ABSTRACT

A robust vibration absorber reduces vibrations of a system over a range of frequencies. Passive vibration absorption is achieved with a tuned mass and spring (known as a dynamic vibration absorber (DVA)). This is a simple, low-cost method of reducing vibrations at a single frequency; however, in order to achieve robust absorption a heavy mass is required. Active vibration control is achieved with an actuator capable of applying a force to the system at any range of frequencies, but this method is expensive. Combining active and passive absorbers allows for robust absorption with minimum passive absorber mass. Various methods of combined absorption have already been proposed for systems with a single resonant frequency.

This paper investigates hybrid robust vibration control of a primary system with frequency components which are the first three harmonics of the excitation of the system. The hybrid absorber consists of a three mass-spring system. Passive absorption is achieved with three masses and joining springs, the active absorption is modelled as springs in parallel with the passive springs but with variable stiffness. Variations in frequency harmonics are proportional; absorption is maintained at a range of frequencies by altering the active spring stiffnesses.

1. INTRODUCTION

A dynamic vibration absorber (DVA) is a simple mass-spring system. When a primary system is subjected to a harmonic force, its vibration at a single frequency can be passively absorbed by attaching a tuned DVA [1]. The presence of the DVA causes an anti-resonance in the frequency response of the coupled system at the tuned resonant frequency of the absorber. The frequency range either side of the anti-resonant frequency where the primary system has a low response increases as the absorber mass increases, therefore in order to obtain a robust enough vibration absorber to provide vibration absorption in a finite frequency range, a large absorber mass is required which creates a heavy, undesirable system [2]. Robust vibration absorption is required in a number of situations, mainly where the excitation frequency of the primary system is variable, but also if the properties of the absorber system cannot be developed accurately or alter due to operating conditions causing an unpredictable absorber resonant frequency.

Active vibration absorption is provided in the form of actuators, which can include piezoelectric actuators, magnetostrictive actuators, electric motors or pneumatic springs, in conjunction with a control system. The active absorber can, theoretically, exert forces on the primary system at any range of frequencies so absorption is provided regardless of external factors which may affect system properties or operating speed. This type of absorption requires complex, expensive actuator and control arrangements [3].

Combining active absorption provision with a passive DVA enables a simple, low-cost, robust absorber system to be developed. The combined absorber is capable of operating at a range of frequencies, without the need for a heavy absorber mass, and without the expense incurred from full active control [4].

Principles of vibration absorption can also be extended to consider absorbing the response of a primary system due to excitation forces with multiple frequencies. This is a concept that is useful in many situations as machinery
often experiences excitations that comprise of a fundamental frequency as well as integer harmonics of that frequency.

Combined active and passive robust absorption has been considered by Kidner et al. [5] who used a beam-like structure as a vibration absorber mounted on a primary system. Swept sinusoidal excitation was applied to the primary system initially at the passive tuned resonant frequency of the absorber; this was altered to equal the resonant frequency of the primary system. Robust absorption was provided by changing the beam geometry and consequently altering the beam stiffness and resonant frequency, fuzzy logic control enabled this geometry change to track the excitation frequency.

Hill et al. [6] developed a passive absorber which absorbed the response of the primary system at three excitation frequencies and in a small frequency range either side of these frequencies. The absorber consisted of two rods which each supported a mass either side of a centre section, mounted on the primary system. This configuration displayed six distinct, predictable mode shapes. The mass, material properties, rod diameters and lengths were tuned so that two absorber resonant frequencies occurred either side of each of the three excitation frequencies. This resulted in a passive absorber system which was slightly more robust than an absorber system would be if three absorber resonant frequencies were tuned exactly to the primary system excitation frequencies. Active absorption was attempted to enable absorption of a wider range of frequencies than the passive device alone would enable. Adaptive absorption was achieved with a stepper motor which wound the masses in and out and therefore changed the resonant frequencies of the absorber. In practice the presence of the motor increased the number of absorber resonances and decreased the effectiveness of the absorber at the required frequencies.

May et al. [7] developed an adaptive DVA to absorb the first, second, and third blade pass frequencies (BPFs) in a turboprop aircraft. The frequencies in question were 100Hz, 200Hz and 300Hz, the absorber was also required to exert 29N, 18N and 10N on the primary system at the three BPFs respectively. The authors developed an absorber with passive resonant frequency tuned to 100Hz; the fundamental frequency. The absorber acted actively over the range 50Hz to 400Hz using an active magnetostrictive rod and lever system.

