Optimum Configuration for Vibration Absorbers of a SDOF System Using Genethic Algorithm

M. Najafi, M.R. Ashory, E. Jamshidi
Dept. of Mechanical Eng., Semnan University, P.O.Box: 35195-363, Semnan, Iran

Nomenclature

C: Absorber damping
K: Absorber stiffness
K: Stiffness of the primary system
m: Absorber mass
M: Primary mass
ω: Excitation frequency
ωp: Natural frequency of the primary system
ωa: Natural frequency of the absorber
μ: Mass ratio
ζ: Damping ratio
γ: Frequency ratio (ωa/ωp)
λ: Frequency ratio (ω/ωp)

ABSTRACT

Dynamic vibration absorbers are widely used for passive control of vibration. Classically, a simple dynamic vibration absorber consists of a mass and a spring attached to a vibrating body which vibrates in a narrow-band frequency range. When correctly tuned, the dynamic vibration absorber suppresses the motion of the point to which it is attached. In this paper different models for the vibration absorbers are suggested to find the optimum one for suppressing the vibration of a single degree of freedom system due to the ground motion. The genetic algorithm method is used to optimize the best model of the absorber. It is shown that when one damper and one spring are used between the primary mass and the absorber mass and one damper and one spring between the absorber mass and the ground the optimum results are obtained.

Keywords: Vibration Absorber, Tuned mass damper, Damping, Genetic Algorithm

1 INTRODUCTION

Dynamic vibration absorbers are used for controlling the vibration of structures. A vibration absorber is a passive element designed to suppress the motion of the point it is attached to a vibrating system with harmonic excitation [1, 2, 3]. Adaptive elements can also be used to tune the vibration absorber for different conditions [4, 5]. Passive non-traditional elements are also used for controlling the broad-band vibration of structures [6, 7, 8]. Recently, advances are reported for controlling the vibration of structures using active controlled elements [9, 10, 11] when the condition of the main system changes in time. However, a traditional vibration absorber may provide a cheaper and more convenient solution for controlling the vibrating structure in a narrow-band frequency range.

Classically, an undamped dynamic vibration absorber consists of a mass and spring attached to a primary vibrating system which vibrates harmonically or in a narrow-band frequency range. A damped vibration absorber consists of a damper and a spring which are added to the absorber mass on one side and attached to the primary mass on the other side. The formula for the optimum frequency and damping of the absorber is derived using the fixed-points theory of Den Hartog [1]. Liu [12] used the fixed-points theory and derived the optimum parameters of system leading to the minimum vibration amplitude at resonance. Wong [13] proposed an alternative design for
the damped vibration absorber. He concluded that there are four stationary points at which the response is independent of the damping of absorber. The model proposed by Wong shows the additional reduction of vibration amplitude of 7% compared to the classical model of Den Hartog.

The objective of the present work is to obtain the optimum configuration of a damped vibration absorber for suppressing the vibration of a single degree of freedom system due to the ground motion. Six different configurations are considered (see Figure 1) and the numerical method of Genetic Algorithm (GA) is used to optimize the parameters of the absorber.

Figure 1. Different models of vibration absorbers for suppressing vibration of a single degree of freedom system due to the ground motion.

2 THEORY

Figure 1 shows six different models for a damped vibration absorber attached to a single degree of freedom mass and spring system. Classically, a damped dynamic vibration absorber consists of a mass, spring and damper attached to the primary system which vibrates harmonically or in a narrow-band frequency range (see Figure 1a). This vibration model is called model A and was first proposed by Den Hartog [1]. A variant form of the damped vibration absorber was proposed and optimized by Wong [13] as shown in Figure 1b. This model is called model B. An alternative form of the vibration absorber is shown in Figure 1c for which two dampers are used, one between the absorber mass \( m \) and the ground and one between the absorber mass \( m \) and the primary mass \( M \) and one spring between the absorber mass \( m \) and the primary mass \( M \). This model is called model C. If two springs are used instead of two dampers, one between the absorber mass \( m \) and the ground and one between the absorber mass \( m \) and the primary mass \( M \) and one damper between the absorber mass \( m \) and the primary mass \( M \), then a new model is obtained. This model is called model D. Figure 1e shows another model of vibration absorber for which the damper is considered between absorber mass \( m \) and the ground compared to model D. This new model is called model E. Finally, if one spring and one damper are used between the absorber mass \( m \)
and the ground and one spring and one damper between the absorber mass $m$ and the primary mass $M$, model F is obtained (see Figure 1f).

