STOCHASTIC RESPONSE SURFACE METHOD FOR ROBUST DESIGN

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ABSTRACT:

In this paper, one can propose a method which takes into account the propagation of uncertainties in the finite element models in a multiobjective optimization procedure. This method is based on the coupling of stochastic response surface method (SRSM) and a genetic algorithm of NSGA type. The SRSM is based on the use of stochastic finite element method (SFEM) via the use of the expansion of the polynomial chaos (PC). Thus, one can avoid the use of Monte Carlo (MC) simulation whose costs become prohibitive in the optimization problems, especially when the finite element models are large and with a considerable number of design parameters.

The objective of this study is on one hand to quantify efficacious the effects of these uncertainties on the responses variability which one wishes to optimize and on the other hand, to calculate solutions which are both optimal and robust with respect to the uncertainties of design parameters.

The interest of the proposed robust design methodology and its performances are highlighted on mechanical examples resulting from the numerical simulation.

Key words: Uncertainties, Polynomial Chaos, Multiobjective Optimization, Responses Surface Methodology, Robustness.

1. INTRODUCTION

During simulation, the objectives of the propagation and the quantification of the uncertainties impact are, in one hand, to provide numerical error bars which facilitate the comparison with experimental observations and thus to better evaluate the quality of the used physical models; and to identify the uncertain parameters having the most greater impact on the simulation so that to be measured or checked with more precision. In the other hand, to lead a safety analysis (probability of exceeding critical values) and to measure the degree of confidence that can be accorded to calculations, at the time of the decision-making in design for example. Many methods were proposed in that effect and which are generally classified in three categories: the MC simulation method which is often considered as the reference method [1], the analytical methods (perturbation method) which are based on an Taylor expansion series of the responses around the means of the random variables [2] and the spectral methods that exploits the basic functions of the Hilbert space associated with the random problems. These functions can be orthogonal polynomials in general, and particularly a polynomial chaos [3]. The MC reference method presents a major disadvantage which is the large number of simulations required for reaching at a degree of acceptable confidence of the desired output.

In order to remedy to this major disadvantage, one can propose in this paper the use of the stochastic response surface method (SRSM) [4] which allows to replace the complex finite element model (FEM) by a polynomial simpler to manage. This method allows on the one hand, reducing the number of simulations required for the adequate evaluation of uncertainties, to analyze systematic uncertainties and to provide the degrees of confidence of the estimated model. On the other hand, the coupling of SRSM with the genetic algorithms of non-dominated sorting genetic algorithm type (NSGA) [5] in a multi-objectives optimization
procedure has the advantage to compute quickly optimal solutions in pre-project or project phases of design in mechanical engineering. This advantage is not evident when direct optimization is made using only GAs. In fact, the models complexity and that of associated modelization (finite elements models with a large size, non-linear model behaviour...) makes the procedure of direct optimization long and sometimes not possible due to the cost calculation reasons and the realistic times of design.

The taking into account of uncertainties in the multi-objectives optimization procedures requires the introduction of robustness indicators of the solutions with respect to these uncertainties in order to find the most stable solutions. Classically, the robustness optimal solution is evaluated at the end of the deterministic optimization procedure. This idea supposes that the deterministic design space contains the robust solutions which one must then select while introducing of the stochastic criteria.

Into this paper, we can introduce a different robustness criterion [6]. Indeed, the solution at the same time optimal and robust is not inevitably a solution of deterministic space. For that, the robustness is introduced into the optimization process like an additional function objective to maximize. Consequently, the deterministic problem of optimization is enriched by additional functions of robustness in order to guide calculation towards the areas of the solutions space which are at the same time optimal and robust.

The interest of the SRSM and its performances are highlighted on two mechanical examples resulting from the numerical simulation. The first example is with two costs functions and six uncertain design parameters and the second is with three costs functions and five uncertain design parameters.

2. STOCHASTIC RESPONSE SURFACE METHOD

2.1 UNCERTAINTY ANALYSIS

The analysis of probabilistic uncertainties of MC type, perturbation or SFEM methods, are more employed for characterizing the uncertainties in the physical systems, particularly when the evaluations of the probability distributions of the uncertain parameters are available. These approaches can describe the uncertainty resulting from the stochastic perturbations, the variability conditions and the risky considerations. In these approaches, the uncertainties associated to the inputs of the model are described by probability distributions and the objective is to estimate the outputs probability distributions.