This paper develops a multi degree of freedom (MDoF) DVA which satisfies the absorption requirements introduced in paper [7]: Absorption of the primary system is provided at the fundamental excitation frequency, 2nd harmonic and 3rd harmonic; nominally 100Hz, 200Hz and 300Hz. Modal participation factors are 29, 18 and 10 at the three excitation frequencies respectively.

In Section 2 of this paper the absorption and force requirements at all three excitation frequencies are provided by a 3 degree of freedom (3DoF) passive DVA. In Section 3 the concept of hybrid passive and active MDoF absorption is used. A static, active spring is used to provide vibration absorption when the three excitation frequencies vary in proportion to each other. This is effective over a small frequency range around the three nominal frequencies. Section 4 illustrates the characteristics and limitations of the active spring. Section 5 introduces a dynamic, active spring to provide robust vibration absorption over a wide frequency range around the three nominal frequencies when the frequencies vary in proportion to each other.

2. MULTI DEGREE OF FREEDOM PASSIVE ABSORPTION

A Three DoF DVA is developed to satisfy the following requirements:

- The first, second and third resonant frequencies occur at 100Hz, 200Hz and 300Hz respectively.
- The sum of the three masses is 1kg.
- The modal participation factors for mass3 are 29 in mode 1, 18 in mode 2 and 10 in mode 3.

The modal participation factors represent the force exerted by the absorber on the primary system in each of the three mode shapes of the absorber. These values are taken from the mass normalised mode vectors of each mode for mass 3.

The equations of motion for this system are:
\[ [M_0] \ddot{x} + [K_s] x = f \]

\[ x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad [K_0] = \begin{bmatrix} K_{12} & -K_{12} & 0 \\ -K_{12} & K_{12} + K_{23} & -K_{23} \\ 0 & -K_{23} & K_{23} + K_{3} \end{bmatrix} \quad f = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} \quad [M_0] = \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix} \]

The 3DoF DVA is shown in Figure 1.

MatLab code was developed to solve this problem; the code required estimated values for the unknown parameters; \( M_1, M_2, M_3, K_{12}, K_{23}, K_3 \). The eigenvalue problem was solved with these values. The residuals of the calculated modal participation factors and resonant frequencies against the required values were found. The sensitivity of the residuals to changes in initial values was calculated and from this information numerical differentiation was carried out, which enabled values of the required parameters to be found to high precision.

The absorber system parameters were found to be:
\[ M_1 = 0.0040606 \text{ kg} \quad M_2 = 0.161353 \text{ kg} \quad M_3 = 0.798040 \text{ kg} \]
\[ K_{12} = 82,355.016 \text{ Nm}^{-1} \quad K_{23} = 329,265.970 \text{ Nm}^{-1} \quad K_3 = 427,111.991 \text{ Nm}^{-1} \]

Figure 2; the frequency response function for this system shows that the resonant peaks of the absorber system are extremely narrow, if this system was attached to a primary system, vibrations of the primary system would be absorbed only on the exact resonant frequencies of the absorber. A slight deviation in excitation frequency away from these frequencies could result in no absorption of the vibrations of the primary system or may even cause the system to vibrate at resonance; a case much worse than if no absorber were present.

\[ \text{Figure 2: Three Degree of Freedom DVA} \]

3. ROBUST HYBRID MULTI DEGREE OF FREEDOM VIBRATION ABSORPTION

This section illustrates how robust vibration absorption can be achieved by combining active vibration provision with the passive absorber system.

In a situation where a primary system experiences excitation which consists of a force with a fundamental frequency and integer harmonics of that frequency; variations in the fundamental excitation frequency cause proportional changes in the frequencies of the other harmonics.
A three mass-spring system must be developed to provide robust vibration absorption at and around all three nominal resonant frequencies. The three harmonics of excitation frequency of the primary system can vary in proportion to each other. The absorber maintains absorption of the resulting vibration of the primary system.

It is not possible for a mass-spring system to comply with the requirements that the resonant frequencies vary in proportion to each other and also maintain the modal participation factors required in Section 2.