The formula for the optimum frequency and damping of model A is derived using the fixed-points theory of Den Hartog. According to [1] the amplitude ratio $\left| \frac{X_1}{Y} \right|_{A}$ is:

$$\left| \frac{X_1}{Y} \right|_{A} = \left| \frac{(k - m \omega^2 + jc \omega)K}{[(K - M \omega^2)(k - m \omega^2) - m \omega^2k] + jc \omega(K - M \omega^2 - m \omega^2)} \right|$$

$$= \sqrt{\frac{(\gamma^2 - \lambda^2)^2 + (2 \zeta \gamma \lambda)^2}{[(1 - \lambda^2)(\gamma^2 - \lambda^2) - \mu \gamma^2 \lambda^2]^2 + (2 \zeta \gamma \lambda)^2(1 - \mu \lambda^2 - \lambda^2)^2}}$$

(1)

where:

$$\omega_p = \sqrt{K/M}, \quad \omega_a = \sqrt{k/m}, \quad \lambda = \omega_a/\omega_p, \quad \gamma = \omega_a/\omega_p, \quad \mu = m/M, \quad \zeta = c/2\sqrt{mk}$$

(2)

Liu [12] used the fixed-points theory and derived the optimum parameters of system leading to the minimum vibration amplitude at resonance:

$$\gamma_{opt\_A} = \frac{1}{1+\mu}, \quad \zeta_{opt\_A} = \sqrt{\frac{3\mu}{8(1+\mu)}}$$

(3)

Therefore the approximate value of the amplitude ratio at resonance is:

$$\left| \frac{X_1}{Y} \right|_{max\_A} = \sqrt{\frac{2+\mu}{\mu}}$$

(4)

The ratio of the vibration amplitude of the mass $M$ and that of the support for model B is [13]:

$$\frac{X_1}{Y} = \frac{K(k - m \omega^2 + jc \omega) + jk \omega}{(K - M \omega^2 + k)(k - m \omega^2 + jc \omega) - k^2}$$

(5)

Equation (5) may be rewritten with the non-dimensional variables as used in Equation (2):

$$\left| \frac{X_1}{Y} \right|_{B} = \sqrt{\frac{(\gamma^2 - \lambda^2)^2 + (1 + \mu \gamma^2)^2(2 \zeta \gamma \lambda)^2}{[(1 - \lambda^2)(\gamma^2 - \lambda^2) - \mu \gamma^2 \lambda^2]^2 + (2 \zeta \gamma \lambda)^2(1 + \mu \lambda^2 - \lambda^2)^2}}$$

(6)

Wong concluded [13] that there are four stationary points at which the response is independent of the damping of the absorber. Using the fixed-points theory, the optimum tuning frequency was derived as:

$$\gamma_{opt\_B} = \frac{1}{\sqrt{1-\mu}}, \quad \zeta_{opt\_B} = \sqrt{\frac{\mu(3-\mu)}{8}}$$

(7)
The approximate value of resonant amplitude ratio of model B under optimum tuning and damping condition is:

\[ \frac{X_1}{Y_{max,B}} = \sqrt{\frac{2-\mu}{\mu}} \]

(8)

The optimum amplitude ratios between the vibration of the primary system and the ground motion for models A and B with mass ratio \( \mu=0.2 \) show the additional reduction of vibration amplitude of 7\% in using model B compared to model A [13].

