Structural Modifications with Additional DOF - Applications to Real Structures

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Nomenclature

\[ D \] Dynamic stiffness matrix
\[ K \] Stiffness matrix
\[ M \] Mass matrix
\[ H \] Structural damping matrix
\{ f \} Force vector
\{ x \} Displacement vector
\[ \alpha \] Receptance matrix

Subscripts

0 Original structure
mod Modifying structure

ABSTRACT
In structural dynamics, it is important to obtain the dynamic properties of a modified structure from those of the original structure, especially for large systems. In this study, an effective structural modification method for modifications with additional degrees of freedom is applied to a real structure with distributed modifications in order to investigate the performance of the method. In this method, which has been proposed in an earlier study by one of the authors of this paper, the frequency response functions (FRFs) of the modified structure are calculated from those of the original system and system matrices of the modifying structure. The performance of the method is investigated by applying it to GARTEUR SM-AG19 model with modifications in the form of beams attached under the wings acting as stiffeners causing flexural rigidity. The receptances calculated by using the structural modification method are compared with measured ones. A very good agreement is observed between predicted and measured results, and it is concluded that the structural reanalysis method proposed can be successfully and efficiently used for structures with distributed modifications.
1. INTRODUCTION

During the design of all mechanical structures it is required to fulfill certain mechanical criteria. However, any modification applied to a structure has an effect of changing the structural properties such as resonant frequencies, mode shapes and deformation distribution. When, for instance, an aircraft is modified by attaching external payload to it, the dynamic behavior of the aircraft changes and this change can be critical as it may cause serious vibration problems; hence, dynamic behavior of the modified aircraft has to be predicted in the design stage. In order to predict the dynamic behavior accurately, the finite element (FE) model of the modified structure can be constructed. However it may be very difficult and time-consuming to construct a new FE model for every modification. Therefore it will be more practical to predict the dynamic behavior of the modified structure by using dynamic response information of the original structure and dynamic data of the modifying structure.

Structural modification methods focus on the change of dynamic behavior of a structure due to modifications in mass, stiffness and damping properties of the system. Kyprianou, et al. [1] divided the structural dynamic modification problems into two categories: inverse structural dynamic modifications and direct structural dynamic modifications. Direct structural dynamic modification concentrates on the determination of modified structure characteristics due to modification on the original structure. Conversely, inverse structural dynamic modification is an optimization procedure looking for necessary modifications in order to achieve the desired dynamic behavior. Kyprianou, et al. [1, 2] focused on inverse structural dynamic modification. Li and He [3] presented a new approach for structural modifications required to change the dynamic characteristics of an undamped system. Furthermore, Park [4, 5] studied measured frequency response function based inverse structural dynamic modification in order to obtain necessary structural modifications. In a later work, Mottershead, et al. [6] presented an inverse method for assigning natural frequencies and nodes of normal modes of vibration by the addition of grounded springs and concentrated masses. In the direct structural dynamic modification research area different studies were conducted on lumped and distributed structural modifications with or without additional degrees of freedom (DOF). For lumped modification problems, Özgüven [7] proposed a matrix inversion method in order to find receptances of locally damped structures from those of the corresponding undamped structure. Later a recursive solution algorithm was presented in order to avoid the matrix inversion [8]. In a further work [9], Özgüven generalized this approach for reanalyzing a structure subjected to structural modification with or without additional DOF. In this method, the exact FRFs of the modified structure are calculated by using FRF matrix of the original system and mass, stiffness and damping matrices of the modifying structure [9]. Şanlitürk [10] used the same approach, but avoided matrix inversion by employing Sherman-Morrison method. However, this approach, like many others in this category, is for modifications without additional DOF. In the direct structural dynamic modification, the number of studies on distributed structural modification problems is limited due to the difficulties in coupling continuous modifying structures with a structure. D’Ambrogio [11] studied the prediction of frequency response function of the modified structure subjected to modification in the form of rib and plate stiffeners causing flexural rigidity change and presented quasi-local characteristics of the additional dynamic stiffness matrix due to structural modification. Since dynamic properties of an original structure are identified by experimental techniques containing only translational DOF due to the difficulties in measuring rotational DOF, and structural information of modifying structure contains both rotational and translational DOF, reduction or expansion techniques have to be applied in order to obtain consistent dynamic properties for both of the structures. In a later work, D’Ambrogio and Sestieri [12-14] proposed a modeling approach for the distributed modifications and introduced different techniques to have consistent degrees of freedom for original and modifying structures. The difficulty introduced by rotational DOF in structural dynamics modification has been investigated by different researchers. Avitabile, et al. [15] developed and presented a different technique to determine rotational DOF to be used in structural dynamic modification problems. Hang, et al. [16] focused on the distributed structural dynamics modification with additional DOF by using the original relationship developed by Özgüven [9] and modeling method of the distributed modification developed by D’Ambrogio and Sestieri [12-14].

