Experimental Measurement of NC Machine Tool Milling Cutting Forces

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ABSTRACT

Experimental measurement of the cutting forces in milling processes can be a useful tool to validate standard and non-conventional milling models to be adopted in most industrial applications. As an example, such models can be used to design the machine tool components and fixtures, to choose the technological process parameters and to define process monitoring and diagnostic tools such as monitoring of tool wear and surface quality, diagnosis of critical cutting conditions. With respect to the latter case, high speed machining may exhibit self-excited chattering vibrations, which can be hard to predict using conventional theoretical cutting models.

In this paper an experimental technique for cutting force measurement estimate is proposed. A system composed of a mechanical fixture with 4 piezoelectric tri-axial force transducers was designed, to be installed on the machine tool working table. During the milling operation on a workpiece mounted on this fixture, the 12 signals from transducers may be used to estimate the cutting force resultant. A filtering technique is proposed here to reduce error in cutting force experimental measurement estimation. Details of this technique are given, and some application examples are discussed by means of a numerical simulation in the frequency domain. An experimental application on an instrumented 3-axis laboratory machine tool is then illustrated and discussed in detail.

NOMENCLATURE

(bold) array notation
( ) transpose operator
( )\(^{-1}\) matrix inversion operator
( , , ) first and second time derivative operator
\(\cdot \times\) scalar (dot) and vector (cross) product
\(\mathbf{v} = [v_x, v_y, v_z]^T\) column vector of Cartesian global components
\(\mathbf{v}' = [v'_x, v'_y, v'_z]^T\) column vector of Cartesian local components
\(\mathbf{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}\) vector operator: \(\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} 0 & -v_y & v_x \\ v_z & 0 & -v_x \\ -v_y & v_x & 0 \end{bmatrix} \mathbf{v} = \mathbf{v}\)
\(J, J'\) inertia matrix of body in global and local reference
\(I_n\) identity \(nxn\) matrix
\(R\) rotation transformation matrix
\(p, p'\) position vector in global and local reference
INTRODUCTION

In milling operations the elastodynamic behaviour of the machine and of the cutting process is responsible for both the working quality and the lifetime of the cutting tools. In high-speed production, critical vibrations such as unstable and regenerative chatter vibrations may occur in the cutting dynamics. The result is an unwanted and uneven-rough pattern left on the object surface, also followed by high level acoustic noise and tool break. Chatter studies can be found in many references. Paper [1] mainly focuses on cutting force modelling, while other studies [2-7] deal with chatter diagnostics. In [2-4] milling tests were performed to measure the cutting force in chattering conditions and to sketch stability charts. The cutting force was evaluated by means of the Frequency Response Function (FRF) estimate between tool force and transverse displacement, and the tool transverse displacement, measured by a non-contact eddy current transducer. The FRF was experimentally obtained on a dedicated single degree of freedom (dof) oscillating fixture, designed for this purpose. The measured force was used to validate the force model, as well as to develop a criterion to discriminate between stable conditions (where the vibration level might still be high) and instable conditions prone to evolve to chatter behaviour. In [5] a very long and slender tool was used instead; a couple of optical sensors, mounted on the spindle housing, were used to measure the tool displacement. Other authors investigated critical chatter conditions by means of sound pressure spectra measurements, acquired with microphones close to the working area [6,7].

Force measurement appears to be a basic requirement for developing chatter monitoring procedures. This paper deals with proper cutting force measurement. Since it is not possible to directly measure cutting forces, it is common practice to couple the working table on which the workpiece can be mounted to the machine tool frame by means of force sensors. The inertia forces due to workpiece and table vibrations must therefore be taken into account while estimating cutting forces from measured restrain forces. In this paper a filtering procedure is developed, with the basic assumption of unconstrained table rigid body motion, externally loaded by the cutting forces (unknown) and of the restraint (measured) forces. A rigid body table dynamical behaviour was taken into account, neglecting high frequency flexible contribution. In order to estimate the table inertial contribution, an experimental procedure employing six accelerometric measurements was proposed. A table was designed to behave as a rigid body in the frequency range of interest. This table was then instrumented with four tri-axial force cells and six accelerometers. The filtering procedure is explained and discussed in detail, and simulated in the frequency domain. Some results are also reported.