Figure (1) depicts schematically the concept of uncertainty propagation: each point of the response surface (i.e., each calculated output value) of the model to changes in inputs “1” and “2” will be characterized by a probability density function (pdf) that will depend on the pdfs of the inputs.

![Figure 1: A schematic depiction of the propagation of uncertainties](image)

2.2 PRINCIPLE OF THE METHOD

The Stochastic Response Surface Method (SRSM) [4], as the name suggests, can be viewed as an extension to the classical deterministic Response Surface Method (RSM) [9]. The main difference between the SRSM and the RSM is that in the former the inputs are random variables, where as in the latter, the inputs are
deterministic variables. The motivation underlying the use of SRSM is to reduce the number of model simulations required for adequate estimation of uncertainty, as compared to conventional methods. This is accomplished by approximating both inputs and outputs of the uncertain system through series expansions of standard random variables; the series expansions of the outputs contain unknown coefficients that can be calculated from the results of a limited number of model simulations.

Evaluating an SRSM expansion consists of the following steps (Figure 2):

1- Input uncertainties are expressed in terms of a set of standard random variables srvs;
2- A functional form is assumed for selected outputs;
3- The parameters of the functional approximation are determined.

The srvs are selected from a set of independent, identically distributed (iid) normal random variables, \( \{ \xi_i \}_{i=1}^n \), where \( n \) is the number of independent inputs, and each \( \xi_i \) has zero mean and unit variance. When the input random variables are independent, the uncertainty in the \( i \)th model input \( X_i \), is expressed directly as a function of the \( i \)th srv, \( \xi_i \); that is, a transformation of \( X_i \) to \( \xi_i \) is employed. Such transformations are useful in the standardized representation of the random inputs, each of which could have very different distribution properties.

The steps involved in the application of the SRSM are presented in Figure 2.

![Figure 2: Schematic depiction of the steps involved in the application of the Stochastic Response Surface Method](image)

Consider a model \( Y = F(X) \), where the set of random inputs is represented by the vector \( X \), and a set of selected random outputs or output metrics is represented by the vector \( Y \). First, the vector of input random variables is expressed as a function of the form \( X = h(\xi) \), where \( \xi \) denotes the vector of the selected srvs.

Then, a functional representation for outputs, of the form \( Y = f(\xi, a) \), where \( a \) denotes a parameter vector, is assumed.

### 2.3 REPRESENTATION OF STOCHASTIC INPUTS-OUTPUTS

The first step in the application of the SRSM is the representation of all the model inputs in terms of a set of “standardized random variables”. Here, the srvs are selected from a set of independent, identically distributed (iid) normal random variables, \( \{ \xi_i \}_{i=1}^n \) where \( n \) is the number of independent inputs, and each \( \xi_i \) has zero mean and unit variance. When the input random variables are independent, the uncertainty in the \( i \)th model input \( X_i \), is expressed directly as a function of the \( i \)th srv, \( \xi_i \), i.e., a transformation of \( X_i \) to \( \xi_i \) is employed. Such transformations are useful in the standardized representation of the random inputs, each of which could have very different distribution properties [4].
An output of a model may be influenced by any number of model inputs. Hence, any general functional representation of uncertainty in model outputs, should take into account uncertainties in all inputs. For a deterministic model with random inputs, if the inputs are represented in terms of the set \[ \{ \xi_i \}_{i=1}^n \], the output metrics can also be represented in terms of the same set, as the uncertainty in the outputs is solely due to the uncertainty of the inputs [4]. This work addresses one specific form of representation, the series expansion of normal random variables, in terms of Hermite polynomials; the expansion is called “polynomial chaos expansion” [3]. When normal random variables are used as srvs, an output can be approximated by a polynomial chaos expansion on the set \[ \{ \xi_i \}_{i=1}^n \], given by:

\[
\hat{y} = a_0 + \sum_{i=1}^{n} a_i \Gamma_i (\xi_i) + \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} a_{ij} \Gamma_2 (\xi_i, \xi_j) + \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} a_{ijk} a_{ijk} \Gamma_3 (\xi_i, \xi_j, \xi_k) + \ldots
\]