In this paper, using the original formulation of Özgüven [9], an approach is presented for predicting the dynamic response of a structure with distributed modifications from the response of the original structure itself and dynamic flexibility matrix of the modifying structure. In this approach the frequency response function of the original structure can be obtained either experimentally from modal testing or theoretically by using finite element method (FEM), and the modifying structure is modeled in such a way that consistent DOF are present at the connection nodes. The method proposed is validated by different case studies. The effect of modal truncation made in calculating FRFs of the original structure on the accuracy of the predicted FRFs is also investigated. In order to demonstrate the performance of the method when used for real structures, the scaled aircraft test structure
GARTEUR SM-AG19 [17] is modified by attaching beams acting as stiffeners under the wings, and theoretically calculated FRFs are compared with experimentally measured ones.

2. THEORY

2.1. Modeling Approach for Distributed Modifications

In a structural modification problem the additional dynamic stiffness matrix due to structural modification is given by

\[
\Delta D = [D] - [D_0]
\]

where \([D]\) and \([D_0]\) are the dynamic stiffness matrices of the modified and original structures, respectively. For lumped modifications \([\Delta D]\) corresponds directly to dynamic stiffness matrix of the modifying structure. However, for distributed modifications, it has to be calculated by using Eq. (1) which may not correspond to the dynamic stiffness matrix of the modifying structure. In order to apply Eq. (1), the dynamic stiffness matrices of the original and modified structures should be available. This requires the computation of the FE models for original and modified structures. However, if such FE models were available, the advantage of using structural modification method would be limited. D’Ambrogio and Sestieri [12] overcame this drawback by using quasi-local characteristics of additional dynamic stiffness matrix due to structural modification, \([\Delta D]\). In order to obtain the additional dynamic stiffness matrix due to structural modification, bounded region which covers the modifying area is modeled for both original and modified structures.

The approach proposed by D’Ambrogio and Sestieri has two drawbacks which are due to the inaccurate modeling of the structure and quasi-local characteristics of the additional dynamic stiffness matrix due to structural modification. The former error is due to not representing the modified structure accurately: Let us consider the beam in Figure 1, which is modeled using beam elements; if it is modified by adding a shorter beam on it as shown in Figure 2, as the neutral planes of the thinner and thicker parts of the modified beam will not coincide, the finite element model will not represent the real structure accurately. Modeling by using beam elements introduces certain errors independent from the quality of the mesh. D’Ambrogio and Sestieri discussed this error by modeling both original and modifying structures using beam and brick elements in the FE model [12]. The latter error directly depends on the size of the bounded region covering the modifying area. When the size of the bounded region which covers the modifying area is larger, the error introduced to \([\Delta D]\) will be smaller. In order to avoid such errors, a different approach for the application of the structural modification technique is used in this paper. If distributed modification is applied to an original structure in such a way that additional DOF is introduced, then it is not necessary to use Eq. (1) in order to calculate the additional dynamic stiffness matrix due to structural modification, as the problem will be a structural coupling problem. In that case, the additional dynamic stiffness matrix due to structural modification will be equal to the dynamic stiffness matrix of the modifying structure which can directly be used in the structural modification method.

