Detection Feature Influence on Optimized Excitations for Structural Health Monitoring

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NOMENCLATURE

\[ z \]  Vector of state variables for an ordinary differential equation (ODE) system
\[ \dot{z} \]  Vector of time derivatives of the state variables for an ODE system
\[ \mu \]  Vector of parameters for an ODE system
\[ \varepsilon \]  Speed parameter
\[ z_1 \]  Time series vector corresponding to the first ODE state variable
\[ z^* \]  Normalized excitation time series vector
\[ N \]  Length of the excitation time series vector (number of samples)
\[ x_b \]  First mass displacement response vector of spring-mass system in baseline condition
\[ x_{b1} \]  First noise-corrupted baseline response
\[ x_{b2} \]  Second noise-corrupted baseline response
\[ x_d \]  First mass displacement response vector of spring-mass system in damaged condition
\[ x_c \]  Noise-corrupted damaged response vector
\[ w_1 \]  Vector of Gaussian noise used to produce \( x_{b1} \) from \( x_b \)
\[ w_2 \]  Vector of Gaussian noise used to produce \( x_{b2} \) from \( x_b \)
\[ w_3 \]  Vector of Gaussian noise used to produce \( x_c \) from \( x_d \)
\[ \text{SNR} \]  Signal-to-noise ratio
\[ \sigma^2_{corrupting} \]  Variance of the Gaussian distribution from which samples of \( w_1, w_2, \) and \( w_3 \) are drawn
\[ \sigma^2_{signal} \]  Variance of either \( x_b \) or \( x_d \) depending on context
\[ x \]  Observed response time series
\[ X \]  Delay matrix created from \( x \)
\[ M \]  Dimension (number of delay vectors) used to create \( X \)
\[ T \]  Delay used to create \( X \)
\[ N^* \]  Number of points on the delay matrix \( X \), also number of points on an embedded attractor
\[ U \]  \( M \times M \) matrix of left-singular column vectors \([u_1 \ u_2 \ u_3 \ldots \ u_M]\)
\[ S \]  \( M \times M \) diagonal matrix of scalar singular values
\[ V \]  \( N^* \times M \) matrix of right-singular column vectors \([v_1 \ v_2 \ v_3 \ldots \ v_M]\)
\[ T_{nat} \]  First natural frequency delay
\[ \omega_i \]  \( i \)-th natural frequency in Hz
\[ f_s \]  Sampling rate
\[ M^* \]  Embedding dimension (number of columns of \( X \) used to create an attractor)
\[ \bar{X}_{scaled} \]  \( N^* \times M^* \) matrix corresponding to submatrix formed by the first \( M^* \) vectors in \( V \), an attractor
\[ \bar{X}_b \]  A baseline attractor
\[ \bar{X}_c \]  A comparison attractor
\[ \bar{X}_b(n = f) \]  A fiducial point on the baseline attractor given by index \( f \)
\[ \bar{X}_c(n = f) \]  A fiducial point on the comparison attractor given by index \( f \)
\[ \psi_{ij} \] Distance between points on an attractor with indices \( i \) and \( j \)

\[ \Psi^b_f \] Set of distances between fiducial point with index \( f \) and all other points on the baseline attractor, ordered from smallest to largest

\[ \Phi^b_f \] Set of nearest neighbors to the fiducial point with index \( f \) on the baseline attractor

\( P \) Number of nearest neighbors selected for each fiducial point

\( h \) Theiler window

\( s \) Number of time steps that each nearest neighbor is evolved, the prediction horizon

\[ c^b_f \] Geometric centroid of a set of time-evolved neighbors for a fiducial point on baseline attractor

\[ c^c_f \] Geometric centroid of a set of time-evolved neighbors for a fiducial point on comparison attractor

\[ \gamma^f \] Prediction error corresponding to a given fiducial point with index \( f \)

\( F \) Number of randomly selected fiducial points for a comparison between two attractors

\( \mu_b \) Mean of baseline prediction error distribution gathered from baseline-baseline comparison