(1)

Where, \( \hat{y} \) is any output metric (or random output) of the model, the \( a_i \)’s are deterministic constants to be estimated, and the \( \Gamma_i \)’s are multi-dimensional Hermite polynomials of degree \( p \):

\[
\Gamma_p (\xi_i, ..., \xi_p) = (-1)^p \frac{e^{\xi_i^T/2} \xi_i^T \Gamma_p e^{\xi_i^T/2} \xi_i}{\xi_i^T e^{\xi_i^T/2} \xi_i e^{\xi_i^T/2} \xi_i^T} \]

(2)

Where \( \xi \) is the vector of \( n \) iid normal random variables \( \{ \xi_i \}_{i=1}^n \) that are used to represent input uncertainty. Hermite polynomials on \( \{ \xi_i \}_{i=1}^n \) are random variables, since they are functions of the random variables \( \{ \xi_i \}_{i=1}^n \).

Furthermore, the Hermite polynomials defined on \( \{ \xi_i \}_{i=1}^n \) are orthogonal with respect to an inner product defined as the expectation of the product of two random variables [4]. Thus,

\[
E[\Gamma_p \Gamma_q] = 0 \text{ iff } \Gamma_p \neq \Gamma_q
\]

(3)

### 2.4 RESPONSE OF A DYNAMIC MODEL

The stochastic equilibrium equation of the structure subjected to a deterministic harmonic excitation is written:

\[
( -\omega^2 M(\theta) + (1 + j\eta) K(\theta) ) y(\omega, \theta) = f_e(\omega)
\]

(5)

Where: \( M(\theta) \approx M_0 + \sum_{i=1}^{n} M_i \xi_i(\theta) \), \( K(\theta) \approx K_0 + \sum_{i=1}^{n} K_i \xi_i(\theta) \), \( y(\omega, \theta) \) and \( f_e(\omega) \) represent respectively the mass matrix, stiffness and the exact response stochastic vector of the structure as well as the applied forces vector.

By introducing the relation (3) into the equation (5) of the model and by projecting this equation on polynomials \( \Gamma_m \) (\( m = 0, ..., N \)), one obtains the following linear system:

\[
\sum_{a=0}^{g} (-\omega^2 M_0 + (1 + j\eta) K_0) a_a \langle \Gamma_n \Gamma_m \rangle + \sum_{a=0}^{g} \sum_{i=1}^{n} (-\omega^2 M_i + (1 + j\eta) K_i) a_a \langle \xi_i \Gamma_n \Gamma_m \rangle = f_e \langle \Gamma_m \rangle
\]

(6)

The equation (6) is considered for \( m = 0, ..., N \), and leads to a system of \( (P + 1) \) linear matrix equations, the solution of which corresponds to vectors \( a_a \).

\[
(D + A)a_a = b
\]

(7)

Where \( D \) is a diagonal matrix by blocks and \( A \) is a hollow matrix, such that:

\[
D_{ii} = (-\omega^2 M_0 + (1 + j\eta) K_0) \langle \Gamma_i \rangle^2 ; \quad A_{ij} = \sum_{\epsilon=1}^{n} (-\omega^2 M_0 + (1 + j\eta) K_0) \langle \xi_{\epsilon} \Gamma_i \Gamma_j \rangle
\]

### 3. ROBUST MULTIOBJECTIVE OPTIMIZATION

In mechanical engineering, the optimization problems are often multi-objectives. The costs functions are complex (multimodal, non convex...), and generally present conflicts between them. For this, it is necessary to choose a multi-objectives optimization strategy able to propose the best alternatives among several. Two steps are necessary to make multi-objectives optimization:
• **Calculation of the objectives functions to be optimized:**

The calculation and the evaluation of these costs functions can be done in two manners, either directly during the optimization on the exact model, or in the forms of polynomial interpolation by using the SRSM method. In this latter case, the passage towards physical space (\(X_i\)) for a normal law for example is:

\[X_i = \mu_{X_i} + \sigma_{X_i} \xi\]

(8)

Where \(\mu_{X_i}\) and \(\sigma_{X_i}\) are respectively the mean and the standard deviation of \(X_i\).

