Damage detection using frequency measurements

Laxmikant Kannappan, Graduate student
Krishna Shankar, Senior lecturer
Sreenatha G Anavatti, Senior lecturer
Aerospace Civil and Mechanical Engineering,
University of New South Wales, Australian Defence Force Academy,
Canberra, ACT, AUSTRALIA 2600

ABSTRACT

Monitoring the integrity of aerospace structures is expensive, especially for aging aircraft still in service which need thorough and frequent inspection since the probability of structural integrity issues arising in aircraft increase exponentially with age. With advances in technology, the industry is now exploring the adaptation of on-line health monitoring systems, which can monitor structures continuously for detection and assessment of damage progression. Vibration based damage monitoring techniques offer the advantages of being whole field, low cost, easily applicable and adaptable to computer based online expert systems.

This paper deals with a method to identify cracks in beams using the change in its natural frequency before and after damage. Here cracks are modelled as rotational spring with stiffness K. The natural frequencies for this beam are determined using Euler - Bernoulli theory. Working back on this theory, the location (β) and extent (a/h) of crack can be found using the frequencies of the cracked structure.

Finite element analysis (FEA) was conducted to demonstrate the validity of this theory. In FEA, part-thickness, through-width cracks were modelled as discontinuities using singular elements. Damage was also modelled as part-width, through-thickness cracks. Both FEA and experimental modal analysis were performed on beams with part width cracks to verify the theory.

INTRODUCTION

The traditional method of structural damage monitoring using non-destructive inspection techniques such as dye penetrant, radiography, magnetic particle, ultrasonic and eddy current testing, involves considerable expense, human effort and aircraft down time. With both Civilian and military operators seeking more cost effective and efficient means of aircraft maintenance, the new approach to airframe structural maintenance involves implementation of on-line health monitoring systems, which offer the advantages of zero down time and real time damage detection and identification. Since the traditional non-destructive inspection techniques, which were developed for ground based maintenance regimes, are mostly unsuitable for online implementation, a number of emerging techniques, such as the use of fibre optic sensors and acoustic emission are being explored for implementation as on-line health monitoring systems.

One of the most promising techniques for on-line health monitoring is the measurement of vibration parameters which are sensitive to the initiation and progression of damage in the structure. Vibration monitoring can be implemented easily and at low cost with sensors such as piezoelectric films. Further they could be employed without an excitation source, since the vibrations of the airframe structure caused in-flight can be monitored to detect changes caused by the onset or progression of damage. It is quite well known that a crack or any damage in a structure changes its dynamic characteristics, viz. natural frequencies, mode shapes and damping. The changes in these dynamic properties are influenced by the location and size of damage. Theoretically, by monitoring the change in any of these parameters, damage location and severity can be identified. Damage assessment through monitoring changes in mode shapes or their derivatives such as slopes and curvatures is laborious and time consuming since it involves taking measurements over a large number of points distributed across the surface of the structure. The presence of noise and the influence of sensor locations
and distribution on the measured data, further reduce the reliability and applicability of these techniques to real structures. On the other hand, changes in resonant frequencies can be measured with a single sensor located any where on the structure with the least amount of uncertainty and with greater reliability and repeatability Doebling et. al. [1]. Hence frequency based damage monitoring techniques appear to be more promising for practical applications.

In 1975 Vandiver [2] demonstrated the use of frequency based damage monitoring by employing changes in measured resonant frequencies of an offshore light station tower to identify the presence of damage. Cawley and Adams [3] gave a detailed formulation to detect damage in a structure using differences between damaged and undamaged resonant frequencies for each mode in comparison with finite element results. On of the main disadvantages is that this method relies on an accurate numerical model. Chondros and Dimarogonas [4] modelled crack as a rotational spring with stiffness, k_r. They simulated damage to be at the fixed end of the cantilever beam, so that the boundary conditions at that end changed. By finding k_r the depth of crack was obtained. Rizos et. al. [5] modelled crack at any location (β=x/L) along the length of the beam by dividing the beam into two segments. They assumed all boundary conditions were same to the left and right of the crack except for an increase in the slope. Applying these boundary conditions a characteristic equation (8X8 matrix) was derived which was a function of crack location. They solved the inverse problem using Newton-Raphson method.