Figure 1. Original Beam

Figure 2. Modified Beam
For instance, the dynamic stiffness matrices of the original and modifying structures for the modified beam in Figure 2 are given by

\[
[D_0] = [K_0] - \omega^2 [M_0] + i[H_0]
\]
\[
[D_{\text{mod}}] = [K_{\text{mod}}] - \omega^2 [M_{\text{mod}}] + i[H_{\text{mod}}]
\]

where \([K_0], [M_0]\) and \([H_0]\) represent stiffness mass and structural damping matrices of the original structure, and similarly \([K_{\text{mod}}], [M_{\text{mod}}]\) and \([H_{\text{mod}}]\) are stiffness, mass and structural damping matrices of the modifying structure. They all can be obtained directly from the FE models of the original and modifying beams. When additional DOF are introduced to the original structure, the dynamic stiffness matrix of the modified structure can be obtained by assembling the dynamic stiffness matrices of the original and modifying structures. Similarly, for distributed modifications, if additional DOF are introduced to the original structure there is no need to use Eq. (1); instead, additional dynamic stiffness matrix due to structural modification can directly be obtained from the FE model of the modifying structure.

### 2.2. Structural Modification Method with Additional DOF

In this section, the formulation given by Özgüven [9] is summarized. FRF matrix of a modified system can be partitioned as; DOFs which correspond to original structure only (superscript a), DOFs at connection points (superscript b), and DOFs that belong to modifying structure only (superscript c). Then the following equations can be written for original and modifying structures.

\[
\begin{bmatrix}
\alpha_0^a \\
\alpha_0^b \\
\alpha_0^c
\end{bmatrix}^{-1} =
\begin{bmatrix}
\alpha_0^{aa} & \alpha_0^{ab} & \alpha_0^{ac} \\
\alpha_0^{ba} & \alpha_0^{bb} & \alpha_0^{bc} \\
\alpha_0^{ca} & \alpha_0^{cb} & \alpha_0^{cc}
\end{bmatrix}^{-1} - [K_0] - \omega^2 [M_0] + i[H_0]
\]

\[
\begin{bmatrix}
\alpha_0^a \\
\alpha_0^b \\
\alpha_0^c
\end{bmatrix}^{-1} =
\begin{bmatrix}
\alpha_0^{aa} & \alpha_0^{ab} & \alpha_0^{ac} \\
\alpha_0^{ba} & \alpha_0^{bb} & \alpha_0^{bc} \\
\alpha_0^{ca} & \alpha_0^{cb} & \alpha_0^{cc}
\end{bmatrix}^{-1} - [K_{\text{mod}}] - \omega^2 [M_{\text{mod}}] + i[H_{\text{mod}}]
\]

\[
\begin{bmatrix}
\alpha_0^a \\
\alpha_0^b \\
\alpha_0^c
\end{bmatrix}^{-1} =
\begin{bmatrix}
\alpha_0^{aa} & \alpha_0^{ab} & \alpha_0^{ac} \\
\alpha_0^{ba} & \alpha_0^{bb} & \alpha_0^{bc} \\
\alpha_0^{ca} & \alpha_0^{cb} & \alpha_0^{cc}
\end{bmatrix}^{-1} - [K_{\text{mod}}] - \omega^2 [M_{\text{mod}}] + i[H_{\text{mod}}]
\]

where \([\alpha_0^a]\) and \([\alpha]\) represent the receptance matrices of the original and modified structure, respectively. Pre-multiplying Eq. (4) by