• **Choice of the research method of optimal solutions:**

In this work, we exploit the genetic algorithms NSGA type [5], which enable to explore more the design space and to cover the whole front of optimal Pareto.

Generally, a multi-objective optimization problem is expressed by the equation (9):

\[\min F(x) = (f_1(x), f_2(x), \ldots, f_k(x))^T \quad ; \text{s.t.} \quad x \in S\]

(9)

Where \(f_1(x), f_2(x), \ldots, f_k(x)\) are cost functions, \(x = (x_1, x_2, \ldots, x_n)^T\) is the vector of \(n\) optimization parameters, \(S \in \mathbb{R}^n\) represent the whole realizable solutions and \(F(x)\) is the vector of the functions to be optimized. The whole Pareto optimal solutions, is that formed by the solutions which are not dominated by others. If we consider a minimization problem where \(y, z \in S\) are two vectors of \(n\) optimization parameters, \(y\) dominates \(z\) noted \(y > z\) if and only if \(\forall \ i \in [1, \ldots, n] : f_i(y) \leq f_i(z) \) and \(\exists \ j \in [1, \ldots, n] : f_j(y) < f_j(z)\).

The physical parameters used to describe a structure are often uncertain. These uncertain parameters are generally identified as random variables and are introduced in the resolution approach of the problem such as optimization.

In the optimization field, taking into account of the solutions robustness is essential in the search of an optimal design, because it is well-known that a deterministic optimal solution can prove to be in adapted in practice and the committed mistakes during the manufacturing for example do not allow to obtain the values of the design variables with a sufficient precision. A suboptimal solution but stable with towards the manufacturing tolerances will be much more interesting to the subject concerned with the designer.

Generally, the realization of a robustness function is based on the mean and the standard deviation. Ait Brik and al. [6] have suggested writing the robustness function \(f^*\) of an objective function \(f\) by the ratio between the mean and the standard deviation:

\[f^* = \frac{\mu_f}{\sigma_f}^{-1}\]

(10)

The ratio \(\sigma_f/\mu_f\) indicates the dispersion (or the vulnerability function) de \(f\). Where \(\mu_f\) and \(\sigma_f\) are respectively the mean and the standard deviation of the whole samples \(f(x)\) function. \(N\) represents the number of simulation of Monte Carlo.

The robust multi-objective optimization problem (RMOP) is built by optimizing simultaneously the initial cost functions and their robustness: If the initial multi-objective optimization problem (OMP) is defined under the form:

\[\min_{x} F(x) = (f_1(x), f_2(x), \ldots, f_m(x))\quad \text{avec} \quad x \in C\]

(11)

The robust multi-objective optimization problem (RMOP) is then raised in the following way:

\[\min_{x} F^*(x) = (f_1(x), f_1^*(x), f_2(x), f_2^*(x), \ldots, f_m(x), f_m^*(x))\quad \text{avec} \quad x \in C^*\]

(12)

With \(f_i^*(x)\) is the robustness of the objective function \(f_i(x)\) and \(1 \leq i \leq m\).
The robust solutions with respect to uncertainties are those which make it possible to minimize simultaneously the original cost functions \( (f_1(x), f_2(x), ..., f_m(x)) \) and to maximize their robustness \( (f'_1(x), f'_2(x), ..., f'_m(x)) \).

In this paper, the calculation of the various robustness functions is made in two manners, that is to say directly by MC or SRSM, the determination of the mean and the standard deviation of each cost function.

4. NUMERICAL SIMULATION

To validate the method, we propose the study of bi-dimensional structures modelled with a beams elements and 288 degrees of freedom (dof) (figure 3), the other is spatial structure representing “the helicopter landing gear” with 2500 DOF.

4.1 COUPLED BEAMS

The considered structure (Figure 3) is a system of three beams connected between them by two linear springs \( K_1 \) and \( K_2 \) to the points F, G and H. The structure is embedded with the frame, directly at the points A, B and C and by two linear springs \( K_3 \) and \( K_4 \) at the points D and E. The total structure (beams + springs) is represented by six random design parameters which are defined in table 1. The uncertainties levels on these parameters are of weak, average and strong type.