TABLE RIGID BODY MODELLING

The spatial motion of a rigid body can be described in a fixed Cartesian frame \((X_0, Y_0, Z_0)\), also called global, while it can be useful to describe other quantities in a local reference \((X', Y', Z')\) centered in the rigid body centroid. By this procedure [8], the position of the rigid body in the frame \((X_0, Y_0, Z_0)\) can be described by six unknowns \(q = [x, y, z, \phi, \beta, \psi]^T\), centroid actual position vector \(x = [x, y, z]^T\) and three successive Euler angles \(\phi = [\phi, \beta, \psi]^T\) as body rotations along local reference axis \((X', Y', Z')\). A generic global position vector \(p_i\) can be obtained from local position vector \(p'_i\):

\[
p_i = x + R \cdot p'_i,
\]

where:

\[
R = R_x(\psi) \cdot R_y(\beta) \cdot R_z(\phi) = \begin{bmatrix}
\cos(\psi) - \sin(\psi) \\
\sin(\psi) \cos(\psi) \\
1
\end{bmatrix} \begin{bmatrix}
\cos(\beta) & \sin(\beta) & 1 \\
-\sin(\beta) & \cos(\beta) & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
\cos(\phi) - \sin(\phi) \\
\sin(\phi) \cos(\phi) \\
\sin(\phi) \sin(\phi)
\end{bmatrix}
\]

With a similar transformation, a local vector \(v'_i\) on the rigid body can be transformed to the global referenced \(v_i\):

\[
v_i = R \cdot v'_i,
\]

When dealing with vibrations, the assumption of small generalized displacements can be adopted. Assuming that \(\gamma = \gamma_0 + \Delta \gamma\), where rotation \(\Delta \gamma\) being associated to vibrations is small with respect to rest value \(\gamma_0\), a linear rotation operator can be defined instead for each single rotational transformation [9-11]:

\[
R_a(\gamma) = R_a(\gamma_0) + \frac{\partial R}{\partial \gamma}(\gamma_0) \cdot \Delta \gamma = R_a(\gamma_0) + R'_a(\gamma_0) \cdot \Delta \gamma
\]
Since the rotational transformation matrix between two reference systems can be defined as the product of three successive rotations along the reference axes, it follows:

\[
\begin{align*}
R(\phi, \beta, \psi) &= R^\circ + \Delta \phi R^\circ + \Delta \beta R^\circ + \Delta \psi R^\circ \\
R^\circ &= R_x(\psi_0) R_y(\beta_0) R_z(\phi_0) \\
R^\circ' &= R_z(\psi_0) R_y(\beta_0) R_x(\phi_0) \\
R^\circ'' &= R_z(\psi_0) R' (\beta_0) R_x(\phi_0) \\
R^\circ''' &= R_z(\psi_0) R_y' (\beta_0) R_x(\phi_0)
\end{align*}
\tag{5}
\]

With the assumption of small generalized displacements, the unknowns expression can be written as \( q = q_0 + \Delta q \), where the dofs \( \Delta q = \{ \Delta x', \Delta \Phi^T \} \) are linear variation from the kinematical generalized position of interest \( q_0 = \{ x_0', \Psi_0 \}^T \). Equation (1) can then be rewritten under this linearization as follows:

\[
p_a = p_0 + C_r(q_0, p^*_r) \Delta q
\tag{6}
\]

with the constants:

\[
p_a = x_0 + R^\circ p^*_r, \\
C_r(q_0, p^*_r) = C_r = \begin{bmatrix} I & |R^\circ p^*_r| & |R^\circ'' p^*_r| & |R^\circ''' p^*_r| \end{bmatrix}
\tag{7}
\]

It also follows, by time derivation of Eq.(6), that:

\[
\dot{p}_r = C_r(q_0, p^*_r) \Delta q
\tag{8}
\]

\[
\ddot{p}_r = C_r(q_0, p^*_r) \Delta \dot{q}
\tag{9}
\]