\( \sigma_b \) Standard deviation of baseline prediction error distribution from baseline-baseline comparison

\( \mu_c \) Mean of comparison prediction error distribution gathered from baseline-damaged comparison

\( \sigma_c \) Standard deviation of comparison prediction error distribution from baseline-baseline comparison

\( F_s \) Scaling factor used for the differential evolution algorithm

\( CR \) Crossover constant used for the differential evolution algorithm

**ABSTRACT**

We have recently demonstrated that evolutionary algorithms can be used to modify the excitation applied to a structure such that damage detection resolution on the output will be improved. Recent results indicate that optimized inputs are highly sensitive to the damage detection feature employed in the optimization, implying that the excitation and feature type should be closely matched if improved detection sensitivity is desired. This study compares two variations of the prediction error feature used to discern damage-induced differences in structural response. In particular, we show that the inclusion of time-dependent information in the prediction error feature forces the evolutionary optimization procedure to settle on chaotic excitations for improved detection sensitivity. This contrasts with the multi-tone input preferred for a geometrically-based version of prediction error.

1. **INTRODUCTION**

Within the field of structural health monitoring (SHM) much effort is focused on improving the ability to extract information pertaining to damage from structural response data. In this sense, damage detection is a matter of change detection and the development of detection features that remain sensitive to damage while rejecting false positives arising from noise or environmental effects is a thriving research area within the field. Typical features include autoregressive models, modal models, and wavelet analysis. Reviews of these and other methods are provided in [1-2] and will not be discussed in this work. In recent years, the use of a state-space formulation has led to the development of features such as phase space warping [3], dynamic continuity [4], and prediction error [5-7]. A state-space representation of the structural response is intriguing because of the delay-embedding theorems [8-11]; a theoretical edifice that allows the complete steady-state dynamic behavior of the structure to be extracted by simply observing a single structural response variable. In theory, any relevant dynamical information will be represented in the state-space and consequently, changes in the structural dynamics (possibly induced by damage) will be represented as well. Thus, features that have been developed to detect changes in state-space dynamics can be used to infer the presence of damage in a structure.

Feature development typically assumes that information pertaining to damage is present in the structural response and will need to be extracted given the corrupting effects of noise and other information unrelated to damage. In essence, the identification of damage in a structural response is a matter of detection; a subject with roots in the development of radar [12]. What has not been considered as carefully is the idea that the input used to excite the structure can be matched to a given structure, detection feature, and damage condition. We have recently shown that the excitation applied to a structure can be modified such that an improvement in detection sensitivity is observed [13].
In particular, we use an evolutionary algorithm to modify the parameters that govern the dynamical evolution of a system of ordinary differential equations (ODEs) such that excitations produced by the system yield improved detection sensitivity when applied to a structure. Integration of the system for a given set of parameters yields a time series that can be used to activate the structure and the wide variety of dynamic behavior — ranging from harmonic to chaotic — that can be produced by the system helps to ensure that an improved excitation can be found by the evolutionary algorithm. Each generated excitation is tested on a model of the structure in the damaged and undamaged condition and the ability to detect the presence of damage is quantified. Thus, each parameter set selected by the evolutionary algorithm yields an excitation that can be characterized by how well damage is detected given the structural response. In this manner, a portion of the space of possible excitations available to the ODE system is searched by the evolutionary algorithm and selection pressure leads to the discovery of excitations that improve damage detection.

The nature of the excitation selected by the optimization routine will naturally be tailored to the specific structure and damage condition. For example, the frequency characteristics of the structure or whether damage manifests as a reduction in stiffness or the appearance of nonlinearity are structure- and damage-specific characteristics that will affect the character of the improved excitation. In addition, the type of detection feature will influence the nature of the selected excitation. We show that a minor modification in the calculation of prediction error that takes time-indexing into account changes the nature of selected excitations from multi-harmonic to chaotic.