Similar approach was used by Liang et. al. [6]. Here they solved the inverse problem by separating the matrix of the characteristic equation with known frequencies on one side and unknown β and k_r on the other side. Hence, there was no need for any non-linear solver, thus reducing computational time. Nandwana and Maiti [7] applied this method for detecting cracks in stepped cantilever beams. Again Nandwana and Maiti [8] applied this theory to detect the presence of inclined edge and inclined internal cracks in cantilever beams.

In this paper, the theory of modelling crack as a rotational spring is implemented for a beam with through-width, part-thickness (edge cracks) and through-thickness, part-width cracks (centre cracks). Finite Element Analysis (FEA) data is used to validate the theory in case of edge cracks and experimental data is used in case of centre cracks. Location and extent (a/h) of cracks in both the cases is determined. The location of the crack is determined with 1% accuracy.

**THEORY**

Cracks can be modelled as change in flexibility which again can be represented by a massless, rotational spring with stiffness, k_r (fig.1 and fig 2). Bigger the damage more the flexibility [8], and lesser the stiffness of the spring. The crack is assumed to have a uniform depth along the whole width [5]. Where ever there is a crack, the beam is segmented, but connected by the spring.
The theory of crack localisation and characterisation is applied to a cantilever beam. By Euler-Bernoulli vibration theory, the deflection of the two segments to the left and right of the crack are given as

\[ X1(x) = a_1 \cos \lambda x + a_2 \cos 2\lambda x + a_3 \cos 3\lambda x + a_4 \cos 4\lambda x \]  
(1)

\[ X2(x) = b_1 \cos \lambda x + b_2 \cos 2\lambda x + b_3 \cos 3\lambda x + b_4 \cos 4\lambda x \]  
(2)

where, \( \lambda \) gives the frequency of the cracked beam

The boundary conditions in the fixed and free end can be obtained from any basic vibration engineering textbook Inman [9]. For the boundary conditions along the crack location, it is assumed that the displacement, bending moment and shear force continue to be same on both sides of crack but there is an increases in slope.

Across the location of crack, at \( \beta = x/L \)

- **Deflection**, \( w_1 = w_2 \)  
(3)

- **Bending moment**, \( EI \frac{\partial^2 w_1}{\partial x^2} = EI \frac{\partial^2 w_2}{\partial x^2} \)  
(4)

- **Shear force**, \( \frac{\partial}{\partial x} \left[ EI \frac{\partial^2 w_1}{\partial x^2} \right] = \frac{\partial}{\partial x} \left[ EI \frac{\partial^2 w_2}{\partial x^2} \right] \)  
(5)

but,

\[ \text{Slope}, \frac{\partial w_2}{\partial x} = \frac{\partial w_1}{\partial x} + \frac{EI}{k_r} \frac{\partial^2 w_1}{\partial x^2} \]

Substituting \( K = \frac{k_r L}{EI} \) and \( \beta = x/L \), we get

\[ \text{Slope}, \frac{\partial w_2}{\partial \beta} = \frac{\partial w_1}{\partial \beta} + \frac{1}{K} \frac{\partial^2 w_1}{\partial \beta^2} \]  
(6)

From equation 6 it is evident that if \( K = \alpha \) (infinity), then the second part added to represent change, \( \frac{1}{K} \frac{\partial^2 w_1}{\partial \beta^2} \) will become zero and thus there is no change in slope. This means, bigger the K value smaller the damage and vice-versa. A characteristic equation is obtained solving equation 1 and 2 with boundary conditions 3-6 (Appendix 1). It can be rewritten to a form

\[ |\Delta| = \left| \frac{K}{\lambda} |\Delta_1| + |\Delta_2| \right| \]  
(7)

\( \Delta_1 \) and \( \Delta_2 \) are shown in the Appendix2. K values are determined for each mode and at each point on the beam. Now K is plotted against \( \beta \ (x/L) \) for every mode. Irrespective of the mode of vibration, the assumed spring constant K will be the same. Hence, the point of intersection of the curves gives the location of the crack. The actual spring constant, kr, is calculated from the non dimensional spring constant, K, obtained from the plot. These spring constant values are a function of the extent of the crack.


\[ K = \frac{k_r L}{EI} \Rightarrow k_r = \frac{KEI}{L} = \frac{1}{c} \]  
(8)

where E is the modulus of elasticity, I is second moment of area of cross section, L is length of the beam and c is the compliance defined as,

\[ c = \frac{5.346}{EI} f(a/h) \]  
(9)

