General Dynamics Company
5200 Springfield Pike Suite 200, Dayton, OH 45431-1289
Joseph A. Pellettiere and Nathan L. Wright
Human Effectiveness Directorate, Air Force Research Laboratory
2800 Q Street, Wright-Patterson AFB, OH 45433-7947

NOMENCLATURES

\[ A_j \] Approximation at level \( j \)
\[ c_{jk} \] Scaling function coefficients
\[ d_{jk} \] Wavelet function coefficients
\[ D_j \] Detail at level \( j \)
\( j \) Level index
\( k \) Time shift
\( R_j \) Validation metric
\( R_c ) ( \) Threshold value for \( R_j \)
\( R_{xx}(\tau) \) Autocorrelation function
\( R_{xy}(\tau) \) Cross-correlation function
\( S, S(t) \) Biodynamic response or signal
\( T \) Time duration of impact
\( T_x \) Length of a record
\( \alpha_1 \) Weighting factor for correlation coefficient
\( \alpha_2 \) Weighting factor for time shift
\( \alpha_3 \) Weighting factor for amplitudes
\( \Delta \) Sampling interval
\( \mu \) Mean value
\( \rho_{xy}(\tau) \) Correlation coefficient function
\( \tau \) Time delay
\( \phi_j(t) \) Scaling function at level \( j \)
\( \psi_j(t) \) Wavelet function at level \( j \)

ABSTRACT

Computational modeling and simulation has become a widely used tool for the study of biodynamics. Validation is a primary way to assess the accuracy and reliability of computational modeling and simulation. One approach for
the model validation is the test-simulation correlation. In this paper, simulation results and test data are decomposed on a wavelet basis. Instead of comparing two signals in the time domain directly or in the frequency domain via the Fourier transform, the discrepancy or agreement between two signals is viewed in the time-frequency domain. The test-simulation correlation analysis is performed at consecutive decomposition levels so that the details at different scales can be taken into account through the multi-resolution analysis provided by wavelet decompositions. A metric is developed for quantitative evaluation of the test-simulation agreement, which includes the correlation coefficient, timing shift, and amplitude difference. A procedure is devised for automatic test-simulation comparison at multiple levels.

1. INTRODUCTION

Computational modeling and simulation of biodynamics has gained rapid development in the past few decades, especially in recent years. The models used for computational modeling and simulation of biodynamics can be classified as lumped-parameter models, rigid multi-body models, finite element (FE) models, and integrated models [1]. These models provide different levels of abstraction to a system, and thus have different applications. Since only FE models are able to provide the required detail description of the human body that is necessary for stress analysis [2], the finite element method has become a common tool for biodynamical modeling. The FE models with various complexity, scale, and detail have been developed for organs, segments, parts, or the entire human body. Advanced and sophisticated commercial software is available for the FE modeling and simulation.

For a computational model to be practically useful for engineering analysis and design, it is necessary to validate the model, since an un-validated model may produce results containing unknown and unbounded errors. The model validation is a process of determining the degree to which a computer simulation is an accurate representation of the real world, from the perspective of the intended uses of the model [3,4]. The validation is also a process of assessing and improving confidence in the usefulness of a computational model for a particular application. As biodynamical modeling is concerned, the model validation lags far behind the model development. A major reason is that experimental data or test data that can be used for the model validation are very limited or even unavailable, but ineffective execution of the model validation is also an important factor.

Biodynamical modeling is often used to simulate the human responses to dynamic or impact loading. The human responses can be the acceleration or deformation in certain regions of the body, such as the head acceleration and chest deformation. Therefore, the validation of biodynamical modeling is usually performed by comparing the simulation results with the test data for each respective human response. The comparison can be categorized into [5]:

- Point-to-point comparison, which is the comparison between two time history records and includes the comparison of peak value, peak timing, and pulse shape. This comparison is rigorous and comprehensive and is usually required for important responses.
- Peak value comparison, which compares one or few peak values between test data and simulation results. This comparison is coarse and loose in terms of the agreement between two signals, and is usually applied to the responses of less importance.
- Injury criterion comparison, which compares the values of an injury criterion calculated from a simulation signal and a test signal. This comparison can be stricter than the peak-value comparison, but is looser than the point-to-point comparison.
- Corridor comparison, which checks if simulation results fall within a corridor defined by test data. While this comparison also checks the agreement between two signals in terms of pulse shape, duration, and amplitude, it is much looser than the point-to-point comparison.

The point-to-point comparison is frequently used in the model validation. It is rigorous and comprehensive. However, it is not efficient or even not feasible to conduct exact point-to-point comparison, since the data of simulations and tests can be very large in size and may be contaminated with noise. It can also be unnecessary, as only important features that are embedded in the data are of interest. Therefore, efficient approaches for the point-to-point comparison are required.