\[
\begin{bmatrix}
\alpha_0^a \\
\alpha_0^b \\
\alpha_0^c
\end{bmatrix}^{-1} =
\begin{bmatrix}
\alpha_0^{aa} & \alpha_0^{ab} & \alpha_0^{ac} \\
\alpha_0^{ba} & \alpha_0^{bb} & \alpha_0^{bc} \\
\alpha_0^{ca} & \alpha_0^{cb} & \alpha_0^{cc}
\end{bmatrix}^{-1} - [K_0] - \omega^2 [M_0] + i[H_0]
\]

and post-multiplying by \([\alpha]\) gives

\[
\begin{align*}
\begin{bmatrix}
\alpha_0^a \\
\alpha_0^b \\
\alpha_0^c
\end{bmatrix}^{-1} &=
\begin{bmatrix}
\alpha_0^{aa} & \alpha_0^{ab} & \alpha_0^{ac} \\
\alpha_0^{ba} & \alpha_0^{bb} & \alpha_0^{bc} \\
\alpha_0^{ca} & \alpha_0^{cb} & \alpha_0^{cc}
\end{bmatrix}^{-1} - [K_0] - \omega^2 [M_0] + i[H_0]
\end{align*}
\]

After some matrix manipulations the receptance submatrices of the modified system can be obtained as

\[
\begin{align*}
\alpha_{\text{ba}} &= [I_0 - \alpha_0^{bb} [D_{\text{mod}}^{-1} [\alpha_0^{ba}]]
\alpha_{\text{ca}} &= [I_0 - \alpha_0^{cc} [D_{\text{mod}}^{-1} [\alpha_0^{ca}]]
\alpha_{\text{bc}} &= [I_0 - \alpha_0^{bc} [D_{\text{mod}}^{-1} [\alpha_0^{bc}]]
\alpha_{\text{bb}} &= [I_0 - \alpha_0^{bb} [D_{\text{mod}}^{-1} [\alpha_0^{bb}]]
\alpha_{\text{cc}} &= [I_0 - \alpha_0^{cc} [D_{\text{mod}}^{-1} [\alpha_0^{cc}]]
\end{align*}
\]
\[
\begin{pmatrix}
\alpha^{aa}
\end{pmatrix} = 
\begin{pmatrix}
\alpha_0^{aa} - \alpha_0^{ab}
\end{pmatrix}
\begin{bmatrix}
0
\end{bmatrix}
\begin{bmatrix}
D_{mod}
\end{bmatrix}
\begin{pmatrix}
\alpha^{ba}
\alpha^{ca}
\end{pmatrix}
\]

(9)

\[
\begin{pmatrix}
\alpha^{ab} \\
\alpha^{ac}
\end{pmatrix} = 
\begin{pmatrix}
\alpha_0^{ab} \\
\alpha_0^{ac}
\end{pmatrix}
\begin{bmatrix}
0
\end{bmatrix}
\begin{bmatrix}
I
-[D]
\end{bmatrix}
\begin{pmatrix}
\alpha_{cb}
\alpha_{cc}
\end{pmatrix}
\]

(10)

It should be noted that the order of the matrix to be inverted is equal to the DOF of the modifying structure, which is usually much less than then the total DOF of the structure.

3. NUMERICAL EXAMPLES

3.1. Free-Free Beam

The aim of this example is to demonstrate the accuracy of the structural modification method when applied to distributed systems. In this example a free-free beam shown in Figure 3 is modified by attaching a smaller beam under the original one. Original beam was divided into 24 brick elements (20 nodes per element) with 3 DOF per node yielding total DOF of 723. The modifying structure was divided into 4 brick elements with 3 DOF per node yielding total DOF of 153. The geometrical and material properties of the original and modifying beams are given in Table 1. The predicted direct point FRF for the modified beam at node 64 in Z direction is shown in Figure 5. The FE model of the modified structure is also constructed, and the FRF of the same coordinate is obtained and compared with that obtained by the structural modification method (Figure 5).