2. EXCITATION SOURCE, STRUCTURAL MODEL, AND EMBEDDING PROCEDURE

The optimization routine is comprised of six major elements: (1) an excitation source that can be modified to yield excitation time series; (2) a system/structure that can be tested in the baseline and damaged condition; (3) a means of conditioning or pre-processing the response data; (4) the selection and calculation of a feature from the observed data that is sensitive to the appearance of damage; (5) a metric that allows comparison between the collected features; and (6) an evolutionary algorithm that can be used to modify the excitation source given information about the fitness of a set of candidate solutions. The schematic in Figure 1 depicts the general optimization routine and includes the particular details of this study.

![Schematic of the optimization procedure](image)

Figure 1: Schematic of the optimization procedure. The excitation source used in this study is a system of ordinary differential equations governed by a vector of parameters, \( \mu \). The system is integrated and one of the state variable time series is used to excite the spring-mass system in its baseline and damaged condition. \( x_{b1} \) and \( x_{b2} \) are two baseline time series differing only by additive white noise while \( x_c \) is the damaged time series. Each time series is embedded to produce two baseline attractors and a damaged attractor. Prediction error is used to compare between the attractors and yields two nominally Gaussian distributions. Fitness is calculated from these distributions and stored along with the ODE parameter vector. Differential evolution used to search the space of parameter vectors for excitations that improve damage detection.

The general form of the ODE system (oscillator) used as an excitation source is given by

\[
\dot{z} = F(z; \mu)
\]

where the left-hand side is a vector of time derivatives of the state-variables given by \( z \). Each of the parameters given in \( \mu \) are the variables modified by the evolutionary algorithm. The particular ODE system used in this study is a generalized version of the Lorenz system.
\[
\begin{align*}
\ddot{z}_1 &= (\mu_1 z_2 - \mu_2 z_1)e \\
\ddot{z}_2 &= (-\mu_1 z_1 z_3 + \mu_1 z_1 - \mu_3 z_2)e \\
\ddot{z}_3 &= (\mu_6 z_1 z_2 - \mu_7 z_3)e
\end{align*}
\]  
(2)

where the speed parameter, \( \varepsilon \), is used to scale the frequency band of the system without changing the character of the dynamics. The speed parameter is appended to \( \mu \) and is subject to modification by the evolutionary algorithm.

Once a parameter set is selected, a time series corresponding to each of the state variables is produced by integrating the system with a fourth-order Runge-Kutta routine. In this work, a time-step of 0.001 s is used to integrate the system from an initial condition of \([0 1 0]\) for a total of 70 s and the first 30 s of the \( z_t \) time series is discarded to eliminate transients. The remaining 50 s time series is normalized by its largest absolute value to produce \( z^* \) and used to excite the model 2DOF system at the last mass as shown in Figure 2.

Figure 2: 2DOF spring mass system. \( m=0.0001, c=0.001, k_1 = k_2 = 1 \) in the undamaged case, \( k_1 = 0.97 k_2 \) in the damaged case.

Thus, for a given parameter vector an excitation, \( z^* = z^*(n), n = 1, 2, ..., N \), is used to force the system in the baseline condition yielding a vector, \( x_b \), corresponding to the first mass displacement response. Following the creation of this pristine response, two separate instances of zero-mean Gaussian noise are added to \( x_b \) to create two noise-corrupted baseline responses \( x_{b1} \) and \( x_{b2} \). That is to say, two vectors \( w_1 \) and \( w_2 \) are each created by selecting \( N \) samples from the normal distribution, \( N(0, \sigma_{\text{corrupting}}^2) \), where the corrupting variance is related to the variance of \( x_b \) and the desired signal-to-noise ratio (SNR) by

\[
\sigma_{\text{corrupting}}^2 = \frac{\sigma_{\text{signal}}^2}{\text{SNR}}
\]

(3)

where SNR = 100 for all optimizations in this work.