For the absorber system to achieve robust absorption while complying with the constraint that the sum of all masses must be equal to 1kg; Passive absorption is required at the fundamental excitation frequency of the primary system and at the second and third harmonics. A small amount of active provision combined with the passive system enables the absorber properties to vary. Therefore the resonant frequencies of the absorber also vary. Frequency variations can be tuned to track the excitation frequencies of the primary system.

The active provision takes the form of a spring; the stiffness of this spring is variable by a factor \( \sigma \). The overall stiffness of the active spring is denoted by \( \sigma dk \). This `active' spring is placed in parallel with spring \( K_3 \), which joins mass 3 to the primary system.

The product \( \sigma dk \) represents the `active' spring stiffness, \( dk \) is constant, \( \sigma \) is variable. When \( \sigma = 0 \), the resonant frequencies of the absorber are the nominal frequencies.

Altering the stiffness of the active spring alters the resonant frequencies of the absorber system; increasing the stiffness increases the resonant frequencies, decreasing the stiffness decreases the resonant frequencies. Therefore, variations in excitation frequency can be tracked by altering the active stiffness of the hybrid absorber system and vibration suppression can be maintained.

To comply with the requirement that all three resonant frequencies vary in proportion to each other, they must vary linearly with respect to \( \sigma \). The rate of change of the natural frequencies with respect to \( \sigma \) is therefore:

\[
\frac{df_1}{d\sigma} = 1, \quad \frac{df_2}{d\sigma} = 2, \quad \frac{df_3}{d\sigma} = 3,
\]

(2)

This is the equivalent of

\[
\begin{bmatrix}
U_1 & 0 & 0 \\
0 & U_2 & 0 \\
0 & 0 & U_3
\end{bmatrix}
\begin{bmatrix}
K_1 & 0 & 0 \\
0 & K_1 & 0 \\
0 & 0 & K_3
\end{bmatrix}
\begin{bmatrix}
U_1 & 0 & 0 \\
0 & U_2 & 0 \\
0 & 0 & U_3
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 0 \\
0 & 4 & 0 \\
0 & 0 & 9
\end{bmatrix}
\]

where \( [K_1] = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & dk
\end{bmatrix} \) (3)

U, U2 and U3 are mass normalised mode shape vectors. Equation (3) also gives the modal participation factors of the system designed in Section 2. The derivation of equation (3) is carried here:

Take \([K]\) and \([M]\) to be the mass and stiffness matrices of the hybrid passive and active system:

\[U^T[K]V = [\lambda], \text{ where } [K] = [K_o] + \sigma[K_1]. \quad U^T[M]V = [I], \text{ where } [M] = [M_o] + \sigma[M_1].\]  

(4a), (5a)

U and V are mass normalised mode shape vectors and \([\lambda] = \begin{bmatrix}
\lambda_1 & 0 & 0 \\
0 & \lambda_2 & 0 \\
0 & 0 & \lambda_3
\end{bmatrix}, \quad [I] = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}\].

Differentiate (4a) and (5a) with respect to \( \sigma \) and indicate this differentiation with dot notation:


(4b), (5b)

Define \( B = V^{-1}\dot{V} \) and \( A = U^{-1}\dot{U} \), (4b) and (5b) become: \( A^T\lambda + U^TK_1V + \lambda B = \dot{\lambda}, \quad A^T + U^TM_1U + B = 0 \)  

(4c), (5c)

(4c) – \( \lambda_1(5c) \) simplifies to: \( U^T[K_1]V = [\ddot{\lambda}] \)

(6)

So if the rate of change of frequencies \( f_1, f_2 \) and \( f_3 \) with respect to \( \sigma \) are 1, 2, and 3; the rate of change of eigenvalues \((\lambda_1, \lambda_2 \text{ and } \lambda_3)\) are 1, 4 and 9. Substitute this information into (6) to get the result shown in (3).
Matlab code from section 2 was adapted to find a system that had:

- Resonant frequencies of 100Hz, 200Hz and 300Hz when $\sigma = 0$;
- $M_1 + M_2 + M_3 = 1$ kg;
- $\frac{U_1^T[K_1]U_3}{U_1^T[K_1]U_1} = \frac{9}{1}$ and $\frac{U_2^T[K_1]U_2}{U_1^T[K_1]U_1} = \frac{4}{1}$

The resulting absorber parameters are:

- $M_1 = 0.194363$ kg
- $M_2 = 0.333333$ kg
- $M_3 = 0.472303$ kg
- $K_{12} = 179,040.507$ Nm$^{-1}$
- $K_{23} = 435,068.433$ Nm$^{-1}$
- $K_3 = 870,135.921$ Nm$^{-1}$
- $d_k = 0.944606$ Nm$^{-1}$

This section searches for a linear solution to a non-linear problem, therefore the resulting system provides absorption which is approximate to the requirements.