The governing equations of motion of model C shown in Figure 1c, are:

\[ M\ddot{x}_1 + K(x_1 - y) + k(x_1 - x_2) + c_1(\dot{x}_1 - \dot{x}_2) = 0 \]

(9)

\[ m\ddot{x}_2 + c_2(\dot{x}_2 - \dot{y}) - c_1(\dot{x}_1 - \dot{x}_2) - k(x_1 - x_2) = 0 \]

(10)

Assuming a harmonic motion of the ground \( y = Ye^{-j\omega t} \), the responses are:

\[ x_1 = X_1e^{-j\omega t} \text{ and } x_2 = X_2e^{-j\omega t} \]

(11)

Equations (9) and (10) become:

\[ -M\omega^2 X_1 + K(X_1 - Y) + k(X_1 - X_2) - j\omega c_1\omega X_1 - X_2 = 0 \]

(12)

\[ -m\omega^2 X_2 - jC_2\omega(X_2 - Y) + j\omega c_1\omega(X_1 - X_2) - k(X_1 - X_2) = 0 \]

(13)

Solving these equations, the ratio of amplitudes can be obtained as:

\[ \frac{X_1}{Y} = \frac{K(m\omega^2 + j\omega c_1\omega + j\omega c_2\omega - k) + c_1c_2\omega^2 + j\omega c_2 k\omega - k}{-mM \omega^4 - j(Mc_1 + mc_1 + Mc_2)\omega^3 + (MK + Mk + mk + c_1c_2)\omega^2 + j(c_2K + c_1K + c_2 k)\omega - Kk} \]

(14)

Equation (14) can be written in the non-dimensional form as:

\[ \frac{|X_1|}{Y} = \frac{(\gamma^2 - \lambda^2 - 4\mu\xi_2\gamma^2\lambda^2\lambda^2) + (2\xi_1\gamma\lambda + 2\xi_2\gamma\lambda + 2\mu\xi_2\gamma^3 \lambda^2)}{(\lambda^2 - \lambda^2 - \mu\gamma^2\lambda^2 - 4\mu\xi_1\xi_2\gamma^2\lambda^2 + \gamma^2) + (2\xi_1\lambda^3 + 2\xi_2\gamma^3 + 2\xi_2\gamma^3 - 2\xi_1\gamma^3 - 2\xi_2\gamma^3 - 2\xi_2\gamma^3 - 2\xi_2\gamma^3 \lambda^2)} \]

(15)

where:

\[ \omega_p = \sqrt{K/M}, \omega_d = \sqrt{k/m}, \lambda = \omega_p/\omega_d, \gamma = \omega_d/\omega_p, \mu = m/M, \xi_1 = 1/2\sqrt{mk}, \xi_2 = c_2/2\sqrt{mk} \]

(16)

In the same manner, the ratio of amplitudes for model D can be obtained as:

\[ \frac{X_1}{Y} = \frac{K(m\omega^2 + j\omega c_1\omega - k - k_2) + j\omega c_2\omega - k_1k_2}{-mM \omega^4 - j(M + m)c\omega^3 + (m(K + k_1) + M(k_1 + k_2))\omega^2 + j(K + k_2)c\omega - K(k_1 + k_2) - k_1k_2} \]

(17)

For model E the ratio of amplitudes is:
\[
X_1 = \frac{K(m\omega^2 + jc\omega - k_1 - k_2) + jck_1\omega - k_1k_2}{-mM\omega^4 - jMc\omega^3 + (m(K + k_1) + M(k_1 + k_2))\omega^2 + j(K + k_1)c\omega - K(k_1 + k_2) - k_1k_2}
\]  

(18)

And finally for model F the ratio of amplitudes is:

\[
X_1 = \frac{K(m\omega^2 + jc\omega + jc\omega - k_1 - k_2) + c_jc\omega^2 + j(c_jk_2 + c_jk_1)\omega - k_1k_2}{-mMe^4 - j(M + m)c_i + Mc_j\omega^3 + (m(K + k_1) + M(k_1 + k_2) + c_jc)\omega^2 + j(K + c_j + c_jk_2 + c_jk_1)\omega - K(k_1 + k_2) - k_1k_2}
\]  

(19)