The two vectors \( w_1 \) and \( w_2 \) are then added to the pristine baseline to produce \( x_{b1} = x_b + w_1 \) and \( x_{b2} = x_b + w_2 \). These two baselines represent two separate observations of the system where damage has not occurred to the structure. A third time series, \( x_c \), is collected from the structure in its damaged condition and corrupted with noise to form the comparison time series \( x_c = x_d + w_3 \). Note that the variance corresponding to \( w_3 \) is calculated from \( x_d \) not \( x_b \). Once the three time series are generated, comparisons between the \( x_{b1} \) and \( x_{b2} \) as well as \( x_{b1} \) and \( x_c \) are made. Good excitations are those that improve the ability to discern between the baseline-baseline comparison and the baseline-damaged comparison for a given feature and metric.

Damage detection is achieved by embedding the excitations in state space via singular value decomposition (SVD) and using prediction error (PE) to compare between the embedded attractors. Specifically, the embedding procedure is similar to the manner described in [14] but is slightly modified as described in [13]. SVD is useful for the optimization procedure used in this work because there is no need to explicitly calculate the dimension [15] and delay [16] typically associated with the more traditional delay-embedding procedure. SVD also has the added benefit that it automatically reduces the noise on the signal and normalizes the response amplitude by scaling out the variance associated with each state space dimension when used as described in [13]. A brief overview of the procedure is provided to inform the discussion of prediction error.

Given an observed time series \( x = x(n), n = 1, 2, ..., N^* \), a delay matrix, \( X \), is created by choosing a dimension, \( M \), and a delay, \( T \) such that

\[
X(n) = \begin{bmatrix} x(n) & x(n+T) & \cdots & x(n+(M-1)T) \end{bmatrix}, \quad n = 1...N^*
\]

(4)

A singular value decomposition of the delay matrix exists such that

\[
X^T = USV^T
\]

(5)
where $U_{M \times M}$ is an orthogonal matrix of left-singular column vectors, $S_{M \times M}$ is a diagonal matrix of singular values ordered from largest to smallest, and $V_{N^* \times M}$ is a matrix composed of right-singular column vectors.

Time series are embedded in this study by using only the first $M^*$ vectors in $V$ that correspond to the deterministic portion of the data.

$$\bar{X}_{\text{scaled}} = V_{N^* \times M^*} = \overline{V}$$

where the overbar represents data associated with the deterministic component as adopted in [14] and the delay for generating $X$ is set as $T = 1$ with $M = 52$. This choice for $M$ is based on twice the delay that would best unfold the structure’s first natural frequency if it were a single tone. Specifically, the first natural frequency delay is given by $T_{nf} = f_s/(4\omega_1) \sim 25.4$ Samples for $f_s = 1000$ S/s and selecting $M = 52$ helps ensure that frequency information occurring at the time scales of the structure’s first natural frequency will be maximally unfolded in a three-dimensional space ($M^* = 3$). Once an attractor has been embedded for a given response, it can be compared to another embedded response via prediction error.

3. PREDICTION ERROR AND FITNESS

Prediction error (PE) is a measure of how well the dynamics of one attractor predict the dynamics of another attractor; if the two are similar, the dynamics on one will provide information about the dynamic evolution of the other and the prediction error will be low. If the dynamic similarity of the attractors is reduced, then the prediction error will increase. The local dynamics on a given attractor are probed by selecting a single fiducial (reference) point and some set of nearest neighbors to that point. The evolution of the selected points on the attractor can then be compared to a similar evolution of points on the comparison attractor. Repeated application of this comparison procedure provides an averaged view of how the attractors are dynamically related.

In detail, the first step in the calculation of prediction error is to select a single fiducial point, $\bar{X}_b(n = f)$, from the baseline attractor, where $f \in \{1, 2, \ldots, N^*\}$ is uniformly selected. Given the selection of this fiducial point, the Euclidean distance between the fiducial point and any other point on the attractor can be determined. Let this distance be labeled $\psi_{ij}$ where $i = f$ is the index of the fiducial point and $j \in \{1, 2, \ldots, N^*\}$ where $j \neq f$ is the index of another point, $\bar{X}_b(n = j)$, on the attractor. Let $\Psi^b_j$ be the set of all distances between the selected fiducial point with index $f$ and the other points on the baseline attractor ordered from smallest to largest. The set, $\Phi^b_j$, of $P$ nearest neighbors to the fiducial point is created by selecting the first $P$ distances from $\Psi^b_j$ corresponding to points on the attractor that also satisfy the condition $|j - f| > h$, where $h$ is the Theiler window [17] used to remove from consideration nearest neighbors that are temporally correlated to the fiducial point.