![Figure 3. The FE model of the original beam](image)

![Figure 4. The FE model of the modified beam](image)

<table>
<thead>
<tr>
<th>Table 1. Geometrical and material properties of the original and modifying beams</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Original Beam</strong></td>
</tr>
<tr>
<td>Young’s Modulus</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
</tr>
<tr>
<td>Density</td>
</tr>
<tr>
<td>Length</td>
</tr>
<tr>
<td>Width</td>
</tr>
<tr>
<td>Thickness</td>
</tr>
</tbody>
</table>

As can be seen from Figure 5, the predicted FRF with structural modification technique matches exactly with the FRF calculated from finite element analysis (FEA) of the modified structure, which is an expected result as the method is an exact one.
3.2. Cantilever Plate
In this second example, the effect of modal truncation made in the computation of the FRFs of the original structure on the accuracy of the structural modification method proposed is demonstrated. In this example, the cantilever plate shown in Figure 6 is modified by attaching the plate shown in Figure 7 to the free edge of the original structure. Original plate was divided into 64 shell elements with 6 DOF per node yielding total DOF of 488. The modifying structure was divided into 16 shell elements with 6 DOF per node giving total DOF of 144. The geometrical and material properties of the original and modifying plates are given in Table 2. The direct point FRFs at node 60 in Z direction are predicted for the modified structure by employing the method proposed and by using different number of modes in calculating frequency response function of the original structure. The results are shown in Figure 9 to 11. The effect of truncation made in obtaining FRF of the original structure can be observed in higher frequencies. Using higher number of the modes in the computation of the FRF of the original structure increases the accuracy of the FRF predicted for modified structure.
Table 2. Geometrical and material properties of the original and modifying plates

<table>
<thead>
<tr>
<th></th>
<th>Original Beam</th>
<th>Modifying Beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's Modulus</td>
<td>71 GPa</td>
<td>71 GPa</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>Density</td>
<td>2770 kg/m³</td>
<td>2770 kg/m³</td>
</tr>
<tr>
<td>Length</td>
<td>300mm</td>
<td>75mm</td>
</tr>
<tr>
<td>Width</td>
<td>300mm</td>
<td>300mm</td>
</tr>
<tr>
<td>Thickness</td>
<td>2mm</td>
<td>2mm</td>
</tr>
</tbody>
</table>

Figure 8. The FE model of the modified plate

Figure 9. Comparison of FRFs at 60Z60Z (15 modes are used in the calculation of FRFs of the original structure)

Figure 10. Comparison of FRFs at 60Z60Z (30 modes are used in the calculation of FRFs of the original structure)
3.3. GARTEUR SM-AG19 Model

In this part, in order to show the performance of the structural modification technique, GARTEUR SM-AG19 model (Figure 12) is modified with beams under the wings which act as stiffeners causing flexural rigidity (Figure 13). The GARTEUR SM-AG19 model constructed differs slightly from the original GARTEUR SM-AG19 model. Viscoelastic tape which is placed on the upper surface of the wings in the original model is not used in the present study. In order to apply the structural modification technique, the finite element models of the original and modifying structures are constructed. The solid brick elements are used in the finite element model of GARTEUR SM-AG19 model which has 1380 nodes with 3 DOF per node yielding total DOF of 4140. Two beams each having dimensions of the 200mmx100mmx10mm are used as modifying structures. They are modeled with solid brick elements and each beam has 70 nodes with 3 DOF per node resulting total DOF of 210 for each modifying beam.