\( f(a/h) \) is computed from the strain energy density function as

\[ f(a/h) = 1.8624(a/h)^2 - 3.95(a/h)^3 + 16.375(a/h)^4 - 37.226(a/h)^5 + 76.81(a/h)^6 - 126.9(a/h)^7 + 172(a/h)^8 - 143.97(a/h)^9 + 66.56(a/h)^10 \]  
(10)

Alternatively, Ostachowicz and Krawczuk [12] derived a similar equation to find the extent of damage using decrease in the elastic deformation energy and stress intensity factor. Based on the above said theory, a MATLAB code was written to compute the location and extent of damage using natural frequencies of the cracked beam.
CORRECTION FACTOR FOR $\lambda$

This theory uses theoretical undamaged weighted frequency ($\lambda_{UT}$) which is slightly different from the experimental undamaged weighted frequency ($\lambda_{UE}$). Hence $\lambda_{UT}$ cannot be used directly in the theory. In order to overcome this problem, percentage difference between $\lambda_{UT}$ and $\lambda_{UE}$ (experimental uncracked weighted frequencies) is calculated. This same difference is incorporated to the cracked frequencies also [8].

POINT OF INTERSECTION

To find the point of intersection, the distance between the curves is calculated at each point along the length of the curve using Eq. 10. The location where the root of sum of square of the distances between the curves becomes minimum, is considered to be the point of intersection. From this location obtained, the spring stiffness is determined by interpolating it with the K Vs $\beta$ curves. Another important point to note is that the maximum $\beta$ values used in finding the distance is restricted to 80% of the beam length. After this length, the curves tend to meet, irrespective of the presence of the damage, since the stiffness at the free end of the beam reduces to zero.

$$d_k = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{m} (k_i - k_j)^2}$$

where $d$ is a vector containing distances of $k$ points and $n$ is the number of modes used

RESULTS FOR THROUGH-WIDTH, PART-THICKNESS CRACKS (from FEA)

For performing Finite Element Analysis (FEA), ANSYS 9.0 was used. Damage was modelled as through width, part thickness cracks (Fig. 3). The beam is considered to be made of steel. The dimensions of the beam are 600mm x 25mm x 12mm. The Modulus of Elasticity of the steel used is 210GPa and density is 7800Kg/m3.

![Fig3. Vibration of beam with Edge crack](image)

FEA was performed, changing the location ($\beta = x/L$) and the crack size ($a/h$). First three natural frequencies were used as input to the algorithm to extract the damage characteristics. The first three natural frequencies of uncracked and different cases of cracked beam are shown in the table below (Table 1). Also the results from the algorithm determining the location and extent are given in tables 2 and 3. Typical K Vs $\beta$ curves for damage cases 1, 6, and 8 are plotted (Figs. 4a, 4b, 4c).

<table>
<thead>
<tr>
<th>Damage case</th>
<th>Mode 1 (Hz)</th>
<th>Mode 2 (Hz)</th>
<th>Mode 3 (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undamaged beam</td>
<td>57.94</td>
<td>360.19</td>
<td>995.81</td>
</tr>
<tr>
<td>1. $\beta=0.05$; $a/h=0.05$</td>
<td>57.83</td>
<td>359.74</td>
<td>995.04</td>
</tr>
<tr>
<td>2. $\beta=0.05$; $a/h=0.2$</td>
<td>56.28</td>
<td>353.66</td>
<td>984.91</td>
</tr>
<tr>
<td>3. $\beta=0.1$; $a/h=0.05$</td>
<td>57.85</td>
<td>359.98</td>
<td>995.73</td>
</tr>
<tr>
<td>4. $\beta=0.1$; $a/h=0.2$</td>
<td>56.51</td>
<td>357.05</td>
<td>994.29</td>
</tr>
<tr>
<td>5. $\beta=0.15$; $a/h=0.2$</td>
<td>56.72</td>
<td>359.21</td>
<td>995.35</td>
</tr>
<tr>
<td>6. $\beta=0.15$; $a/h=0.4$</td>
<td>52.89</td>
<td>356.27</td>
<td>993.73</td>
</tr>
<tr>
<td>7. $\beta=0.2$; $a/h=0.2$</td>
<td>56.92</td>
<td>360.15</td>
<td>990.67</td>
</tr>
<tr>
<td>8. $\beta=0.2$; $a/h=0.4$</td>
<td>53.64</td>
<td>359.94</td>
<td>974.09</td>
</tr>
</tbody>
</table>
Table 2. Error between actual and predicted location of damage