Once a neighborhood of points has been selected for a given fiducial point, each point in the neighborhood is evolved $s$ time steps along the trajectory of the attractor. In other words, if the point $\bar{X}_b(n = j)$ has been selected as a neighbor, then the time-evolved point will be $\bar{X}_b(n = j + s)$. Once each point in the set $\Phi^b_j$ has been evolved, the geometric centroid of the neighborhood is calculated to yield $c^b_j$. This procedure of gathering the time-evolved centroids of the neighborhood of points around a given fiducial point is repeated on the comparison attractor to produce a corresponding centroid $c^c_j$. The Euclidean distance between the baseline and comparison centroids is then defined as the prediction error, $\gamma_b$, corresponding to that fiducial point.

Two versions of prediction error are considered in this work and the difference between the two is in the manner of how the fiducial points selected on the baseline attractor are compared to the test attractor. The geometric method of transference uses the exact point that was selected on the baseline as the fiducial point on the comparison attractor. In practice, this point will not exist on the reconstructed attractor, $\bar{X}_{\text{scaled}}$, but this does not prevent the accumulation of neighbors from around the geometric location of the point. The temporal method of fiducial point transference simply selects the time index, $n = f$, of the fiducial point on the baseline attractor as the...
time index for the fiducial point, \( X_c(n = f) \), on the comparison attractor. We show in Section 4 that this minor change in the calculation of prediction error completely changes the nature of the excitation that should be applied to the structure for improved detection sensitivity.

Each comparison between attractors involves the accumulation of \( F \leq N^* \) fiducial points, corresponding neighborhood sets, time-evolved centroids, and ultimately, prediction errors. In other words, \( F \) samples of the underlying prediction error distribution associated with the comparison between two attractors are observed. The nature of this distribution is unknown and therefore a resample-and-average procedure is carried out on the observed samples in order to induce normality via the Central Limit Theorem. In this work, 30% of the observed samples are uniformly selected (with replacement) and averaged to yield a new sample of the new distribution and the procedure is repeated \( F = 4000 \) times to yield \( F \) samples of the nominally Gaussian distribution.

The Gaussian distribution produced for each comparison is what is ultimately used to determine the fitness of a given excitation. As mentioned in Section 2, two baseline structural responses and a damaged response are collected from the structure for each tested excitation. Thus, a comparison between the two baseline attractors will produce a baseline-baseline distribution and a baseline-damaged comparison will yield a corresponding distribution. The fitness is calculated from these distributions as

\[
\text{fitness} = \frac{(\mu_c - \mu_b)^2}{\sqrt{\sigma_c \sigma_b}} = \frac{\Delta \mu^2}{\sqrt{\sigma_c \sigma_b}}
\]

where \( \mu_c \) is the mean of the baseline-comparison distribution with a corresponding standard deviation \( \sigma_c \), and \( \mu_b \) and \( \sigma_b \) correspond to the baseline-baseline distribution. If \( \Delta \mu \) is negative before it is squared, the negative sign is kept after the squaring operation. In essence, a “fit” or improved excitation will force the two Gaussian distributions apart. The distribution of prediction error associated with the baseline-baseline comparison should be centered about a smaller average prediction error than the baseline-damaged distribution and an excitation that increases the distance between the two is more desirable than an excitation where this separation is not as pronounced.