Defining \( Q = m \Delta \dot{x} \) as the linear momentum and \( K_0 = J^\circ \Delta \Phi^T \) as the angular momentum of the rigid body, where \( m \) is the mass and \( J^\circ \) is the expression of the local inertia tensor, the expression of the kinetic energy can be written as follows:

\[
E = \frac{1}{2} \Delta \dot{x}^T Q + \frac{1}{2} \Delta \Phi^T \cdot K_0 \tag{10}
\]

Dynamical equilibrium equations can be obtained from the analytical expression of \( L = E \) system kinetic energy, and assuming generalized external forces \( f_\text{e} \):

\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = f_\text{e}
\tag{11}
\]

By the linearized approach adopted, it follows that:

\[
\frac{d}{dt} \left( \frac{\partial E}{\partial \Delta \dot{q}} \right) = m \Delta \dot{x} + J^\circ \Delta \dot{\Phi}^T = M \Delta \dot{q} = f_\text{e} = \begin{bmatrix} f_\text{r} \\ M f_\text{u} \end{bmatrix}
\tag{12}
\]

with \( J \) as the inertia tensor in global coordinates and \( M \) as the generalized inertia properties matrix:

\[
J = \begin{bmatrix} R^0 \end{bmatrix}^T J^\circ R^0, \quad M = \begin{bmatrix} mI_3 & 0 \\ 0 & (R^0)^T J^\circ R^0 \end{bmatrix}, \quad E = \frac{1}{2} \Delta \dot{q}^T M \Delta \dot{q}
\tag{13}
\]

In this paper, the generalized external force vector \( f_\text{e} \) contains two contributions \( f_\text{r} \) and \( f_\text{u} \). The former comes from the restrain forces \( F_j^r, j=1...12 \), measured by the transducers located at \( p_j^0 \); the latter is due to the tool cutting force equivalent system.
FILTERING INERTIA CONTRIBUTIONS FROM MEASURED FORCES

In order to get good measurement estimates of the cutting forces, inertia contributions should be taken into account. In Eq. (13), while \( \mathbf{M} \) is constant and can be evaluated by computer aided solid modelling procedures, \( \Delta \mathbf{q} \) is unknown.

Equation (9) can be used to obtain the relationship between the acceleration \( \mathbf{\ddot{p}}_i \) of a point \( i \) on the rigid body and the generalized accelerations of the centroidal dofs \( \Delta \dot{\mathbf{q}} = \{\Delta \mathbf{x}', \Delta \mathbf{\phi}'\}^T \) through the 3x6 linearized transformation matrix \( \mathbf{C}_{ij}(\mathbf{q}, \mathbf{p}') \). Point acceleration components can be measured by translational accelerometers placed on the table. Each accelerometer \( j \), located at \( \mathbf{p}'_j \) on the dynamometric table, measures the accelerations \( \mathbf{\ddot{p}}_j^a \) in its own local Cartesian reference frame \( (X'_j, Y'_j, Z'_j) \). The transformation to the global reference, as in Eq. (3), can be obtained by the rotation \( \mathbf{R}_j \), so the global accelerations become:

\[
\mathbf{\ddot{p}}_j = \mathbf{R}_j \mathbf{\ddot{p}}_j^a = \mathbf{C}_{ij}(\mathbf{q}, \mathbf{p}') \Delta \dot{\mathbf{q}} \Rightarrow \mathbf{\ddot{p}}_j^a = \mathbf{R}_j^T \mathbf{C}_{ij}(\mathbf{q}, \mathbf{p}') \Delta \dot{\mathbf{q}} = \mathbf{\hat{C}}_j \Delta \dot{\mathbf{q}}
\]

(14)

Six dofs are needed to fully describe the motion of a rigid body, so six is the minimal number of experimental dofs to be used in the estimation of \( \Delta \dot{\mathbf{q}} \). Defining the new dof set as:

\[
\mathbf{\hat{a}}_k = \mathbf{\hat{p}}_k = \mathbf{\hat{C}}_k \Delta \dot{\mathbf{q}}, \quad k \in \{1, 2, 3\}, \quad \mathbf{\hat{C}}_k \Delta \dot{\mathbf{q}} = \left\{ \begin{array}{c} \dot{\mathbf{C}}_{h1} \Delta \mathbf{q}_1 \\ \dot{\mathbf{C}}_{h2} \Delta \mathbf{q}_2 \\ \dot{\mathbf{C}}_{h3} \Delta \mathbf{q}_3 \\ \dot{\mathbf{C}}_{h4} \Delta \mathbf{q}_4 \\ \dot{\mathbf{C}}_{h5} \Delta \mathbf{q}_5 \\ \dot{\mathbf{C}}_{h6} \Delta \mathbf{q}_6 \end{array} \right\}
\]