Equations (17), (18) and (19) can not be changed to non-dimensional forms as equations (1), (5) and (14) are written. On the other hand, the parameters of equations (17), (18) and (19) are required to be scaled for using in the GA optimization process. The scaled parameters for equations (17) and (18) can be written as:

\[
\gamma_1 = \frac{\omega_{m_1}}{\omega_p}, \quad \gamma_2 = \frac{\omega_{m_2}}{\omega_p}, \quad \lambda = \frac{\omega}{\omega_p}, \quad \zeta = \frac{c}{2\sqrt{m(k_1 + k_2)}}
\]  

(20)

where:

\[
\omega_{m_1} = \sqrt{\frac{k_1}{m}}, \quad \omega_{m_2} = \sqrt{\frac{k_2}{m}}, \quad \omega_p = \sqrt{\frac{K}{M}}
\]

For equation (19):

\[
\gamma_1 = \frac{\omega_{m_1}}{\omega_p}, \quad \gamma_2 = \frac{\omega_{m_2}}{\omega_p}, \quad \lambda = \frac{\omega}{\omega_p}, \quad \zeta_1 = \frac{c_1}{2\sqrt{m(k_1 + k_2)}}, \quad \zeta_2 = \frac{c_2}{2\sqrt{m(k_1 + k_2)}}
\]  

(21)

where:

\[
\omega_{m_1} = \sqrt{\frac{k_1}{m}}, \quad \omega_{m_2} = \sqrt{\frac{k_2}{m}}, \quad \omega_p = \sqrt{\frac{K}{M}}
\]

3 GENETIC ALGORITHM OPTIMIZATION PROBLEM

In this section the implementation of GA to the optimization of a linear vibration absorber system is given. As the variables of optimization of the linear vibration absorber are continuous, floating-point numbers are more logical to be used. This is also known as real-valued GA [14]. The precision of binary GA is limited to the allocated number of bits. On the other hand, the precision of floating-point numbers is limited to the machine precision. Moreover, the real-valued Genetic Algorithm is faster than the binary GA because the numbers do not have to be decoded. The chromosome is an array of design variables or genes and can be written as:

\[
\text{chromosome} = [g_1, g_2, g_3, ..., g_n]
\]  

(22)

where \(g_i\) is \(i^{th}\) gene and \(n\) is the number of design variables. The objective function or cost function shows the fitness value of the chromosome.

\[
\text{cost} = f(\text{chromosome}) = f(g_1, g_2, g_3, ..., g_n)
\]  

(23)

Equations (22), (23) and the constraint equations constitute the optimization problem. A population is an \(N \times n\) matrix of random variables. Each row constitutes one chromosome. All variables have to be normalized to have values between 0 and 1 as a random number is generated in this range. All chromosomes are evaluated using
fitness function. The fittest chromosomes survive to reproduce children in the next generation. Crossover is the process of combining two chromosomes to form offsprings. Adewuya [15] demonstrated some methods of crossover. Uniform crossover [16] is used for the optimization of a linear vibration absorber. If

\[
\text{parent1} = [g_{m1}, g_{m2}, g_{m3}, g_{m4}, g_{m5}, g_{m6}, \ldots, g_{mn}]
\]

\[
\text{parent2} = [g_{d1}, g_{d2}, g_{d3}, g_{d4}, g_{d5}, g_{d6}, \ldots, g_{dn}]
\]

then:

\[
\text{offspring1} = [g_{m1}, g_{d2}, g_{d3}, g_{d4}, g_{d5}, g_{m6}, \ldots, g_{dn}]
\]

\[
\text{offspring2} = [g_{d1}, g_{m2}, g_{m3}, g_{m4}, g_{m5}, g_{d6}, \ldots, g_{mn}]
\]

for which each gene of offspring is chosen randomly from genes of parents. As only the combination of the genes has changed and not the genetic material, mutation is required. Mutation makes small random changes in the chromosomes of the population to provide a diversity to search a broader space. A normally distributed random number is used mostly to the variable selected for mutation

\[
p'_n = p_n + \sigma N\text{,n}(0,1)
\]

where \(\sigma\) is the standard deviation of the normal distribution and \(N\text{,n}(0,1)\) is the standard normal distribution (mean = 0 and variance = 1). This technique is not used in the present work because the variable may exceed its bounds. Instead, generating of random directions that are adaptive with respect to the last successful or unsuccessful generation is considered [16].