Once the fitness has been calculated for a given excitation, it is paired with the ODE parameter vector that produced the excitation and stored for use by the evolutionary algorithm. The algorithm used in this study is differential evolution with the scaling factor \( F_s = 0.9 \) and the crossover constant \( CR = 0.5 \). The details of the algorithm are left to the references [18]. In essence, improvements in fitness are observed when the individual parameters of parameter vectors comprising a population of randomly initiated vectors are exchanged between members of the population (crossover) and randomly changed (mutated). Selection pressure from the fitness function helps to ensure that good genetic material, i.e. good parameter combinations, is maintained in the population and that the average fitness of the population increases over a certain number of generations. A generation is defined as one pass through the population where each member of the population is subject to crossover and mutation. The population was composed of 10 individual parameter vectors and 600 generations were executed for each optimization run. We take a moment to note that it is impossible to determine whether an excitation produced by an optimization run is a local or global optimum or whether it is locally optimal at all. In this regard, the optimization procedure is more of an improvement procedure although it will still be called an optimization throughout this paper.

4. RESULTS AND DISCUSSION

Five optimization runs were performed for each of the two versions of prediction error described in Section 3. While the optimization routine does not produce exactly the same excitation for each run, the selected excitations all belong to the same class of input. In previous work [19], we showed that the class of input preferred for the geometric version of prediction error is multi-tonal with two of the tones in the selected excitation being the most important: a probe-tone that is situated near one of the poles of the system transfer function in the frequency domain and an observation-tone that is situated at some multiple of the probe-tone and is of similar power. A larger change in prediction error is observed if damage forces an exchange between which of the two tones is more powerful in the structural response as shown in Figure 3b.
Figure 3: (a) Power spectral density (PSD) of a two-tone input and its relationship to the transfer function for a 2DOF structure. The effect of damage on the output PSD in the baseline and damaged condition is shown in (b). The observation tone power is greater than the probe tone power in the baseline condition and damage forces an exchange of power between the two tones. (c) A two-tone response will appear as a torus in the state space where the power of the low-frequency component governs the ring diameter and the tube diameter is governed by the high-frequency component. (d) An exchange of power in this case forces an exchange between a “spindle-torus” where the tube diameter dominates the ring diameter to a “ring-torus” where the ring diameter dominates. The excitations produced by an ODE system with optimized parameters will usually have additional tones in addition to the probe and observation tones.

An optimization run of an ODE system when geometric PE is used as a feature will usually converge to an excitation with such two-tone behavior if allowed to run for a sufficient number of generations. The convergence process of the algorithm is fairly typical for a given optimization run. First, a multi-tone or chaotic excitation is generated that allows at least the first natural frequency to dominate the response (figure 4b). In this manner information about the poles of the structure’s transfer function are encoded in the gene pool. From this point the best solutions will tend to converge to at least one pair of dominant tones with one of the pair situated near a natural frequency (usually the first resonance) and the second residing at a higher or lower frequency. Once a dominant tone pair is established, the tones will move in relation to each other until the observation tone resides close to a harmonic or sub-harmonic of the probe tone (the reason for this preference is unknown). At the same time, the probe tone might move nearer to the pole or to either side of the pole depending on which tone has more power and whether the damage has a hardening or softening effect on stiffness. Once the probe and observation tone have stabilized, the relative power of the two tones is adjusted by the algorithm such that a power exchange occurs on the output. Most optimization runs produce at least two-tone excitations although an excitation that exhibits an exchange of power is more difficult for the algorithm to find in the search space.

In this study, the best excitation produced by the five geometric prediction error optimization runs does not force an exchange of power. Nevertheless, the fitness of the excitation has been significantly improved. In this case, there are two observation tones that reside at harmonics of the probe tone: one at half the frequency of the probe tone and the other at 1.5 times the probe tone frequency. The relative power balance between these three tones is strongly represented in the state space dynamics as shown in Figure 4c and a change in power balance due to the softening spring damage condition is easily detected by prediction error.