(15)

it follows that:

\[
\mathbf{\dot{a}} = \int \mathbf{\dot{a}}_k \Delta \dot{\mathbf{q}} = \mathbf{T} \Delta \dot{\mathbf{q}}, \quad \mathbf{T} = \int \mathbf{\hat{C}}_k \Rightarrow \Delta \dot{\mathbf{q}} = \mathbf{T}^{-1} \mathbf{\dot{a}}
\]

(16)

Fig. 1 – Dynamometric table: (a) FE model; (b) Force sensor fixture assembly; (c) Real implementation.

Fig. 2 – NC milling machine tool: (a) FE model; (b) Tool support sledges model; (c) Real implementation.
The position of the accelerometers affects the condition number of the 6x6 geometrical transformation matrix $T$, so care must be taken when locating the sensors (\[12\]).

Introducing the experimental dofs set in Eq.(16) to Eq.(10):

$$E = \frac{1}{2} \Delta \dot{q}^{T} \cdot M \cdot \Delta q - \frac{1}{2} a^{T} \cdot T^{-T} \cdot M \cdot T^{-1} \cdot \dot{a}$$

an equivalent system of 6 inertia force components $f_{i}^{k}$, $f = \bigcup_{k=1}^{6} f_{i}^{k}$, whose direction $d_{k}, k = 1...6 \ (|d_{0}|=1)$ coincides with the direction of $\dot{a}$ accelerations, measured in points $p_{k}^{u}$, follows:

$$f_{i} = -\frac{d}{dt} \left( \frac{\partial E}{\partial a} \right) = -T^{-T} \cdot M \cdot T^{-1} \cdot \ddot{a} = -M_{T} \cdot T^{-1} \cdot \ddot{a} = M_{T} = T^{-T} \cdot M \cdot T^{-1}.$$  

The equivalent tool force system $f_{u} = [F_{u}, M_{u}]^{T}$, where $F_{u}$ is the tool force resultant whose direction crosses $(X_{o}, Y_{o}, Z_{o})$ global frame origin $O$, and $M_{u}$ is the moment resultant, can be obtained from the following D'alembert dynamical equilibrium equation:

$$f_{u} = \begin{bmatrix} F_{u} \\ M_{u} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{12} F_{i}^{R} \\ \sum_{i=1}^{12} p_{i}^{u} \times F_{i}^{R} \end{bmatrix} - \begin{bmatrix} \sum_{i=1}^{6} F_{i}^{K} \times d_{k} \\ \sum_{i=1}^{6} p_{i}^{u} \times \dot{F}_{i}^{K} \times d_{k} \end{bmatrix} = -\sum_{i=1}^{12} \begin{bmatrix} l_{i}^{R} \\ l_{i}^{R} \end{bmatrix} F_{i}^{R} - \sum_{i=1}^{6} \begin{bmatrix} l_{i}^{K} \\ l_{i}^{K} \end{bmatrix} F_{i}^{K} \times \dot{d}_{i}.$$  

Fig. 3 – Modeshapes of the full machine tool assembly: (a) mode 1, $f = 53$ Hz; (b) mode 2, $f = 60$ Hz; (c) mode 3, $f = 84$ Hz; (d) mode 6, $f = 127$ Hz; (e) mode 14, $f = 202$ Hz; (f) mode 15, $f = 214$ Hz.
DYNAMOMETRIC TABLE DESIGN

The experimental instrumentation of Fig.1 was designed under Eq.(19) basic assumptions (rigid body motion). The mechanical fixture and the workpiece mounted on the table should, in the coupled arrangement, exhibit the first flexible eigenmode outside the frequency range of interest. Referring to a milling process, the tool spindle angular velocity not exceeding 12 krpm and considering a maximum number of ten cutting edges, a [0-2kHz] frequency range can be assumed. Merging these requirements with the manufacturing constraints, the simple design of Fig.1a was proposed and built as Fig.1c, after FE model optimization. In it, four tri-axial piezoelectric force transducers were adopted (PCB force sensors, model 260A03), packed as in Fig.1b after proper calibration. The dots in Fig.1a show potential positions for tri-axial accelerometers, used later on for the simulations. The table was thus instrumented by a total amount of eighteen channels.