2. CORRELATION ANALYSIS BASED ON WAVELET DECOMPOSITIONS

For the point-to-point comparison of two signals or time series, if both signals meet certain requirements [6], conventional correlation analysis is an effective tool to determine the overall relationship or similarity between
them. Biodynamic responses including impact responses, however, usually occur in short time durations. They are transient and strongly localized in the time domain. In biodynamic tests, the measurements are often contaminated with noise. Likewise, the simulation results also contain computational noises. As a consequence, conventional correlation analysis may be neither appropriate nor efficient for the analysis of biodynamic responses.

As an illustration, Fig. 1 displays the responses of an automobile frontal impact at two locations of the vehicle [7]. The test data and simulation results using an FE model are plotted in the figure curve-by-curve. It is shown that these responses are transient and contaminated with noises. To determine the relationship between the test data and the simulation results, conventional correlation analysis is directly used to obtain the correlation coefficient for each pair of responses. According to the values of these coefficients, the agreement between the test and the simulation was found to be poor for the responses at the location of the engine top and very poor for that at the location of the right-rear cross member. However, as shown later, sound agreement exists between the test data and the simulation results in a certain frequency range; in other words, the computational model can provide reasonable representation of the actual vehicle at a certain level of abstraction.

As a new tool for signal analysis, wavelets are localized in both frequency and time domain, which match the major characteristics of biodynamic responses. Therefore the wavelet analysis will be used for the correlation analysis of biodynamic responses and in the following study of the test-simulation correlation for the model validation.

As shown in Fig. 2 (a), on a wavelet basis, a signal $S(t)$ can be decomposed as [8-10]

$$S = A_J + \sum_{j<J} D_j,$$  

where

$$A_J(t) = \sum_k c_{jk} \phi_j(t-k),$$  

and

$$D_j(t) = \sum_k d_{jk} \psi_j(t-k).$$  

Here, $A_J$ and $D_j$ are referred to as the approximation and the detail of $S$ at level $j$; $\phi_j(t)$ and $\psi_j(t)$ are the scaling function and the wavelet function at level $j$ for reconstruction; and $c_{jk}$ and $d_{jk}$, given by wavelet
transforms, are scaling function coefficients and wavelet coefficients at level \( j \) and time shift \( k\), respectively. As an example, the decomposition of the acceleration response at the engine top from an impact test, which is displayed in Fig. 1 (a), is decomposed on a wavelet basis (db4) and is shown in Fig. 2 (b).

![Wavelet decomposition](image)

**Figure 2. Wavelet decomposition**

A study was performed by the authors on the use of wavelets for the correlation analysis of impact responses \([11,12]\) and a methodology was developed, which is described as follows. Decompose two signals \( x(t) \) and \( y(t) \) at a certain level \( j \) using wavelets. The approximations \( x_a^j(t) \) and \( y_a^j(t) \), which retain the low frequency components of the two original signals, describe the underlying gross motions of respective responses. They can be treated as deterministic or narrow-band signals. The classic correlation coefficient \([6]\) can be used to describe the linear relationship or similarity between the two approximations. That is,

\[
\rho^j_a(\tau) = \frac{R_{x_a^j,y_a^j}(\tau) - \mu_{x_a^j} \mu_{y_a^j}}{\sqrt{R_{x_a^j,x_a^j}(0) - (\mu_{x_a^j})^2 \left[R_{y_a^j,y_a^j}(0) - (\mu_{y_a^j})^2\right]}},
\]

where \( \mu_{x_a^j} \) and \( \mu_{y_a^j} \) are the mean values, and \( R_{x_a^j,x_a^j} \), \( R_{x_a^j,y_a^j} \), and \( R_{y_a^j,y_a^j} \) are the auto- and cross-correlation functions, which, in general, are given by

\[
R_{xy}(\tau) = \lim_{T_s \to \infty} \frac{1}{T_s} \int_0^{T_s} x(t)y(t+\tau)dt,
\]

where \( T_s \) is the length of record, and \( \tau \) is the time shift, which is allowed to vary in a small range between the two approximations to account for the phase shift between them. A quantity

\[
\rho_{am}^j = \rho_a^j(\tau_m) = \max_{\tau} \{\rho_a^j(\tau)\},
\]

which is the maximum correlation coefficient between the two approximations, can be used to evaluate the discrepancy or agreement between the two approximations, where \( \tau_m \) can be used to measure the time shift between them.