A striking difference in the class of selected excitation is observed when temporal PE is used during the optimization: chaotic excitations are preferred. This result confirms the hypothesis that the detection feature and the type of preferred excitation are strongly coupled and demonstrates that care should be taken when choosing the excitation in an active sensing application. Figure 5d illustrates the nature of the structural response attractors when excited by the best discovered excitation from the five temporal prediction error optimizations.
Figure 4: (a) Plot of maximum population fitness vs. generation for an optimization that employs geometric prediction error. (b) A 2D projection of the chaotic input attractor for one of the early solutions produced by the optimization. The broadband frequency spectrum of the chaos allows the natural frequencies of the structure to be identified as shown by the power spectral density (PSD) plot of the baseline structural response. Eventually an excitation with at least one pair of probe and observation tones will be selected. The best solution selected during this optimization run produces a structural response that is characterized by three similar power tones as shown by the response PSD plot (d). If damage forced an exchange of which tone was dominant in the output, the fitness of the excitation would be further improved. Although damage does not force an exchange of power, the existence of two observation tones (near 5 and 15 Hz) for the single probe tone (near 10 Hz) helps to improve detection. Changes in the relative power between the two observation tones and the probe tone are easily observed in the state space (c).

Figure 5: (a) Plot of maximum population fitness vs. generation for an optimization that employs temporal prediction error. (b) Multi-tone excitations with similar power frequency components are often observed in the beginning stages of the optimization and effectively identify at least one of the structure’s natural frequencies as shown by the dominance of the tone near 10 Hz in the response power spectrum. (c) A 2D projection of the chaotic input attractor selected by the optimization routine. (d) The structural response attractors corresponding to the chaotic input as well as the power spectrum of one of the baseline responses (e).

Let the best excitations produced by the geometric and temporal optimizations be called the geometric and temporal excitations, respectively. The response attractors corresponding to each excitation are generated and
each version of prediction error is used to calculate 10 fitness values for each of the excitations. If the average of these fitness calculations is compared we find that temporal PE performs better than geometric PE on an attractor produced by the temporal excitation. As expected, the reverse is true when the geometric excitation is used. Both features perform well if the attractors were produced by the excitation corresponding to the opposite feature but they are of reduced performance. Fitness was also calculated using both features for the structural response to the typical chaotic Lorenz excitation with $\varepsilon = 100$ ($\mu_1 = \mu_2 = \sigma = 10$, $\mu_3 = \rho = 28$, $\mu_5 = \beta = 8/3$, $\mu_4 = \mu_6 = 1$). From this comparison we find that the two features have similar performance but that the fitness of the typical Lorenz excitation is significantly diminished relative to the tailored inputs. The PE histograms corresponding to the lowest of the 10 fitness calculations for each feature—excitation combination are provided in Figure 6.

We hypothesize that the reason why chaos is preferred for temporally-optimized inputs is because time dependences are explicitly captured by the feature. In this formulation, the sensitive dependence on initial conditions that characterizes a typical chaotic signal is thought to help with the observation of change in the system. Small changes in the stiffness of the structure will lead to significant changes in structural response. Temporal PE increases as trajectories that formerly evolved to similar locations in the state-space begin to diverge significantly due to damage. On the other hand, a chaotic response may appear more noise-like to a feature that only captures geometric relationships as the noise-induced trajectory changes may be on the order of damage-induced changes without temporal information to provide reference to an initial condition on the trajectory.

The reduction in fitness when temporal PE is used on the geometric excitation is not as pronounced as the reduction when geometric PE is used on the temporal excitation. The small but noticeable reduction in fitness when temporal PE is used on a geometric excitation is probably due to one reason: when temporal PE is used, the comparison neighborhood will always be gathered from a region on the comparison trajectory that is similar to the region on the baseline trajectory where the neighborhood was collected. Changes in frequency change the path of the trajectory in the state space, but the change is often not significant in local regions of the state space and even large changes effectively act as phase shifts. Comparison neighborhood evolution will either lead or lag baseline neighborhood evolution with the addition of some offset. This type of error will be on the same order.