4 APPLICATION OF GENETIC ALGORITHM TO THE DAMPED VIBRATION ABSORBER PROBLEM

GA is applied to obtain the optimal design of a damped vibration absorber of a SDOF system due to the ground motion by minimizing the maximum of relative displacement \(X/Y\) versus \(\lambda\) curve for the models shown in Figure 1. For instance for model F, \(\zeta_1, \zeta_2, \gamma_1\) and \(\gamma_2\) are the design variables which construct the genes of chromosome in genetic algorithm procedure. For better and faster convergence of GA procedure and in order to prevent convergence of the optimization problem to trivial solutions, the design variables are limited to the following bounds:

\[0<\zeta_1<1, 0<\zeta_2<1, 0<\gamma_1<1.5\text{ and }0<\gamma_2<1.5\]

GA runs for 2000 generations. In each generation two individuals which are the best in terms of their fitness value are selected as Elite children and survive to the next generation. The other children are created by combining two individuals using uniform crossover. Mutation children are created by introducing random changes, or mutation, to a single parent, considering bounds on design variables. In this problem, the initial population contains 120 individuals. All the individuals in the population lie in the range which is compatible with design variable bounds.

Table 1 indicates the optimum values of the non-dimensional parameters of the six models shown in Figure 1, using the values: \(M=1.0\text{ kg}, K=100\text{ N/m}, m=0.2\text{ kg and }\mu=0.2\). The natural frequency of the primary system is \(\omega_n=10\text{ rad/s}\). Table 2 indicates the optimum values of the parameters of models shown in Figure 1. The amplitude ratios between the vibration of the primary system to the ground motion for models A, B, C, D, E and F under optimum tuning and damping with mass ratio \(\mu=0.2\) were calculated using equations (1), (6), (14), (17), (18), (19) and the results are plotted in Figure 2. Figure 3 shows the comparison of the maximum amplitude ratio of the primary mass with the ground for each model shown in Figure 1. It can be concluded that if model F is optimized, the maximum suppression of the vibration of the primary system can be obtained compared to the other models. However, there is not a significant difference between models F, C and almost model E in suppressing the vibration amplitude of the primary system. Figure 3 indicates the behavior of optimum models A, B, C, D, E and F in resonance area. It can be concluded that models F and C are almost the same in their optimum forms.
Table 1. Optimum values of the non-dimensional parameters of models shown in Figure 1, $K=100$ N/m, $M=1.0$ kg, $\mu=0.2$.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\gamma_A=0.833$</th>
<th>—</th>
<th>$\zeta_A=0.25$</th>
<th>—</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model B</td>
<td>$\gamma_B=1.118$</td>
<td>—</td>
<td>—</td>
<td>$\zeta_B=0.2646$</td>
</tr>
<tr>
<td>Model C</td>
<td>$\gamma_C=1.138$</td>
<td>—</td>
<td>$\zeta_{1C}=1.15\times10^{-4}$</td>
<td>$\zeta_{2C}=0.2969$</td>
</tr>
<tr>
<td>Model D</td>
<td>$\gamma_{1D}=0.8334$</td>
<td>$\gamma_{2D}=0.00048$</td>
<td>$\zeta_D=0.2507$</td>
<td>—</td>
</tr>
<tr>
<td>Model E</td>
<td>$\gamma_{1E}=1.143$</td>
<td>$\gamma_{2E}=0.00089$</td>
<td>—</td>
<td>$\zeta_E=0.304$</td>
</tr>
<tr>
<td>Model F</td>
<td>$\gamma_{1F}=1.1374$</td>
<td>$\gamma_{2F}=1.72\times10^{-5}$</td>
<td>$\zeta_{1F}=1.56\times10^{-5}$</td>
<td>$\zeta_{2F}=0.296$</td>
</tr>
</tbody>
</table>

Table 2. Optimum values of the parameters of models shown in Figure 1, $K=100$ N/m, $M=1.0$ kg, $\mu=0.2$.