### 3. A METRIC AND A PROCEDURE FOR MODEL VALIDATION
If only correlation coefficient is used to measure the similarity between two signals, the measurement is not complete [13], since the definition of correlation coefficient has shortcomings that lead to the failure of identifying differences in some special cases. These special cases are shown in Fig. 3 (a), (b), and (c), respectively. In Fig. 3 (a), two signals are in the same shape and the same phase but with different amplitudes. The correlation coefficient between these two signals is 1. In this case, the correlation coefficient only measures the similarity in shape and phase of the two signals but fails to reveal the difference in the amplitudes between them. Figure 3 (b) illustrates two signals with different shapes, yet the same amplitude and phase. In this case, the correlation coefficient with the value of 1 only deals with the amplitude and phase but fails to tell the difference in the shapes between the two signals. The third case, as shown in Fig. 3 (c), displays two signals differing only in phase. As phase shift is allowed in the calculation of correlation coefficient, as given by Eq. (6), the value of the coefficient is still 1 and the phase shift is not detected.

Therefore, in order to provide a comprehensive comparison between two signals, as correlation coefficient is used as a primary measure, additional measures are needed to avoid the failures of correlation coefficient in those special cases. A metric was thus developed [14], which is defined as

\[
R^j = \left[ \alpha_1(1 - \rho_{am}^j) + \alpha_2 \frac{T_m}{T} + \alpha_3 \left| \frac{A_m^j - A_t^j}{A_t^j} \right| \right] \times 100\%, \tag{7}
\]

where \( \alpha_1, \alpha_2, \) and \( \alpha_3 \) are weighting coefficients, \( 0 \leq \alpha_1, \alpha_2, \alpha_3 \leq 1, \) and \( \alpha_1 + \alpha_2 + \alpha_3 = 1; \) \( \rho_{am}^j \) is given by Eqs. (4) and (6), \( T \) is the pulse duration; and \( A \) is the amplitude of the approximation with subscripts \( t \) for the test signal and \( s \) for the simulation signal. By definition of Eq. (7), \( R^j \geq 0. \) The smaller the \( R^j, \) the better the agreement between the test data and simulation results in terms of the pulse shape, timing shift, and amplitude.

Dyadic wavelet decomposition provides a multi-resolution analysis. If the resolution for the original signal (which is at level 0) is \( \frac{1}{\Delta} \), the resolution at level \( j \) will be \( \frac{1}{(2^j \Delta)} \) [9]. The resolution decreases as the level increases. The larger the resolution, the smaller and finer the details that can be revealed. Conversely, as the decomposition level \( j \) increases, the content of the original signal retained by the approximation at level \( j \) becomes less and coarser. In terms of frequency band, the decomposition of a signal using dyadic orthogonal wavelets is a quadratic sub-band filtering [8]. An ideal filter bank cuts the frequency band in half, but an actual filter bank has a transition band [8]. However, it is true that the approximation and details are narrow-banded sub-signals of the original signal. Therefore, the metric given by Eq. (7) can be used to evaluate the agreement between a test signal and a simulation signal at different scales and different frequency bands, and thus can be used to determine the extent to which a model is valid, or the level at which a model can best represent the actual system.

When a biodynamic signal is decomposed on a wavelet basis, it is important to choose an appropriate type of wavelet and the right level of decomposition. Unlike the Fourier analysis where only sinusoidal waveform is used, the wavelet analysis can be performed with various waveforms (wavelet functions). The selection of an appropriate wavelet function is important and may affect the effectiveness of the methodology. A general rule is not available, but several factors may be involved in the selection and need to be considered [9]; those factors include orthogonality, support width, fast algorithm, etc. The maximum decomposition level is determined such
that, at that level, the approximation of an acceleration response basically represents the gross motion of the corresponding point in the human body or of the corresponding structural component during impact. The three weighting factors provide different emphasis on the correlation coefficient, time shift, and amplitude, respectively. The selection of appropriate values for them depends upon particular situations.

A procedure devised for automatic comparison at multiple levels is shown below.

![Flowchart](image)

**Figure 4.** The procedure for automatic multi-level comparison

### 4. ILLUSTRATION OF APPLICATION

To illustrate the application of the validation method and metric developed in this paper, a 1997 Honda Accord FE crash model [7] will be validated in a case of full frontal impact. The responses from the simulation of a full frontal impact with this model will be compared with the test data. The corresponding actual crash test is a full frontal impact test of a two-door 1997 Honda Accord, which is recorded as Number 2475 in the vehicle crash test database of the National Highway Traffic Safety Administration and will be referred to as test 2475 in the following analysis. Specifically, the crash signals from the test and the simulation at the locations of the engine top and the right-rear cross member are selected to be compared. The time histories of the acceleration responses at these
locations are displayed in Fig. 1, where the number of data points is 1,000, the time duration is 100 ms, and the amplitude is in g that is equal to 9.8 m/s².