Figures 3, 4 and 5 show the first, second and third resonant frequencies as $\sigma$ changes. The dashed lines show the required frequencies and the solid lines are the resonant frequencies of the actual system with the above parameters.

<table>
<thead>
<tr>
<th>Deviation</th>
<th>$f = 100$Hz</th>
<th>$f = 200$Hz</th>
<th>$f = 300$Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>f - 0.1%</td>
<td>$3.1031 \times 10^{-4}$</td>
<td>$1.1061 \times 10^{-4}$</td>
<td>$-3.6703 \times 10^{-5}$</td>
</tr>
<tr>
<td>f + 0.1%</td>
<td>$6.6933 \times 10^{-4}$</td>
<td>$4.7403 \times 10^{-4}$</td>
<td>$-1.6650 \times 10^{-5}$</td>
</tr>
<tr>
<td>f - 1%</td>
<td>$-0.04410$%</td>
<td>$-0.06249$%</td>
<td>$-0.10310$%</td>
</tr>
<tr>
<td>f + 1%</td>
<td>$-0.04860$%</td>
<td>$-0.07003$%</td>
<td>$-0.12740$%</td>
</tr>
</tbody>
</table>

Table 1: Error in Resonant Frequency with Deviation from Nominal Frequency
As the absolute value of \( \sigma \) increases; the error in system resonant frequencies increases. Table 1 shows that when the required frequency deviates by 0.1\% from the nominal frequency, the error in resonant frequency of the designed system is only of magnitude 1x10\(^{-6} \)\%. When the deviation in resonant frequency is at 1\% the magnitude of error in resonant frequency of the designed system is much larger; between 0.04\% and 6.7\%.

The results of this section show that robust vibration absorption is possible for a multi degree of freedom system with combined active and passive absorption, however all resonances cannot be controlled exactly proportionally with a single active spring for large variations from the nominal frequencies. Modal participation factors cannot be constrained in this case.

4. PROPERTIES OF A THREE DEGREE OF FREEDOM SYSTEM WITH AN ACTIVE SPRING

The resonant frequencies of the absorber system against values of the active spring denoted by \( K_C \) are plotted in Figure 6 and Figure 7. Here, \( K_C \) is equal to edk in Section 3. These graphs can be separated into regions; in each region the response of the system is different and can be explained:

- \( K_C << -K_3 \)
  For large negative values of active stiffness, \( K_C \), the value of the equivalent stiffness \( K_3 + K_C \) is near zero. The first resonant frequency goes to zero and the system behaves like a 2DoF system, the second and third resonant frequencies are at their lowest values.

- \( K_C = -K_3 \)
  The response of the system changes abruptly from a 2DoF to a 3DoF system.

- \(-K_3 < K_C < K_3 \)
  This is a wide region where changes in \( K_C \) have little effect on the response of the system because the magnitude of \( K_C \) is much lower than the other spring stiffnesses.

- \( K_C = K_3 \)
  As the magnitude of \( K_C \) reaches the magnitude of the other springs in the system, the rate of change of frequencies with respect to \( K_C \) is high. Veering occurs: the mode shapes related to the second and third resonant frequencies switch over.

- \( K_C >> K_3 \)
  The third natural frequency goes to infinity with further increase in \( K_C \). Infinite \( K_C \) is the equivalent of a rigid rod and so the system behaves like a 2DoF system again. Further increases in \( K_C \) have no effect on the 1\(^{st} \) and 2\(^{nd} \) resonant frequencies. The 1\(^{st} \) and 2\(^{nd} \) resonant frequencies in this region are at their maximum.