![Excitation and observation point](image)

**Table 1:** Uncertain Parametric Modifications.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Probability density function</th>
<th>Uncertainty level</th>
<th>Nominal values</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E )</td>
<td>Normal</td>
<td>2%, 5% et 10%</td>
<td>2.1 \times 10^{11}</td>
<td>Young’s Modulus of the structure [N/m²]</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Normal</td>
<td>2%, 5% et 10%</td>
<td>7800</td>
<td>Density of the structure [Kg/m³]</td>
</tr>
<tr>
<td>( K_1 )</td>
<td>Normal</td>
<td>2%, 5% et 10%</td>
<td>10^3</td>
<td>Stiffness of the spring 1 [Nm]</td>
</tr>
<tr>
<td>( K_2 )</td>
<td>Normal</td>
<td>2%, 5% et 10%</td>
<td>10^2</td>
<td>Stiffness of the spring 2 [Nm]</td>
</tr>
<tr>
<td>( K_3 )</td>
<td>Normal</td>
<td>2%, 5% et 10%</td>
<td>10^3</td>
<td>Stiffness of the spring 3 [Nm]</td>
</tr>
<tr>
<td>( K_4 )</td>
<td>Normal</td>
<td>2%, 5% et 10%</td>
<td>10^3</td>
<td>Stiffness of the spring 4 [Nm]</td>
</tr>
</tbody>
</table>
Through this example firstly, the impact of uncertainties of the design parameters on the desired excitation (displacement at the point D) has been studied. In the second time, the whole of the robust optimal solutions by the consideration of the following PMOR have been determined:

\[
\min_x F^*(x) = \left[ f_1(x), f_1^*(x), f_2(x), f_2^*(x) \right]
\]

\[
\text{avec } x \in [E, \rho, K_1, K_2, K_3, K_4, K_5]
\]

Where: \(f_1\) and \(f_2\) represent respectively the total mass and the displacement of the structure at the point D to be minimized; \(f_1^*\) and \(f_2^*\) represent respectively their functions of vulnerability to be minimized.

The figures (4: a - b - c) respectively illustrate the variability of displacement at the point D according to the various uncertainties levels 2%, 5% and 10% on the parameters of design.

**Figure 4:** The pdf and the cdf of the displacement, Uncertainty level (a) : 2% - (b) : 5% -(c) : 10%
It is noted that the cumulative density function and the probability density of displacement of the structure are well represented by second order of the SRSM method compared to MC reference. Moreover, the displacement follows a normal law what proves the dependence with the inputs laws. On the other hand, the response resulting from the application of method RSM-MC (method based on classical response surfaces RSM) coincide not well with the response of reference resulting from direct MC simulation. This is mainly with the quality of the prediction of the RSM method since the determination coefficient $R^2 = 0.987$.

To highlight the interest of this method, one can compare the CPU time with those of the MC and the RSM-MC methods. The examination of the table (2) shows and justifies the interest of the use of the SRSM.

**Table 2 :** CPU comparison - SRSM, MC and RSM-MC

<table>
<thead>
<tr>
<th></th>
<th>MC 10000 samples</th>
<th>SRSM ordre 2 10000 samples</th>
<th>RSM-MC 10000 samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPU Time (min)</td>
<td>42</td>
<td>13</td>
<td>25</td>
</tr>
</tbody>
</table>

Robust multi-objectives optimization is carried out in two manners: either starting from the exact responses resulting from the MEF and the use of AG, or starting from the responses resulting from metamodel (SRSM) and the use of AG for the determination of the optimal robust Pareto solutions. It is noted that the results of multiobjective optimization by SRSM are convincing. By way of example, the table (3) illustrates 3 types of robust solutions obtained with the corresponding parameters of design.

The table (4) of metric shows that the two frontiers obtained are close one to the other (Mahalanobis distance [10]: $D^2 = 0.1514$). Moreover, the distribution of the points of the frontier (spacing metric [11]) resulting from the “SRSM_AG” is definitely better than that of the points of the frontier resulting from the use of AG alone.