Figure 6: Baseline-baseline and baseline-damaged prediction error histograms calculated for various excitation-feature combinations. Each histogram is nominally Gaussian due to the resample-and-average procedure described in Section 3. In addition, each histogram has been normalized by the largest prediction error value in the baseline-damaged histogram. The fitness values shown in the figures reflect this normalization. There are 4000 PE values for each histogram and each abscissa is composed of 500 bins. (a), (b), and (c) have been calculated using temporal prediction error and (d), (e), and (f) have been calculated with geometric prediction error. The best excitation produced out of the five optimization runs that employed geometric prediction error was used to excite the system for figures (b) and (e). The best temporal excitation was used for (c) and (f). The typical Lorenz excitation was used for (a) and (d).
whether temporal or geometric PE is used to observe the attractors. In some regions of the state space, however, the local trajectory of the baseline attractor is evenly positioned between two damaged trajectories. When a comparison neighborhood corresponding to a baseline fiducial point in this region is accumulated, the neighbors may be selected from either of the trajectories closest to that point. In these cases, the prediction error will be significantly higher as neighbors evolve to completely different regions of the state space than they would have if they had been gathered from a fiducial point anchored to a single trajectory.

Another question raised by the preference for chaotic excitations is what differentiates one chaotic excitation from another. Part of the explanation for the difference seems to be related to the frequency structure of the excitation: the top two temporal optimizations have produced excitations where the power applied near the first natural frequency is minimized. The third-best excitation applies the most power near the first natural frequency yet has significantly reduced fitness compared to the top two excitations (58.8 versus 180.6 and 157.0 when normalized with the procedure described in Figure 6). Minimization of power at the first natural frequency was observed by the authors when band-limited Gaussian excitations were optimized in [19]. Excitation fitness was increased when spectral power was reduced in the vicinity of the first natural frequency while the total power of the excitation was held constant. In the chaotic case, the algorithm may be trying to produce an excitation where the first and second natural frequencies are of similar power and dominate the response. If so, the natural frequencies might act like a pair of probe and observation tones with relative power between the tones significantly affecting the dynamics.

5. CONCLUSION

This study builds on previous work that has shown that excitations can be tailored to a structure such that damage discrimination is improved on the output. Such excitations will be specific to the structure and type of damage used during the evolutionary optimization procedure. We show that the type of damage detection feature will significantly affect the class of excitation that should be applied to a structure for improved detection sensitivity. Multi-tone excitations with at least one pair of probe and observation tones are preferred when geometric prediction error is used and chaotic excitations are preferred for temporal prediction error.

Geometric prediction error offers the best discrimination out of all the excitation-feature combinations when used with the geometric excitation. The temporal excitation—while not as powerful as the geometric excitation—also offers significant detection sensitivity with the added benefit of a broadband frequency spectrum. The broadband nature of a chaotic input should allow for lower model fidelity during the optimization because exact natural frequencies are not required from the optimization model. The geometric optimization, however, requires better model fidelity because the resultant excitation is sensitive to how closely probe and observation tones reside to natural frequencies. In addition, broadband inputs should be preferred purely from the perspective of exciting the entire response bandwidth. The broadband frequency structure coupled with the deterministic aspect of chaos helps to improve discrimination and temporal prediction error is a feature capable of leveraging this determinism.

Temporal prediction error can be used with an excitation that has been optimized for geometric PE without significantly reducing discrimination capability. On the other hand, discrimination capability is significantly reduced when geometric PE is employed on a temporal excitation. While geometric PE is capable of discerning damage when a chaotic input is employed it does not perform nearly as well as the temporal version. Both features lose discriminating power when an un-tailored Lorenz excitation is used to excite the structure. Even with an un-tailored Lorenz input, temporal PE performs better than geometric PE performs with the temporal excitation; illustrating the importance of temporal information in a feature when chaotic inputs are used.

In short, excitations can be tailored to a structure for improved damage detection. The improvement in damage discrimination capability is significant; however, the optimization procedure used to produce the improved inputs is dependent on the structural model, damage type, and the detection feature. Care should be taken when selecting a feature to use for the optimization.

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