<table>
<thead>
<tr>
<th>Damage case</th>
<th>Actual</th>
<th>Predicted</th>
<th>Error</th>
<th>%Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.05</td>
<td>0.051</td>
<td>-0.001</td>
<td>-2.1</td>
</tr>
<tr>
<td>2</td>
<td>0.05</td>
<td>0.051</td>
<td>-0.001</td>
<td>-2.1</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td>0.102</td>
<td>-0.002</td>
<td>-2</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>0.102</td>
<td>-0.002</td>
<td>-2</td>
</tr>
<tr>
<td>5</td>
<td>0.15</td>
<td>0.15</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>0.15</td>
<td>0.15</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>0.2</td>
<td>0.202</td>
<td>-0.002</td>
<td>-1</td>
</tr>
<tr>
<td>8</td>
<td>0.2</td>
<td>0.2</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3. Error between actual and predicted a/h

<table>
<thead>
<tr>
<th>Damage case</th>
<th>Actual</th>
<th>Predicted [10]</th>
<th>%Error</th>
<th>Predicted [12]</th>
<th>%Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.05</td>
<td>0.0545</td>
<td>-8.9158</td>
<td>0.0488</td>
<td>2.3271</td>
</tr>
<tr>
<td>2</td>
<td>0.20</td>
<td>0.2269</td>
<td>-13.4392</td>
<td>0.2012</td>
<td>-0.5847</td>
</tr>
<tr>
<td>3</td>
<td>0.05</td>
<td>0.0556</td>
<td>-11.2085</td>
<td>0.0499</td>
<td>0.2938</td>
</tr>
<tr>
<td>4</td>
<td>0.20</td>
<td>0.2293</td>
<td>-14.6637</td>
<td>0.2034</td>
<td>-1.6771</td>
</tr>
<tr>
<td>5</td>
<td>0.20</td>
<td>0.2293</td>
<td>-12.6272</td>
<td>0.1997</td>
<td>0.1395</td>
</tr>
<tr>
<td>6</td>
<td>0.40</td>
<td>0.4519</td>
<td>-11.9804</td>
<td>0.4075</td>
<td>-1.8710</td>
</tr>
<tr>
<td>7</td>
<td>0.20</td>
<td>0.2216</td>
<td>-10.8060</td>
<td>0.1965</td>
<td>1.7626</td>
</tr>
<tr>
<td>8</td>
<td>0.40</td>
<td>0.4479</td>
<td>-11.9804</td>
<td>0.4075</td>
<td>-1.8710</td>
</tr>
</tbody>
</table>

Fig4a. K Vs \( \beta \) plot for case1

Fig4b. K Vs \( \beta \) plot for case6

Fig4c. K Vs \( \beta \) plot for case8
RESULTS FOR THROUGH-THICKNESS, PART-WIDTH CRACKS (from experiments)

Experimental Modal Analysis (EMA) was conducted on an aluminium beam 600x50x3.2 mm with Young’s Modulus 69.6GPa and density 2882Kg/m³. Damage was induced as rectangular slots 0.2mm wide, 20mm long and full depth using wire-cut Electrical Discharge Machining (EDM) process. Experiments were conducted on three damaged beams changing the location of the damage. These slots were modelled to simulate centre cracked plates in bending (fig. 5).

EMA was conducted using B&K4374 miniature accelerometers and STAR-Modal was used to extract the first natural frequencies (Table 4). FEA was preformed and it was observed that the results differed by less than 0.5%. The results from the experiments were input to the damage detection algorithm. The K value obtained from the algorithm, representing the spring stiffness, can be used as an indicator of extent of damage. A comparison of the actual and predicted location is shown in table 5. Typical K Vs β plots for all damage cases are also shown below (Figs. 6a-6c).