![Figure 5](image)

**Figure 5 :** the first Pareto front of robust solutions – Comparison between the two optimisation approaches – Uncertainty 5%

**Table 3 :** Some robust optimal solutions – Uncertainty 5%

<table>
<thead>
<tr>
<th>Design parameters</th>
<th>Cost functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>$M_0$ (kg)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$D$ (m)</td>
</tr>
<tr>
<td>$K_1$</td>
<td>Nominal values</td>
</tr>
<tr>
<td>$K_2$</td>
<td>$2.1 \times 10^{11}$</td>
</tr>
<tr>
<td>$K_3$</td>
<td>7800</td>
</tr>
<tr>
<td>$K_4$</td>
<td>$10^5$</td>
</tr>
<tr>
<td>$M_0$</td>
<td>$10^8$</td>
</tr>
<tr>
<td>$K_4$</td>
<td>$10^8$</td>
</tr>
<tr>
<td>Nominal values</td>
<td>1560</td>
</tr>
<tr>
<td>Optimal design</td>
<td>2.37e-6</td>
</tr>
<tr>
<td>Optimal solution</td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
<td>1482.18</td>
</tr>
<tr>
<td>2.0035e+11</td>
<td>1.442e-6</td>
</tr>
<tr>
<td>9.26e+7</td>
<td></td>
</tr>
<tr>
<td>9.5e+6</td>
<td></td>
</tr>
<tr>
<td>1.0229e+6</td>
<td></td>
</tr>
<tr>
<td>9.59e+7</td>
<td></td>
</tr>
<tr>
<td>7417</td>
<td></td>
</tr>
<tr>
<td>Case 2</td>
<td>1482.42</td>
</tr>
<tr>
<td>2.0048e+11</td>
<td>1.434e-6</td>
</tr>
<tr>
<td>9.12e+7</td>
<td></td>
</tr>
<tr>
<td>9.52e+6</td>
<td></td>
</tr>
<tr>
<td>1.05e+6</td>
<td></td>
</tr>
<tr>
<td>9.54e+7</td>
<td></td>
</tr>
<tr>
<td>7410</td>
<td></td>
</tr>
<tr>
<td>Case 3</td>
<td>1482.7</td>
</tr>
<tr>
<td>2.0031e+11</td>
<td>1.431e-6</td>
</tr>
<tr>
<td>9.3e+7</td>
<td></td>
</tr>
<tr>
<td>9.53e+6</td>
<td></td>
</tr>
<tr>
<td>1.0104e+6</td>
<td></td>
</tr>
<tr>
<td>9.62e+7</td>
<td></td>
</tr>
<tr>
<td>7420</td>
<td></td>
</tr>
</tbody>
</table>
Table 4: Results of different metrics – Uncertainty 5%

<table>
<thead>
<tr>
<th>Relative metric</th>
<th>Mahalanobis distance</th>
<th>Directe optimization</th>
<th>Optimization with SRSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Directe optimization</td>
<td>( D^2 = 0.1514 )</td>
<td>0.3030</td>
<td>0.1114</td>
</tr>
</tbody>
</table>

Absolute metric | Spacing metric | 0.3030 | 0.1114 |

4.2 THE HELICOPTER LANDING GEAR

The structure presented in figure (6) is a simplified helicopter landing gear model. Its total architecture is as follows:

- The landing gear consists of two skates of circular section (1 and 2) connected to each other by two rollbars (3 and 4), these four elements are "deformable",
- Four rigid beams (5, 6, 7 and 8) connect this structure at gravity center of the model,
- A mass is localized in the helicopter gravity center.

![Figure 6: Global architecture of the simplified helicopter landing gear](image)

(a) Decomposition of the structure by elements type
(b) helicopter landing gear description

The connections between rollbar and skate are of the claming type, whereas those binding the rigid structure to the rollbar are of ball socket type. Lastly, one can model the connection ground/skates by four springs of identical stiffness. This structure has five random variables (Table 5).