The fourth order of Daubechies wavelet, db4 [9,10], was used since it is orthogonal and compactly supported with the length appropriate for the crash signals being analyzed. The maximum decomposition level is determined such that, at that level, the approximation of these acceleration responses basically represents the gross motions of the corresponding structural components during impact. In a conventional analysis, automobile structural acceleration responses are filtered with the CFC-60 filter [15]. This filter is a low-pass filter with the high-pass frequency of 60 Hz, and the cut-off frequency of 100 Hz at which the filter gain equals 70 percent (-3db) [16]. As the original signals are sampled with the sampling rate of 0.1 ms, the time scale of level 6 is 6.4 ms, approximately corresponding to the frequency range from 0 to 78.1 Hz. The approximation at level 6 basically matches the signal filtered with the CFC-60 filter in terms of frequency range. Therefore, the maximum level of 6 is chosen for the decomposition of the signals.

The crash responses are decomposed at six levels on the db4 wavelet basis

$$S = A_6 + D_6 + D_5 + D_4 + D_3 + D_2 + D_1,$$

(8)

The approximations at each level are displayed in Figs. 5 and 6 for the locations of engine top and right-rear cross member, respectively, where the signals from the test are displayed on the left (Fig. 5 (a) and Fig. 6 (a)) and the signals from the simulation are on the right (Fig. 5 (b) and Fig. 6 (b)). The correlation coefficients and peak-timing shifts between the two original signals from the test and the simulation and between the approximations of them at each level are determined using Eqs. (4)—(6), with values given in Table 1. The amplitudes or the peak values of the pulses shown in Figs. 5 and 6 are also given in Table 2. Based on these values, the validation metric defined by Eq. (7) is calculated, with the values given in Table 2.

Figure 5. Original signal and approximations at each level for engine top
Figure 6. Original signal and approximations at each level for right-rear cross member

Table 1 shows that as the decomposition level increases, the correlation coefficient between the two approximations increases; the peak-timing shift has no or very small changes; and the difference between the two amplitudes decreases from the beginning and may increase toward high decomposition levels. Table 2 illustrates that the value of the validation metric depends upon the selection of weighting factors. For the accelerations at the engine top, the decomposition at level 5 allows the simulation to provide the best representation of the test, whereas level 6 is the best level at which the simulation is closest to the test for the acceleration response at the right-rear cross member.

Table 1. Correlation coefficients, peak-time shift, and amplitudes

<table>
<thead>
<tr>
<th></th>
<th>Engine top</th>
<th>Right-rear cross member</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{mn}$</td>
<td>$S(t)$</td>
<td>$A_1$</td>
</tr>
<tr>
<td>$\tau_{mn}$</td>
<td>0.65</td>
<td>0.66</td>
</tr>
<tr>
<td>$\tau_{mn}$</td>
<td>432.1</td>
<td>412.9</td>
</tr>
<tr>
<td>$A_i$</td>
<td>220.9</td>
<td>215.7</td>
</tr>
</tbody>
</table>

Table 2. Metric values based on correlation analysis and peak-value comparison

<table>
<thead>
<tr>
<th></th>
<th>Engine top</th>
<th>Right-rear cross member</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1 = 0.4, \alpha_2 = 0.2, \alpha_3 = 0.4$</td>
<td>$S(t)$</td>
<td>$A_1$</td>
</tr>
<tr>
<td>33.5</td>
<td>32.7</td>
<td>29.2</td>
</tr>
<tr>
<td>$\alpha_1 = 0.5, \alpha_2 = 0.2, \alpha_3 = 0.3$</td>
<td>32.0</td>
<td>31.3</td>
</tr>
<tr>
<td>$\alpha_1 = 0.3, \alpha_2 = 0.2, \alpha_3 = 0.5$</td>
<td>34.9</td>
<td>34.1</td>
</tr>
</tbody>
</table>
5. CONCLUDING REMARKS

Wavelet analysis can be used in the validation of biodynamical modeling. Signals from tests and simulations can be viewed in the time-frequency domain using wavelet decompositions. Based on wavelet decompositions, correlation analysis can be performed between the approximations of the signals from a test and a simulation. A comprehensive evaluation of the agreement between a test and a simulation can be done with a metric that consists of correlation coefficient, timing shift, and amplitude difference. With multi-resolution analysis provided by wavelet analysis, the comparison between a test and a simulation can be conducted at consecutive levels so that details at different scales can be considered. A procedure can be implemented to determine the extent to which a model is valid and the level at which a model can best represent the actual system.

REFERENCES