<table>
<thead>
<tr>
<th>Model</th>
<th>$k_A=13.889$ N/m</th>
<th>—</th>
<th>$c_A=0.833$ N.s/m</th>
<th>—</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model B</td>
<td>$k_B=25$ N/m</td>
<td>—</td>
<td>—</td>
<td>$c_B=1.118$ N.s/m</td>
</tr>
<tr>
<td>Model C</td>
<td>$k_C=25.9$ N/m</td>
<td>—</td>
<td>$c_{1C}=5.23\times10^{-4}$ N.s/m</td>
<td>$c_{2C}=1.3517$ N.s/m</td>
</tr>
<tr>
<td>Model D</td>
<td>$k_{1D}=13.89$ N/m</td>
<td>$k_{2D}=4.705\times10^{-6}$ N/m</td>
<td>$c_D=0.83557$ N.s/m</td>
<td>—</td>
</tr>
<tr>
<td>Model E</td>
<td>$k_{1E}=26.115$ N/m</td>
<td>$k_{2E}=1.585\times10^{-5}$ N/m</td>
<td>—</td>
<td>$c_E=1.3906$ N.s/m</td>
</tr>
<tr>
<td>Model F</td>
<td>$k_{1F}=22.87$ N/m</td>
<td>$k_{2F}=5.925\times10^{-9}$ N/m</td>
<td>$c_{1F}=7.11\times10^{-5}$ N.s/m</td>
<td>$c_{2F}=1.347$ N.s/m</td>
</tr>
</tbody>
</table>

Figure 2. Maximum amplitude ratio of the primary mass to the ground motion for the models shown in Figure 1 at $K=100$ N/m, $M=1.0$ kg, $\mu=0.2$. 
Moreover, a reduction of vibration amplitude of 7% is obtained in using model B (Wong's model) compared to model A (Den Hartog's model) as reported in [13]. The additional reduction of vibration amplitude of 1.3% can be obtained in using models F or C. To investigate whether the vibration absorber in model F is in general superior to the other models, the maximum amplitude ratio of the primary mass to the ground for the optimum models shown in Figure 1 are computed using GA for different values of mass ratio $\mu = 0.05, 0.1, 0.2, 0.3, 0.4$ and $0.5$. This is a common practice to choose $0.05 < \mu < 0.25$ for vibration absorbers [17]. Figure 4 shows the comparison of the amplitude ratio for the models shown in Figure 1 versus mass ratio. The similar comparison between models B and A was reported by Wong [13] showing that the absorber in model B is in general superior to the absorber in model A for vibration suppression.

![Figure 3](image3.png)

Figure 3. Comparison of the vibration amplitude ratio for optimum models A, B, C, D, E and F.

![Figure 4](image4.png)

Figure 4. Vibration amplitude ratio versus mass ratio for models A, B, C, D, E and F, and $\mu=0.05, 0.1, \ldots, 0.5$
CONCLUSIONS

The purpose of the present research was to investigate the potential use of Genetic Algorithm (GA) method in the optimal design of a damped vibration absorber in order to suppress vibration of a single degree of freedom system excited by the ground motion. Six different models are proposed to obtain the best configuration of a damped vibration absorber. Under the optimum condition of absorbers, it is indicated that the largest suppression of resonant vibration amplitude of the primary system is obtained when the vibration absorber of model F is used. However, there is not a considerable difference between the performance of model F and model C. The comparison reveals that model F is superior to the other models for different mass ratios and can provide an alternative design for the traditional dynamic vibration absorbers. Moreover, GA method shows its superiority to the traditional analytical method such as fixed-points theory of Den Hartog, though it should be used with caution. GA method may also be used to design the vibration absorbers for Multi Degrees Of Freedom (MDOF) systems.

REFERENCES