Table 5: Uncertain parametric modifications

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Probability density function</th>
<th>Uncertainty level</th>
<th>Nominal values</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>Normal</td>
<td>5%</td>
<td>( 2.1 \times 10^{11} )</td>
<td>Young's Modulus [N/m²]</td>
</tr>
<tr>
<td>p</td>
<td>Normal</td>
<td>5%</td>
<td>7800</td>
<td>Density [Kg/m³]</td>
</tr>
<tr>
<td>K</td>
<td>Normal</td>
<td>5%</td>
<td>5 \times 10^4</td>
<td>Stiffness [Nm]</td>
</tr>
<tr>
<td>M</td>
<td>Normal</td>
<td>5%</td>
<td>1141.1</td>
<td>Concentrated mass in the gravity center [Kg]</td>
</tr>
<tr>
<td>D</td>
<td>Normal</td>
<td>5%</td>
<td>0.0512</td>
<td>External diameter of skate section [m]</td>
</tr>
</tbody>
</table>

When the helicopter is at the ground, undesirable coupling can occur between the rotors rotational frequency and the landing gear eigenfrequency. For example, if the rotor excites the landing gear according to the clean pendulation mode (rigid body mode) or according to the spacing mode of the skate, (elastic mode), one can imagine the nuisances that can be caused; complete swing of the helicopter in the first case, oscillations maintained vertical components in the second case.

In this example, the study is limited to the impact of 5% uncertainties of the design parameters on the first eigenfrequency of the landing gear. Then, the study relates to the landing gear vibratory behavior optimization by holding account of uncertainties on the design parameters and having like cost functions:

- The minimization of the first eigenfrequency;
- The maximization of the frequential band \( (f_2 - f_1) \);
- The minimization of the global mass of the landing gear.
The robust multi-objectives problem to solve is then:

\[
\min_x F^* (x) = \left( f_1 (x), f_2 (x), f_3 (x), f_4 (x), f_5 (x), f_6 (x) \right)
\]

\[\text{avec } x \in [E, \rho, K, M_c, D]\]

(18)

Figure (7) illustrates the variability of the first eigenfrequency for a 5% uncertainty level. This figure shows that SRSM method coincide well with MC reference. Moreover, its prediction level is definitely better than the RSM-MC. The first eigenfrequency prediction by the RSM is carried out with a determination coefficient \(R^2 = 0.91\).

The table (6) which presents the CPU times of the various methods, shows and justifies the interest of the SRSM.

**Table 6 : CPU comparison- SRSM, MC and RSM-MC**

<table>
<thead>
<tr>
<th>CPU Time (min)</th>
<th>MC</th>
<th>SRSM ordre 2</th>
<th>RSM-MC</th>
</tr>
</thead>
<tbody>
<tr>
<td>10000 samples</td>
<td>3750</td>
<td>15</td>
<td>425</td>
</tr>
</tbody>
</table>

Figure (8) illustrates the robust solutions Pareto frontiers obtained by AG and SRSM-AG methods. The results of the robust multi-objectives optimization by SRSM method are satisfactory.
The metric ones of the table (7) shows that the two frontiers obtained are close one to the other (\(D^2 = 0.251\)). Moreover, the distribution of the frontier points (spacing metric) resulting from the "SRSM_AG" is slightly better than that of the frontier points resulting from the use of AG alone.

<table>
<thead>
<tr>
<th>Table 7 : Results of different metrics – Uncertainty 5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative metric</td>
</tr>
<tr>
<td>Mahalanobis distance</td>
</tr>
<tr>
<td>Directe optimization</td>
</tr>
<tr>
<td>(D^2 = 0.251)</td>
</tr>
<tr>
<td>Optimization with SRSM</td>
</tr>
<tr>
<td>Absolute metric</td>
</tr>
<tr>
<td>Spacing metric</td>
</tr>
<tr>
<td>Directe optimization</td>
</tr>
<tr>
<td>(0.393)</td>
</tr>
<tr>
<td>Optimization with SRSM</td>
</tr>
<tr>
<td>(0.3162)</td>
</tr>
</tbody>
</table>

5 CONCLUSIONS

A method based on stochastic response surfaces is proposed in this paper for the study of the propagation of uncertainties in mechanical engineering. It allows representing the stochastic responses of the complex physical systems by polynomials more easy to be evaluated in nonlinear problems or multi-criteria optimization iterative procedures. It results from this a significant reduction of the calculation costs in comparison with the Monte Carlo or standard response surfaces methods while preserving a good predictivity level of the responses. The integration of the SRSM in a robust multi-objectives optimization procedure gives satisfactory results which it must be confirmed on more complex structures.

REFERENCES