![Figure 6: Resonant Frequencies v K_C, negative values of K_C](image)

**x**: Frequency at the point where \( K_C = -(K_{23} + K_3) \)

![Figure 7: Resonant Frequencies v K_C, positive values of K_C](image)

**x**: Frequency at the point where \( K_C = K_{23} + K_3 \)
The resonant frequencies occur in the following ranges:

\[ 0 \text{Hz} < f_1 < 177.16 \text{Hz}, \quad 177.17 \text{Hz} < f_2 < 290.87 \text{Hz}, \quad 290.87 \text{Hz} < f_3 < \alpha \text{Hz} \]

The resonant frequency limits affect the effectiveness of this absorber system for use with a primary system which has excitation at a fundamental frequency and harmonics of this frequency that vary in proportion to each other.

The lower limit of the 3\textsuperscript{rd} resonant frequency is 290.87Hz this is 3.04% of the nominal frequency. Decreases of 3.04% of the 2\textsuperscript{nd} and 1\textsuperscript{st} resonant frequency are 193.91Hz and 96.96Hz respectively. The upper limit for the whole system is given by the upper limit of the second resonant frequency, which is 290.87Hz. This is 45.44% of the nominal frequency. So, for the 1\textsuperscript{st}, 2\textsuperscript{nd} and 3\textsuperscript{rd} resonances, the absorber can provide vibration absorption from 96.96Hz to 145.44Hz, from 193.91Hz to 290.87Hz and from 290.87Hz to 436.32Hz respectively.

5. DYNAMIC ACTIVE ABSORPTION PROVISION

In Section 3 the force exerted by the passive and active springs \( K_3 \) and \( \sigma dk \) on the primary system is proportional to the extension of the springs. The active spring is described as static because the frequency of the system has no effect on the force that the active spring produces. This section uses a frequency dependent active spring. So for different modes, a different magnitude of force can be exerted by the spring.

The requirements of the combined passive and active absorber system in this section are that within the frequency limits found in Section 4, the resonant frequencies vary in proportion to each other.

The values of mass and the stiffness of the other springs are the same as in Section 2. When no active provision is exerted, the resonant frequencies of the system are 100Hz, 200Hz and 300Hz and the modal participation factors of mass three are 29, 18 and 10.

The addition of the frequency dependent spring adds two non-physical DoFs to the absorber system. The two DoFs have an inertia represented by 1kg in the mass matrix of the current system \([M_2]\). The stiffness matrix of the passive system is the same as \([K_0]\) in Sections 2 and 3 but with two extra DoFs populated with zeros. The stiffness matrix for the active spring is \([K_1]\).

\[
[M_2] = \begin{bmatrix}
M_1 & 0 & 0 & 0 & 0 \\
0 & M_2 & 0 & 0 & 0 \\
0 & 0 & M_3 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}, \quad [K_0] = \begin{bmatrix}
K_{12} & -K_{12} & 0 & 0 & 0 \\
-K_{12} & K_{12} + K_{23} & -K_{23} & 0 & 0 \\
0 & -K_{23} & K_{23} + K_3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}, \quad [K_1] = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & K_C & K_{AC} & K_{BC} & 0 \\
0 & K_{AC} & K_A & 0 & 0 \\
0 & K_{BC} & 0 & K_B
\end{bmatrix}
\]

The overall stiffness of the system \([K_2]\) is: \([K_2] = [K_0] + \sigma[K_1]\)

The equation of motion for this system in the frequency domain is: \( \begin{bmatrix} [K_2] - [M] \omega^2 \end{bmatrix} \begin{bmatrix} x \end{bmatrix} = F \), \( (7) \)

\[
x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_A \\ x_B \end{bmatrix}, \quad F = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ 0 \\ 0 \end{bmatrix}
\]

The presence of the extra DoFs within the frequency dependant spring enables the force that the spring exerts to alter for the three different mode shapes.