Table 4. Natural frequencies of the Al-7076 beam (from Experiment)

<table>
<thead>
<tr>
<th>Damaged case</th>
<th>Mode1</th>
<th>Mode2</th>
<th>Mode3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undamaged</td>
<td>7.11</td>
<td>44.61</td>
<td>125.01</td>
</tr>
<tr>
<td>1:d=100mm (β=0.17)</td>
<td>7.005</td>
<td>44.525</td>
<td>124.715</td>
</tr>
<tr>
<td>2:d=150mm(β=0.25)</td>
<td>7.035</td>
<td>44.54</td>
<td>123.875</td>
</tr>
<tr>
<td>3:d=200mm (β=0.33)</td>
<td>7.04</td>
<td>44.38</td>
<td>123.765</td>
</tr>
</tbody>
</table>

Table 5. Comparison of actual and predicted β (location of damage)

<table>
<thead>
<tr>
<th>Damage case</th>
<th>Actual</th>
<th>Predicted</th>
<th>Error</th>
<th>%Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.17</td>
<td>0.1712</td>
<td>-0.12</td>
<td>-0.71</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
<td>0.2673</td>
<td>-0.73</td>
<td>-2.92</td>
</tr>
<tr>
<td>3</td>
<td>0.33</td>
<td>0.3303</td>
<td>-0.0003</td>
<td>-0.09</td>
</tr>
</tbody>
</table>

Fig 5. Vibration of beam with middle crack

Fig6a. K Vs β plot for case1

Fig6b. K Vs β plot for case2
Conclusion

This identification method is based on the Euler-Bernoulli theory where it is assumed that the beam deflects because of pure bending. The main advantage of this method is the simplicity with which frequency of a structure can be measured in comparison to the measurement of mode shapes which is difficult for complex structures.

The application of the method to damage detection and assessment has been demonstrated for beams with through thickness part width crack and for beams with part thickness full width cracks. The accuracy of predicted damage location is within 3% in both cases, and the accuracy in damage size assessment is of the order of 3%.

Also, this method is very sensitive provided the frequencies of the damaged and undamaged structures can be measured accurately. The Finite Element Modelling shows that cracks with a minimum depth of 5% of the thickness can be detected by the algorithm. Experiments are being performed to determine the sensitivity that can be achieved in practice.

References

Appendix 1

\[
\left[\begin{array}{cccccccc}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\cos \lambda & \cosh \lambda & -\sin \lambda & \sinh \lambda \\
0 & 0 & 0 & 0 & \sin \lambda & 0 & \cosh \lambda & -\cos \lambda \\
\end{array}\right]
\]

\[= 0\]

where, \(\alpha = \lambda \beta\)

Appendix 2

\(\Delta_1\) and \(\Delta_2\) can be written as

\[
\left[\begin{array}{cccccccc}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\cos \lambda & \cosh \lambda & -\sin \lambda & \sinh \lambda \\
0 & 0 & 0 & 0 & \sin \lambda & 0 & \cosh \lambda & -\cos \lambda \\
\end{array}\right]
\]

\[= \frac{K}{\lambda} \left[\begin{array}{cccccccc}
\alpha \lambda & \cosh \lambda & \sin \alpha & \sinh \lambda & -\cos \alpha & -\cosh \lambda & \sin \alpha & -\sinh \lambda \\
\sin \alpha & \sinh \lambda & -\cos \alpha & -\cosh \lambda & \sin \alpha & -\sinh \lambda & \cos \alpha & \cosh \lambda \\
-\sin \alpha & \sinh \lambda & \cos \alpha & \cosh \lambda & \sin \alpha & \sinh \lambda & -\cos \alpha & -\cosh \lambda \\
\end{array}\right]
\]

and

\[
\left[\begin{array}{cccccccc}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\cos \lambda & \cosh \lambda & -\sin \lambda & \sinh \lambda \\
0 & 0 & 0 & 0 & \sin \lambda & 0 & \cosh \lambda & -\cos \lambda \\
\end{array}\right]
\]

\[= \frac{K}{\lambda} \left[\begin{array}{cccccccc}
\alpha \lambda & \cosh \lambda & \sin \alpha & \sinh \lambda & -\cos \alpha & -\cosh \lambda & \sin \alpha & -\sinh \lambda \\
\sin \alpha & \sinh \lambda & -\cos \alpha & -\cosh \lambda & \sin \alpha & -\sinh \lambda & \cos \alpha & \cosh \lambda \\
-\sin \alpha & \sinh \lambda & \cos \alpha & \cosh \lambda & \sin \alpha & \sinh \lambda & -\cos \alpha & -\cosh \lambda \\
\end{array}\right]
\]

Knowing the natural frequencies of the cracked beam, K can be found.

\[K = \frac{\Delta_2}{\Delta_1}\]