Figure 11. Comparison of FRFs at 60Z60Z (102 modes are used in the calculation of FRFs of the original structure)
Modal test is conducted on the original and modified GARTEUR SM-AG19 model by using an impact hammer. During the test, the same measurement and excitation equipment is used for both original and modified structures. Accelerometer positions are shown in Figure 14. In Figures 15 and 16 modifying beam and the general view of the experimental structure are given, respectively. Direct point FRFs measured at point 3 in Z direction (Figure 14) for both original and modified structures are given in Figure 17.
In Figure 18, direct point FRF of point 3 in Z direction obtained from the FE model is compared with that obtained experimentally in order to see the accuracy of the FE model of GARTEUR SM-AG19 model. As can be seen from Figure 18 they are in good agreement. The mismatch in the magnitudes of the FRF at the resonances may be attributed to the constant loss factor used for all modes in the finite element model of the structure to represent the damping in the system. The FRF of the modified structure is obtained by using the structural modification method proposed, and it is compared with the experimentally measured one (Figure 19). Again, a good agreement is observed between the FRF calculated by using the structural modification method and experimentally measured FRF. The discrepancy around the third mode may be due to the slight differences between the theoretical and experimental FRFs of the original structure. The truncation made in the calculation of the FRF of the original structure can be one of the reasons for such slight differences.

**Figure 18.** Theoretical (FE method) and experimental FRF at 3Z3Z for original GARTEUR SM-AG19

**Figure 19.** Theoretical (structural modification method) and experimental FRF at 3Z3Z for modified GARTEUR SM-AG19
The numerical examples given above show that the structural modification technique with additional DOF is an effective method for analyzing distributed modifications. Main advantage of the structural modification method employed is that there is no need to use Eq. (1) for the calculation of the additional dynamic stiffness matrix due to structural modification. Since additional DOF are introduced in modification, the structural modification problem turns into a structural coupling problem and therefore dynamic stiffness matrix of the modifying structure can directly be used in the structural modification method. Moreover, since modification introduces additional DOF to the original structure, a more accurate model is obtained for the modified structure.

4. CONCLUSIONS

In structural dynamic modification applications, difficulties may arise when modification is distributed. The main difficulty is due to the rotational degrees of freedom. Obtaining the additional dynamic stiffness matrix due to distributed structural modification is also another difficulty in structural modification methods. Although there are some approaches to overcome such difficulties by making simplifications and thus introducing inaccuracies; using structural modification methods with additional degrees of freedom, which is equivalent to treating the problem as a structural coupling problem, overcomes at least the latter difficulty mentioned above. Then, there is no need to partially model the modified and the original structure in order to calculate the additional dynamic stiffness matrix due to distributed structural modification, as proposed in some previous studies [12, 16]. In this paper, the method proposed in an earlier study by one of the authors for structural dynamic modifications with additional degrees of freedom [9] is applied to structures with distributed modifications, the main objective being to investigate the performance of the method when used for real structures with distributed dynamic modifications. In the method proposed, the frequency response functions of the modified structure are calculated from those of the original structure and the system matrices of the modifying structure.

Firstly, the validity of the approach proposed is demonstrated by applying it to a beam problem: Theoretically calculated FRFs are compared with those obtained from the FE analysis of the modified structure, and a perfect match was observed as expected, since the method yields exact FRFs when exact values of the FRFs for the original structure are used. It was shown in the second case study that the accuracy of the predictions strongly depends on the accuracy of the FRFs used for the original structure. In order to study the effect of modal truncation made in calculating the FRFs of the original structure on the accuracy of the method, a cantilever plate which was modified by attaching a smaller plate to the free edge was modeled by using FEM. It is observed that the performance of the structural modification technique increases when the number of modes included in the computation of FRFs of the original structure is increased.

The performance of the method when applied to a real structure is also investigated by applying it to GARTEUR SM-AG19 model with modifications in the form of beams attached under the wings acting as stiffeners causing flexural rigidity. The receptances calculated by using the structural modification method are compared with experimentally measured ones. A very good agreement is observed between the predicted and measured results. The discrepancies in the magnitudes of the FRFs at resonances are attributed to the constant loss factor used for all modes in the finite element model of the original structure to represent the damping of the system. It is concluded in this study that the structural reanalysis method proposed can be successfully and efficiently used for structures with distributed modifications, and thus the problems encountered in approach such as the one suggested by D’Ambrogio and Sestieri for distributed structural modification problems can be avoided.

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