The static equivalent of this spring is the spring \( K_C \) used in Section 4 (now denoted \( K_{SC} \)). The relationship between the static and frequency dependent springs can be found by considering the force exerted. For the static spring the force exerted is:

\[
F_3 = K_{SC} x_3
\]

For the frequency dependent spring, the static force exerted is found from the 3\textsuperscript{rd} line in equation (7) when \( \omega = 0 \):

\[
F_3 = K_{SC} x_3
\]
\[ K_C x_1 + K_{AC} x_A + K_{BC} x_B = F_3 \]  
(9)

For the frequency dependent spring, lines 4 and 5 in equation (7) expand to give:

\[ x_A = -\frac{K_{AC} x_3}{K_A - \omega^2}, \quad \text{and} \quad x_B = \frac{K_{BC} x_3}{K_B - \omega^2} \]  
(10)

Substituting (10) back into (9) gives:

\[ F_3 = \left[ \frac{K_C}{K_A - \omega^2} - \frac{K_{AC}^2}{K_A - \omega^2} - \frac{K_{BC}^2}{K_B - \omega^2} \right] x_3 \]  
(11)

The three terms in the brackets of equation (11) are the equivalent of the static spring stiffness \( K_{SC} \), equation (7).

The values of \( K_A \) and \( K_B \) represent the eigenvalues of the controller; the location of the controller resonant frequencies in the frequency response of the absorber determines the contribution from the active spring. For this example the values of \( K_A \) and \( K_B \) are set as linear functions of the fundamental excitation frequency \( f_{ex} \) of the primary system. The values of \( K_C, K_{AC} \) and \( K_{BC} \) are found so that the requirements of the absorber system are met. The functions of \( K_A \) and \( K_B \) are:

\[ K_A = 10779 f_{ex} + 265459, \quad K_B = 56209 f_{ex} - 2110722 \]  
(12), (13)

Figure 8 shows the values of \( K_C, K_{AB} \) and \( K_{BC} \) as excitation frequency changes. The above values for \( K_A, K_B \), combined with values of \( K_C, K_{AC} \) and \( K_{BC} \) give resonant frequencies of the absorber system as shown in Figure 9. The errors in absorber resonant frequency at the limits of operation are given in Table 2. Table 2 clearly shows that the frequency dependent spring enables very accurate control of the absorber.

<table>
<thead>
<tr>
<th>Deviation</th>
<th>( f = 100 \text{Hz} )</th>
<th>( f = 200 \text{Hz} )</th>
<th>( f = 300 \text{Hz} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f - 2% )</td>
<td>( 3.25849 \times 10^{-12} % )</td>
<td>( -1.23565 \times 10^{-11} % )</td>
<td>( -5.36907 \times 10^{-3} % )</td>
</tr>
<tr>
<td>( f + 44% )</td>
<td>( 5.92119 \times 10^{-6} % )</td>
<td>( -7.89492 \times 10^{-2} % )</td>
<td>0 %</td>
</tr>
</tbody>
</table>

Table 2: Error in Resonant Frequency with Deviation from Nominal Frequency

As the excitation frequency deviates from the nominal frequency, Figure 8 shows that the magnitude of control stiffness that is required for the absorber to track the frequency increases. At the upper limit of excitation frequency that can be absorbed, a very large value of \( K_C \) is required.

![Figure 8: Controller Parameters vs Fundamental Excitation Frequency](image1)

![Figure 9: Absorber Resonant Frequency](image2)

The magnitude of the mass and stiffness in the absorber system is linked to the cost of the absorber; heavier systems may be more expensive to run or may be unacceptable in the design requirements of the primary system.
Actuator cost can increase with frequency range. This is because actuator cost is related to reactive power. At the extremes of excitation frequency, in this example, actuator cost may be extremely high.

Reactive power is defined as: Reactive power = spring force x relative velocity \hspace{1cm} (14)
Which is also: Reactive power = spring stiffness x (relative displacement)^2 x frequency \hspace{1cm} (15)

Potential further development of the idea of hybrid absorption could be for a semi-active spring to be introduced into the system, in such a way that the change in fundamental excitation frequency could be tracked by altering a mechanical parameter. Only the higher harmonics would then require active control for total vibration suppression.

6. CONCLUSIONS

This paper has investigated multi degree of freedom absorption. For a passive system, the frequencies that require absorption and the force requirements at these frequencies can be achieved. However, to maintain minimum mass requirements, robustness is unattainable.

Introducing a static active spring enables system robustness. However, maintaining force requirements at frequencies away from the passively absorbed frequencies is not possible. Varying all absorber resonant frequencies in proportion to each other is possible for small deviations from the nominal frequencies.

The stiffness values of a frequency dependent active spring can be tuned so that absorber resonant frequencies can vary in proportion to each other over a wide frequency range. At the nominal, passively absorbed frequencies modal participation factors can be set.

7. REFERENCES