ELASTODYNAMIC MODELING OF THE MILLING MACHINE

The 3 NC axis milling machine in Fig.2c was acquired by DIEM-LAV laboratories for chatter related topic research. The complete assembly (milling machine frame, tool support sledges and dynamometric table) was modeled by FE approach; Fig.2a shows the resulting model. The frame was discretized by shell-type finite elements, while the sledges of Fig.2b were considered, in the frequency range of interest, as rigid bodies interconnected by lumped springs and kinematic constraints. Auxiliary accessories, like a pump and a fluid tank, were added by lumped masses and rigid constraints. The dynamometric table parts were modeled as solid-type finite elements. Attention was paid to properly model the force transducers and their connection to the parts of the table: tri-axial bushings were adopted to simulate the elastic behaviour of the force cells, while rigid constraints and lumped masses were used to model the subparts of Fig.1b and their contacts with the table shells, as can be seen in Fig.1a. Attachment mode superset [13] was calculated.

![Fig. 4 – simulated measurement of the resultant force (dashed line) and of the filtered one (continuous line): rigid table on springs.](image)

![Fig. 5 – simulated measurement of the resultant force (dashed line) and of the filtered one (continuous line): rigid table on flexible tooling machine.](image)
from the FE model of the full assembly with the same input force as used in later simulations. The model exhibited high modal density starting from low frequencies. Figs.3a-f show some low frequency mode shapes, where it appears that the dynamometric table approximately behaves as a rigid body.

SIMULATING THE FILTERING TECHNIQUE

Equation (19) was implemented in a C-language code to simulate filtering effects in the frequency domain. The impulse response was thus numerically simulated, by placing a force with white unitary spectrum at the node close to the table center and by numerically evaluating the displacements at the caps of the force transducers modeled as linear spring elements, and the accelerations in three points of the table as in Fig.2a. Restrain forces \( f^R \) were then estimated by means of previously numerically computed transducer end displacements and known sensor elastic stiffnesses. Real damping assumption (damping ratio=2.5% for all modes) was adopted in synthesizing the displacement and acceleration responses. Only the vertical component of the excitation is shown here.

Three different numerical tests were carried out. The simplest one considered the rigid table only, as a six dofs system restrained by the tri-axial springs of the force cells, the latter being connected to the ground on the other caps. From Fig.4 it is clear that the filtering procedure gives much better results than the simple resultant calculations. Only at the resonant frequency does it not have the unitary force response. Moreover, in this simulation, the simple resultant of the force cell channels is quite far from the white unitary spectrum of the input. A second test was made with a rigid table mounted on the flexible frame of the milling machine with sledges attached. The modal base included modes up to 2kHz. Again, in this simulation whose results are reported in Fig.5, a better behaviour was shown by the filtered force results.
The last test was carried out over a model where both the table and the milling machine were considered as flexible. Results in Fig.6 again reported a superior output quality of the filtered force results. The table dynamical behaviour affects the filtered measurements starting from 1 kHz, as can be seen in Fig.7 where a comparison between the two sets is shown; the set with the flexible table overestimates the cutting tool force of about 6% more than the set with rigid body assumption.

CONCLUSIONS

In this paper a technique to improve the quality of milling machine cutting force measurements is presented. Six accelerometric measurements were added to estimate the inertia forces. The analytical description provides an approach to eliminate table inertial influence from experimental measurements taken with a dynamometric table. Three numerical simulations were carried out in the frequency domain to validate the approach, varying the structure on which to mount the instrumented table. The technique showed promising results in the simulations; in-field experimental measurements are currently in process. Future work will be developed in experimental validation of the analytical model, and in defining an experimental technique to identify or update matrix $M$, taking into account of the working piece and fixture inertial contributes.

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